CHAPTER 5

CONGESTION MANAGEMENT ALLOCATION

5.1 Introduction

We use the multiple-transaction framework to characterize transmission congestion and then determine the contribution to congestion attributable to each transaction. The congestion allocation results are explicitly represented in the IGO's congestion relief problem: the IGO acquires the congestion relief services in the most economic manner to remove the congestion contributed by each transaction from its network.

Chapter 5 consists of three additional sections. Section 5.2 focuses on the development of an allocation mechanism to determine the allocation attributed to each transaction of its contribution to the network congestion. Section 5.3 formulates the IGO's least–cost congestion relief problem. We explore the characteristics and ramifications of the optimal solution of this problem. We also consider the policy implication of the mechanism developed. We devote Section 5.4 to the presentation of numerical results and some discussion of the implementation of the congestion management allocation scheme.

The time frame used here for the consideration of congestion is that typically in use in the realm of ancillary services. We focus on the congestion issue for the next day market. In such cases the analysis is done on an hourly basis. Note that congestion may occur due to the onset of a contingency in the real-time operation of the power system.

However, the real-time congestion relief is outside the scope of the dissertation. The long-term solution to congestion through the transmission system expansion is also not addressed here.

5.2 Overload Congestion Allocation

We use the multiple-transaction network framework in Chapter 2 to study a system of *N+1* buses with the swing bus at bus 0 and the set $\mathcal{M}$ of transactions. We denote by $\mathcal{L}$ the set of transmission lines. We refer to the flows that result from these proposed transactions as the preferred schedule flows. We denote by f_l the flow in line $l \in \mathcal{L}$. We adopt the convention that f_l refers to the real power flow from the *from* bus to the *to* bus and the direction of the net flow f_l is the positive direction. We simplify the representation of the various transmission constraints and model them in terms of line flow limits. Line *l* has the flow limit f_l^{max}. In this model, a congestion situation corresponds to the overloading of one or more transmission lines in the preferred schedule flows. Let $\tilde{\mathcal{L}} \subset \mathcal{L}$ be the subset of overloaded lines, i.e.,

$$\tilde{\mathcal{L}} \triangleq \left\{ l \in \mathcal{L} : f_l > f_l^{max} \right\} \tag{5.1}$$

Let Δf_l denote the overload in line $l \in \tilde{\mathcal{L}}$:

$$\Delta f_l \triangleq f_l - f_l^{max} \tag{5.2}$$

We consider the overloaded line $l \in \tilde{\mathcal{L}}$. We construct an overload model to represent Δf_l explicitly in terms of the proposed transactions. Let line *l* join buses *i* and *j*. We assume that the effects of the shunt elements on the line flow are negligible. Then, this line is represented by its line impedance $r_{ij} + jx_{ij}$. Given the set $\mathcal{M}$ of proposed transactions, the voltage magnitude V_n and angle q_n for buses $n = 0,1,2,...,N$ are determined by Equations (2.7) and (2.9). Then, the line flow f_l from bus *i* to bus *j* is given by

$$f_l = \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}[V_i^2 - V_i V_j \cos(q_i - q_j)] + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}[V_i V_j \sin(q_i - q_j)] \qquad (5.3)$$

We next assume the DC power flow conditions specified in Chapter 2 hold. Then, f_l may be approximately represented by $\tilde{f}_l$, where

$$\tilde{f}_l = \frac{1}{2}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j)^2 + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j) = \left[\frac{1}{2}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}\right](q_i - q_j)$$

(5.4)

Recall that $\hat{\underline{q}} = [\hat{q}_1, \hat{q}_2, ..., \hat{q}_N]^T$, as defined in Chapter 2, is the voltage angle vector computed by the DC power flow. We set the value of $\hat{q}_0$ to 0. It follows from Equation (2.12) in Chapter 2 that

$$\hat{q}_i - \hat{q}_j = \sum_{m \in M} p_{ij}^{(m)} t^{(m)}, \; i, \; j = 0,1,2,...,N, \; i \neq j \qquad (5.5)$$

In order to express the approximation of the line flow in terms of the proposed transactions, we further approximate $\tilde{f}_l$ in Equation (5.4) by $\hat{f}_l$, where $\hat{f}_l$ is given by

$$\begin{aligned} \hat{f}_l &= \left[\frac{1}{2}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}\right](\hat{q}_i - \hat{q}_j) \\ &= \left[\frac{1}{2}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}\right]\sum_{m \in M} p_{ij}^{(m)} t^{(m)} \qquad (5.6) \\ &= \sum_{m \in M}\left\{[\frac{1}{2}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2}]p_{ij}^{(m)}\right\} t^{(m)} \end{aligned}$$

If we define for each $m \in M$

$$j_l^{(m)} \triangleq \left[\frac{1}{2} \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} \right] p_{ij}^{(m)} \tag{5.7}$$

then $\hat{f}_l$ is explicitly written in terms of $t^{(m)}$ as

$$\hat{f}_l = \sum_{m \in M} j_l^{(m)} t^{(m)} \tag{5.8}$$

Note that if we assume that $r_{ij} << x_{ij}$ and that $q_i - q_j$ is small, then $r_{ij} / [2(r_{ij}^2 + x_{ij}^2)](q_i - q_j) << x_{ij} / (r_{ij}^2 + x_{ij}^2) \approx 1 / x_{ij}$. Consequently, $j_l^{(m)} \approx p_{ij}^{(m)} / x_{ij}$ so that the approximation in Equation (5.8) is relatively straightforward to compute. Then, the overload Δf_l may be approximated by

$$\Delta f_l \approx \tilde{f}_l - f_l^{\max} \approx \hat{f}_l - f_l^{\max} = \sum_{m \in M} j_l^{(m)} t^{(m)} - f_l^{\max} \tag{5.9}$$

We use the approximation in Equation (5.9) for the overflow model in terms of the proposed transactions.

From the expression of $\hat{f}_l$ in Equation (5.8), we may consider $j_l^{(m)} t^{(m)}$ as the flow contributed by transaction m in the preferred schedule, where $j_l^{(m)}$ may be either nonnegative or negative. Its sign indicates the direction of the contribution to the flow f_l associated with transaction m. For the overloaded line l, we partition M into two non-intersecting subsets D_l and C_l:

$$D_l \triangleq \left\{ m \in M : j_l^{(m)} \geq 0 \right\} \text{ and } C_l \triangleq \left\{ m \in M : j_l^{(m)} < 0 \right\} \tag{5.10}$$

Thus, D_l is the subset of the transactions whose associated flows contribute to the dominant flow in line l, while C_l is the subset of transactions, if any, whose associated flows contribute to the counter flow in line *l*. Furthermore, we define for each transaction $m \in M$ the subset $\tilde{L}^{(m)} \subset \tilde{L}$

$$\tilde{L}^{(m)} \triangleq \left\{ l : l \in \tilde{L} \text{ and } j_l^{(m)} \geq 0 \right\} \tag{5.11}$$

where $\tilde{L}^{(m)}$ is the subset of overloaded lines which the associated flows of a given transaction *m* contribute to the dominant flows of the lines. Note that for a given *m*, $\tilde{L}^{(m)}$ may be empty but $\bigcup_{m \in M} \tilde{L}^{(m)} = \tilde{L}$.

We define the quantity

$$\Psi_l \triangleq \sum_{m \in C_l} [-j_l^{(m)} t^{(m)}] \tag{5.12}$$

where $\Psi_l = 0$ if $C_l = f$ and positive otherwise. We rewrite the overload Δf_l in Equation (5.9) as

$$\Delta f_l \approx \sum_{m \in D_l} j_l^{(m)} t^{(m)} - \sum_{m \in C_l} (-j_l^{(m)}) t^{(m)} - f_l^{\max} = \sum_{m \in D_l} j_l^{(m)} t^{(m)} - [f_l^{\max} + \Psi_l] \tag{5.13}$$

We may view the effect of the counter flow in line *l* as an increase in the line flow limit from f_l^{max} to $f_l^{max} + \Psi_l$. Each transaction in D_l results in contributions to the dominant flow on the overloaded line *l* and therefore is considered as contributing to that overload. So the line *l* overload is attributed in its entirety to the transactions in D_l. We define

$$\hat{M} \triangleq \bigcup_{l \in \tilde{L}} D_l \tag{5.14}$$

to be the subset of M of transactions that contribute to the overloads in one or more lines. In addition, we assume that the line overload Δf_l is contributed *uniformly* by each transaction in D_l. Then, the contribution to the total overload Δf_l attributed to transaction $m \in D_l$ is

$$\Delta f_l^{(m)} = \frac{j_l^{(m)} t^{(m)}}{\sum_{m' \in D_l} j_l^{(m')} t^{(m')}} \left\{ \sum_{m' \in D_l} j_l^{(m')} t^{(m')} - [\Psi_l + f_l^{max}] \right\} = \frac{j_l^{(m)} t^{(m)}}{\sum_{m' \in D_l} j_l^{(m')} t^{(m')}} \Delta f_l, \; m \in D_l \quad (5.15)$$

The attribution of the overload $\Delta f_l^{(m)}$ to transaction m is based on arguments similar to those used in the determination of TLR for NERC's transmission loading relief [9].

5.3 Overload Congestion Relief

The overload allocation is explicitly represented in the determination of congestion relief actions to remove the congestion by the IGO. We assume that the only congestion relief means available to the IGO is to acquire incremental/decremental injections into the system from every willing participant, be it a generating or a load entity.

We use the notion of real power redispatch to relieve line overloads proposed in [51]. However, in the open access environment the generators and the loads in the network are independent of the IGO and the IGO no longer has the control over these entities that the central operator of the VIU had. Hence, the IGO needs to procure these services from the willing participants. To do so, the IGO runs an auction of adjustments to select the most economic means to accomplish its goal of overload relief. Note that the participants in the adjustment auction need not to be participants in any of the proposed transactions.

5.3.1 Problem formulation

Let us denote by K the set of buses where the participants in the auction are located. The participants in the auction may be generators or loads. We assume that they are rational bidders. The bidder at bus *k* submits an offer with the $ /MW price for the net injection adjustment Δp_k. Note that while a generator may provide Δp_k by increasing or decreasing its production output, a load may also effectively offer Δp_k by varying its demand by $-\Delta p_k$. The bidder at bus k may offer to provide an incremental injection $\Delta p_k > 0$ at a charge of c_k^+ $ / MW. It may also offer to provide a decremental injection by giving a rebate of c_k^- $ / MW to the IGO. In its offer, the bidder also specifies the lower and upper limits of Δp_k within which it is physically capable and/or willing to provide its injection adjustment, i.e., $\Delta p_k \in [\Delta p_k^{min}, \Delta p_k^{max}]$. Note that $c_k^+ > 0$ and $c_k^- >$ 0. A rational bidder submits an offer with $c_k^- \leq c_k^+$. The IGO pays $c_k^+ \Delta p_k$ for $\Delta p_k \geq 0$ to the bidder at bus *k* or receives $-c_k^- \Delta p_k$ for $\Delta p_k < 0$ from the bidder at bus k. The payment by the IGO for the bidder at bus k is illustrated in Figure 5.1.

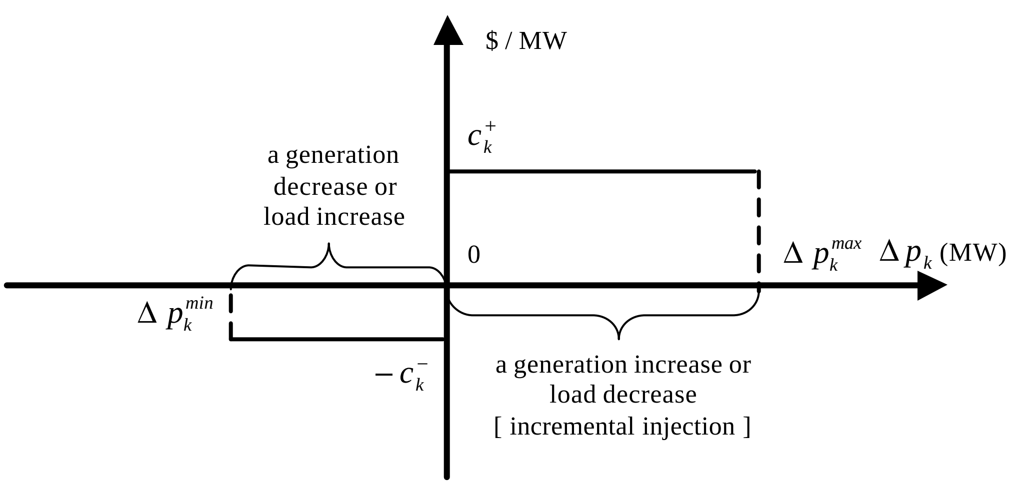

Figure 5.1 The offer of the bidder at bus *k*

The IGO uses the offers submitted by the generators and the loads to determine the most economic congestion relief by minimizing the total costs incurred for removing congestion. The decision variables for the IGO are $\Delta p_k^{(m)}, k \in K, m \in \hat{M}$, where $\Delta p_k^{(m)}$ is the amount of the net incremental/decremental injection acquired from the bidder at bus k for transaction m to relieve the overload burden $\Delta f_l^{(m)}, l \in \tilde{L}^{(m)}$ attributed to the transaction.

The IGO's objective function is

$$\min \; Z = \sum_{k \in K} c_k \Big(\sum_{m \in \hat{M}} \Delta p_k^{(m)} \Big) \tag{5.16}$$

where

$$c_k = \begin{cases} c_k^+, & \text{if } \Delta p_k = \sum\limits_{m \in \hat{M}} \Delta p_k^{(m)} \geq 0 \\ c_k^-, & \text{if } \Delta p_k = \sum\limits_{m \in \hat{M}} \Delta p_k^{(m)} < 0 \end{cases} \tag{5.17}$$

To analyze the effects of the relief actions $\Delta p_k^{(m)}, k \in K, m \in \hat{M}$ on the transmission network, we use a small signal model for the network since $\Delta p_k^{(m)}$ are typically small changes in the net injections at the buses $k \in K$ compared to the injection levels for the proposed transactions. In order to study the changes in the line flows with these $\Delta p_k^{(m)}$ changes, the sensitivities $\partial f_l / \partial p_k$ of the line flow f_l with respect to the net injection p_k at bus k need to be determined. We have derived the approximation $\tilde{f}_l$ of f_l in Equation (5.4). Then, $\partial f_l / \partial p_k$ may be approximated by $\partial \tilde{f}_l / \partial p_k$, where

$$\frac{\partial f_l}{\partial p_k} \approx \frac{\partial \tilde{f}_l}{\partial p_k} = \left[\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} \right] \left(\frac{\partial q_i}{\partial p_k} - \frac{\partial q_j}{\partial p_k} \right) \tag{5.18}$$

We may further approximate $\frac{\partial \tilde{f}_l}{\partial p_k}$ by

$$\frac{\partial \tilde{f}_l}{\partial p_k} \approx \left[\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} \right] \left(\frac{\partial \hat{q}_i}{\partial p_k} - \frac{\partial \hat{q}_j}{\partial p_k} \right) \tag{5.19}$$

Since, from Equation (2.10) in Chapter 2, we have

$$\hat{q}_i - \hat{q}_j = \sum_{k=1}^{N} (d_{ki} - d_{kj}) p_k \tag{5.20}$$

then

$$\frac{\partial \hat{q}_i}{\partial p_k} - \frac{\partial \hat{q}_j}{\partial p_k} = d_{ki} - d_{kj} \tag{5.21}$$

We define

$$y_{l,k} \triangleq \left[\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (q_i - q_j) + \frac{x_{ij}}{r_{ij}^2 + x_{ij}^2} \right] (d_{ki} - d_{kj}) \tag{5.22}$$

Thus,

$$\frac{\partial f_l}{\partial p_k} \approx y_{l,k} \tag{5.23}$$

Thus, $y_{l,k}$ approximates the rate of the change in the line flow f_l with respect to the change in the net injection at bus k.

In addition, we assume that the change in the total system losses due to $\Delta p_k^{(m)}$ is negligibly small. The constraints the IGO congestion relief actions need to satisfy are as follows:

(i) **Power balance**: The total power balance constraint requires that the sum of the adjustments in generation and load be zero:

$$\sum_{k \in K} \left[\sum_{m \in \hat{M}} \Delta p_k^{(m)} \right] = 0 \tag{5.24}$$

(ii) **Removal of preferred schedule flow overloads**: The congestion relief actions $\Delta p_k^{(m)}$, $k \in K$, $m \in \hat{M}$, which are acquired by the IGO from the adjustment auction participant at bus k, need to be set to remove the overload $\Delta f_l^{(m)}$, $l \in \tilde{L}^{(m)}$

$$-\sum_{k \in K} y_{l,k} \Delta p_k^{(m)} = \Delta f_l^{(m)} = \frac{j_l^{(m)} t^{(m)}}{\sum_{m' \in D_l} j_l^{(m')} t^{(m')}} (f_l - f_l^{max}),\ l \in \tilde{L}^{(m)},\ m \in \hat{M} \tag{5.25}$$

(iii) **Assurance of no overload in the IGO-determined schedule**: The existence of $\Delta p_k^{(m')} \neq 0$, $m' \notin D_l$ may impact the flows in one or more of the lines and so there is a need to impose the requirement that the flows in the IGO-determined schedule satisfy the line flow constraints for each line:

$$f_l + \sum_{k \in K} y_{l,k} \left[\sum_{m \in \hat{M}} \Delta p_k^{(m)} \right] \leq f_l^{max}, l \in L \tag{5.26}$$

(iv) **Increment/decrement limits**: The upper and lower limits of each relief action $\Delta p_k^{(m)}$ for removing the overload of transaction m in the preferred schedule flows need to satisfy

$$\Delta p_k^{min} \leq \Delta p_k^{(m)} \leq \Delta p_k^{max},\ k \in K,\ m \in \hat{M} \tag{5.27}$$

(v) **Total increment/decrement limits**: the limits of the inc/dec injection specified by the generators and the loads cannot be violated:

$$\Delta p_k^{min} \leq \sum_{m \in \hat{M}} \Delta p_k^{(m)} \leq \Delta p_k^{max},\ k \in K \tag{5.28}$$

(vi) **Separation of markets**: The generation and the load within each transaction must be kept balanced in the congestion relief. Thus each transaction is kept separate and the IGO is prevented from implicitly arranging new transactions among market participants:

$$\sum_{k\in S^{(m)}\cup B^{(m)}}\left[\sum_{m'\in\hat{M}}\Delta p_k^{(m')}\right]=0\,,\ m\in M \tag{5.29}$$

Thus, Equations (5.16) – (5.29) give the LP OPF formulation of the IGO's least-price congestion relief problem. We can regard this formulation to be, in effect, an optimal power flow problem. Due to the modeling assumptions, this is in standard linear program format with equality and inequality constraints. The equality constraints of Equation (5.25) imply that in order to eliminate the overload $\Delta f_l^{(m)}$ in line l attributed to transaction $m\in D_l$, the IGO acquires the relief actions from the participants in the adjustment bidding to reduce the flow in line l by exactly $\Delta f_l^{(m)}$. Each transaction $m\in\hat{M}$ imposes $\left|\tilde{L}^{(m)}\right|$ such constraints so that there is a total of $\sum_{m\in\hat{M}}\left|\tilde{L}^{(m)}\right|$ equality constraints, where $\left|\tilde{L}^{(m)}\right|$ is the cardinality of the set $\tilde{L}^{(m)}$. The LP formulation with the associated dual variables is given in Equation (5.30).

This congestion management problem formulation is very general; in fact, the formulations that previously appeared in the literature [44] [45] become special cases. For example, if we remove the equality constraint in (5.25) from the formulation in Equation (5.30), then each $\sum_{m\in\hat{M}}\Delta p_k^{(m)}$ in (5.30) may simply be replaced by Δp_k, and consequently (5.30) becomes the formulation of the scheme in [44]. If we also remove the separation of markets constraint in (5.29), then the formulation becomes that in [45]:

$$
\min \quad Z = \sum_{k \in K} c_k \left(\sum_{m \in \hat{M}} \Delta p_k^{(m)} \right)
$$

$$
\begin{cases}
\sum_{k \in K} \sum_{m \in \hat{M}} \Delta p_k^{(m)} = 0 & \leftrightarrow h \\
-\sum_{k \in K} y_{l,k} \Delta p_k^{(m)} = \dfrac{j_l^{(m)} t^{(m)}}{\sum_{h \in D_l} j_l^{(h)} t^{(h)}} (f_l - f_l^{\max}), \; l \in L^{(m)}, \; m \in \hat{M} & \leftrightarrow r_l^{(m)} \\
-\sum_{k \in K} y_{l,k} \sum_{m \in \hat{M}} \Delta p_k^{(m)} \geq f_l - f_l^{\max}, \; l \in L & \leftrightarrow m_l \\
\Delta p_k^{min} \leq \sum_{m \in \hat{M}} \Delta p_k^{(m)} \leq \Delta p_k^{max}, \; k \in K & \leftrightarrow t_k^{\max}, \; t_k^{\min} \\
\Delta p_k^{min} \leq \Delta p_k^{(m)} \leq \Delta p_k^{max}, \; k \in K, \; m \in \hat{M} & \leftrightarrow u_k^{(m),\max}, \; u_k^{(m),\min} \\
\sum_{k \in S^{(m)} \cup B^{(m)}} \left[\sum_{m' \in \hat{M}} \Delta p_k^{(m')} \right] = 0, \; m \in M & \leftrightarrow w^{(m)}
\end{cases}
\tag{5.30}
$$

5.3.2 Optimality analysis

We denote the optimum by $\Delta p_k^{*(m)}, k \in K, m \in \hat{M}$, and at the optimum we have

$$
Z^* = \sum_{k \in K} c_k \left[\sum_{m \in \hat{M}} \Delta p_k^{*(m)} \right] \tag{5.31}
$$

We also denote the values of the dual variables at the optimum by h^*, $r_l^{*(m)}$, m_l^*, $t_k^{*\max}$, $t_k^{*\min}$, $u_k^{*max,(m)}$, $u_k^{*min,(m)}$, and $w^{*(m)}$.

The dual variables $r_l^{*(m)}$ and m_l^* have important economic interpretations. The variable $r_l^{*(m)}$ is the rate of change of the optimum Z^* with respect to a change in the allocation burden $\Delta f_l^{(m)}$ in the preferred schedule overloaded line *l* attributed to transaction *m*, with all other constraints remaining unchanged. Thus, the variable $r_l^{*(m)}$ is the IGO's marginal cost to relieve the congestion contributed by transaction *m* in the overloaded line *l* in the preferred schedule. The term $r_l^{*(m)}$ for an overloaded line *l* in the preferred schedule may be nonnegative or negative since it is associated with an equality constraint. Similarly, for each line $l \in L$, m_l^* is the rate of change of the optimum Z^* with respect to a change in $f_l - f_l^{max}$ with all other constraints remaining unchanged. Because the dual variable is associated with an inequality constraint, $m_l^* \geq 0$.

We consider next the sensitivity of the IGO's least congestion costs Z^* with respect to the line flow capacity $f_l^{\max}$ of a line l or equivalently with the flow f_l of the preferred schedule

$$c_l^* = -\frac{\partial Z^*(f_l^{max})}{\partial f_l^{max}} \tag{5.32}$$

where the evaluation of the derivative is at the optimal solution. Then,

$$c_l^* = \frac{\partial Z^*(f_l)}{\partial f_l} \tag{5.33}$$

We may evaluate c_l^* from the LP formulation in Equation (5.30):

$$c_l^* = \begin{cases} \displaystyle\sum_{m \in D_l} \left[\frac{j_l^{(m)} t^{(m)}}{\sum_{m' \in D_l} j_l^{(m')} t^{(m')}} r_l^{*(m)} \right] + m_l^* & \text{if } l \in \tilde{L} \\ m_l^* & \text{otherwise} \end{cases} \tag{5.34}$$

For a line $l \in \tilde{L}$, one unit decrease in f_l^{max} results in an increase of $j_l^{(m)} t^{(m)} / \sum_{m' \in D_l} j_l^{(m')} t^{(m')}$ in the congestion burden associated with each transaction $m \in D_l$. Thus, the first component $\sum_{m \in D_l} j_l^{(m)} t^{(m)} r_l^{*(m)} / \sum_{m' \in D_l} j_l^{(m')} t^{(m')}$ for an overloaded line l in the preferred schedule is the sum of the additional congestion costs incurred by the IGO in relieving these increased congestion burdens. The additional congestion costs m_l^* are incurred to ensure that there is no overload in the IGO-determined schedule. For a line that was not overloaded in the preferred schedule, only the m_l^* term appears. The expression of c_l^* in (5.34) indicates that $c_l^* = 0$ for a line that is not overloaded in the preferred schedules, and its flow is lower than its flow limit in the IGO-determined schedule.

We may use c_l^* to set up the transmission usage prices charged for flows on each line *l* in the IGO-determined schedules. Since c_l^* is independent of *m*, then each transaction $m \in M$ is charged the same rate for its usage of the line *l*. The charges given by c_l^* are nondiscriminatory. Note, however, that since $r_l^{*(m)}$ may be negative, c_l^* may be negative for some $l \in \tilde{L}$. The physical interpretation of a negative c_l^* is that a decrease in the flow limit f_l^{max} or equivalently an increase in the line flow f_l in the preferred schedule is the optimal decision to minimize the congestion costs Z^* incurred by the IGO. Then, in order to attain the least overall costs in congestion management, the IGO rewards those transactions that have flows associated with the use of line *l*.

We can specify the usage charges for each transaction *m*. In the IGO-determined schedule, the flow $f_l^{*(m)}$ associated with transaction *m* in line *l* is

$$f_l^{*(m)} \approx j_l^{(m)} t^{(m)} + \sum_{k \in K} y_{l,k} \Delta p_k^{*(m)} \tag{5.35}$$

where $j_l^{(m)} t^{(m)}$ is the flow in line *l* contributed by transaction *m* in the preferred schedule, and $\sum_{k \in K} y_{l,k} \Delta p_k^{*(m)}$ is the change in the line flow that is due to the congestion relief actions associated with transaction *m*. The usage charge to transaction *m* on line *l* is

$$C_l^{(m)} = c_l^* f_l^{*(m)} \tag{5.36}$$

5.3.3 Policy implications of the formulation

The formulation of the congestion management problem in Equation (5.30) has a number of important implications for the design of the rules for congestion management. The separation of markets constraint serves to keep each transaction separate and prevent the IGO from implicitly forcing the entities in the preferred transactions into new trades. Thus, the IGO's intervention in the market is limited to the minimum level. This constraint is implemented in the inter-zonal congestion management in the California market.

The rule that the adjustment auction held by the IGO is voluntary allows each individual transaction a great deal of flexibility to decide on its own transmission usage. For example, if a transaction wants to keep intact in the IGO-determined schedule, its participating entities such as the selling generator(s) and the buying load(s) can simply choose not to participate in the adjustment auction. Alternatively, if they do participate in the auction, they may set their offer prices at such a high level as to preclude their offers from being selected. We do not consider the trivial choice for any participant to set $\Delta p_k^{min} = \Delta p_k^{max} = 0$. As a consequence, however, they have to pay whatever usage charges may come out from the auction.

This rule also allows the generators or loads that are not associated with any preferred transaction to participate in the auction. We may treat the incremental or decremental injections chosen from these entities as the IGO-arranged transactions. The IGO plays the role of a market broker in these "fictitious" transactions. Note that this transaction interpretation does not contradict the separation of markets constraint that no new across-transaction trade is allowed, since the entities in these "IGO-arranged" transactions do not participate in the preferred transactions.

However, a transaction's decision according to its own interests may result in insufficient participation in the congestion management market and may negatively impact the overall efficiency of congestion management. On the one hand, the IGO needs to design appropriate signals to compensate generators or loads for the costs due to the redispatch so that they are willing to participate in congestion management; on the other hand, the IGO might need to have the authority to order certain players to participate so as not to disadvantage the entire system. It is particularly true for the situation in which the IGO cannot remove overloads without the participation of a specific generator or load. In this case, the IGO may have to declare this entity a *reliability must run* unit and have to pay its congestion relief service at the price negotiated in the contract with the unit that accepts to serve on a must run basis.

5.4 Numerical Results

The proposed congestion management allocation scheme was implemented and tested on several systems. We discuss the results obtained on Test Systems F and G. We construct three and four transactions in Test Systems F and G, respectively. These two test systems are specified in detail in Appendix E.

We use Test System F to illustrate the capability of the proposed overload allocation scheme to attribute the overloads to the various transactions. The lines whose flows in the preferred schedules are over their limits are lines 3 and 6 that join buses 2 with 4 and 2 with 6, respectively. Then, $\tilde{L} = \{ 3, 6 \}$. The values of the factors $j_l^{(m)}$

computed by Equation (5.6) are given in Table 5.1. Note that the magnitudes and the signs of $j_l^{(m)}$ vary. Take for instance transaction 2, which has $j_l^{(m)}$ with the smallest magnitudes. We interpret that the impacts of transaction 2 on the line flows are the least marked of the three transactions. As another example, consider the $j_l^{(m)}$ associated with transaction 1. The signs indicate that while we attribute flows to the dominant flow in line 3, there is associated flow that contributes to the counter flow in line 6. These effects are due largely to the particular locations of the sellers and the buyers of transactions. The sets D_l associated with lines 3 and 6 are $\{1, 3\}$ and $\{2, 3\}$, respectively, and C_l is simply the complement of D_l in M in each case. The nonempty sets $\tilde{L}^{(m)}$ for each transaction $m = 1, 3$ are $\{3\}$, $\{6\}$, and $\{3, 6\}$, respectively. Note that $\hat{M} = \bigcup_{l \in \tilde{L}} D_l = M = \{1, 2, 3\}$.

Table 5.1 The $j_l^{(m)}$ values for the overloaded lines for Test System F

transaction *m* / overloaded line	1	2	3
3	0.014	-0.006	0.119
6	-0.015	0.007	0.164

We use the allocation scheme in Equation (5.13) to determine the contribution to the congestion attributed to each transaction. The overload allocation results are summarized in Table 5.2. A dash in Table 5.2 indicates that transaction *m* is not associated with a contribution to the overload in the line.

Table 5.2. The overload allocation results in MW for Test System F

overloaded line		$\Delta f_l^{(m)}$ for m =		
number l	overload Δf_l	1	2	3
3	2.70	0.19	-	2.51
6	3.25	-	0.07	3.18

Next, we examine the case in which the generators at buses 2, 13, 22 and 23 decide to participate into the IGO's adjustment auction. Hence, $K = \{ 2, 13, 22, 23 \}$. The offer data of these four generators are tabulated in Table 5.3. The values of the $y_{l,k}$ associated with the overloaded lines and the adjustment auction buses are shown in Table 5.4.

Table 5.3 The offer data in the IGO's adjustment auction for Test System F

offerer at bus k / offer data	participants in transaction 2		Participants in transaction 3	
	22	23	2	13
c_k^+ ($/MWh)	15	10	10	20
c_k^- ($/MWh)	10	7.5	10	15
Δp_k^{max} (MW)	15	30	25	20
Δp_k^{min} (MW)	15	10	20	20

Table 5.4 The $y_{l,k}$ values for Test System F

adjustment bus k / overloaded line l	22	23	2	13
3	-0.21	-0.22	0.07	-0.24
6	0.05	-0.23	-0.26	-0.25

The LP optimal solution $\Delta p_k^{*(m)}, k \in K, m \in \hat{M}$ in megawatts is shown in Table 5.5. These results indicate that transactions 2 and 3 are modified in the IGO-determined schedule. For transaction 2, in which the generators at buses 22 and 23 participate, the output of the generator at bus 22 is decreased by 3.04 MW. Due to the separation of markets constraint for transaction 2 represented by Equation (5.24), the 3.04 MW decrease must be replaced by the only entity in transaction 2 participating in the adjustment auction, the generator at bus 23. As such, in transaction 2, there is a 3.04 MW shift of generation from the generator at bus 22 to that at bus 23. Similarly, in transaction 3, 11.5 MW of generation is shifted from the generator at bus 2 to that at bus 13. Since none of the generators and loads that participate in transaction 1 takes part in the adjustment auction, transaction 1 remains unchanged in the IGO-determined schedule. We may infer that transaction 1 is willing to pay whatever usage charges may result from the auction to keep intact its transaction in the IGO-determined schedule. It does so by simply avoiding to participate in the adjustment auction.

5.5 The optimal solution $\Delta p_k^{*(m)},\ k \in K,\ m \in \hat{M}$ in MW for Test System F

offerer bus k / transaction m	participants in transaction 2		participants in transaction 3	
	22	23	2	13
1	0	-1.63	-7.87	0
2	-9.69	-2.70	-11.5	11.5
3	6.65	7.37	7.87	0
net injection adjustments $\Delta p_k^* = \sum_{m \in \hat{M}} \Delta p_k^{*(m)}$	-3.04	3.04	-11.5	11.5

We use Test System F to illustrate the impacts of the separation of markets constraint in Equation (5.24). We resolve the LP optimization with the separation of markets constraints ignored. We assume that the offerers and their offers do not change.

The net injection adjustments $\Delta p_k^*,\ k \in K$ in MW at the optimum are shown in Table 5.6.

Table 5.6 Values of $\Delta p_k^*,\ k \in K$ in MW of the LP without the separation of markets constraints for Test System F

offerer at bus k	Participants in transaction 2		Participants in transaction 3	
	22	23	2	13
net injection adjustment $\Delta p_k^* = \sum_{m \in \hat{M}} \Delta p_k^{*(m)}$	2.40	23	-9.87	-15.53

The results in Table 5.6 clearly indicate that the IGO implicitly forces the market participants to undertake new transactions which were not contemplated by the market players as evidenced by the submitted preferred schedule. For example, let us consider transaction 3 in which the generators at buses 2 and 13 participate. In the IGO-determined schedule, the outputs of the generators at buses 2 and 13 are reduced by 9.87 MW and 15.53 MW, respectively. The total reduction of 25.4 MW is replaced by the IGO by increasing the output of the generators at bus 22 and bus 23. These two entities, however, do not participate in transaction 3. So the IGO effectively forces the loads in transaction 3 to replace the 25.4 MW reduction through the purchase of an equal quantity from the generators at buses 22 and 23. In effect, the IGO establishes a new transaction that was not entered into by the participants. As such, the IGO, whose responsibility is to remove the congestion in order to maintain system reliability, is seen as intervening in the market. The separation of markets constraint was specifically introduced to prevent the occurrences of such interference.

Next, we turn our attention to Test System G. There are three overloaded lines in the preferred schedule. They are lines 2, 3, and 18 that join buses 2 with 3, 3 with 4, and

3 with 15, respectively. The overload allocation results determined by Equation (5.13) are given in Table 5.7.

Table 5.7 The overload allocation results in MW for Test System G

Overloaded line		$\Delta f_l^{(m)}$ for m =			
Number l	overload Δf_l	1	2	3	4
2	4.9	3.3	1.6	-	-
3	7.5	4.5	1.3	1.7	-
18	3.5	-	1.0	0.6	1.9

We consider that the generators at buses 2, 3, 8, 9 and 12 and the load at bus 6 wish to participate in the IGO's adjustment auction. Hence, $K = \{ 2, 3, 6, 8, 9, 12 \}$. The offer data of the six offerers are given in Table 5.8. Note that the generators at buses 3 and 12 are participants in transaction 3, and the generators at buses 8 and 9 are participants in transaction 4; however, the generator at bus 2 and the load at bus 6 are not involved in any of the four transactions. The values of the $y_{l,k}$ associated with the overloaded lines and the adjustment auction buses are shown in Table 5.9.

Table 5.8 The offer data in the IGO's adjustment auction for Test System G

offerer at bus k / offer data	participants in transaction 3		participants in transaction 4		other participants	
	3	12	8	9	2	6
c_k^+ (\$/MWh)	25	25	30	15	20	22.5
c_k^- (\$/MWh)	10	20	15	7.5	8	10
Δp_k^{max} (MW)	20	15	20	20	10	15
Δp_k^{min} (MW)	-15	-17.5	-10	-25	-15	-5

Table 5.9 The $y_{l,k}$ values for Test System G

adjustment bus k / overloaded line l	3	12	8	9	2	6
2	0.39	-0.15	0.23	0.21	0.13	0.28
3	0.18	-0.04	-0.22	-0.13	0.04	-0.41
18	0.36	-0.11	0.01	0.08	0.09	0.13

The LP optimal solution $\Delta p_k^{*(m)}, k \in K, m \in \hat{M}$ in MW is tabulated in Table 5.10. The optimal results indicate that transactions 3 and 4 are modified in the IGO-determined schedule. These modifications take into consideration the separation of markets constraint for the transactions. In transaction 3, 8.0 MW of generation are switched from the generator at bus 3 to the generator at bus 12, and in transaction 4, 4.0 MW of generation are shifted from the generator at bus 8 to that at bus 9. The generator at bus 2 and the load at bus 6, which are not participants in transactions 3 and 4, cannot participate in the modification of these two transactions.

Table 5.10 The optimal solution $\Delta p_k^{*(m)}$, $k \in K$, $m \in \hat{M}$ in MW for Test System G

offerer bus k / transaction m	participants in transaction 3		participants in transaction 4		other participants	
	3	12	8	9	2	6
1	-1.4	3.0	0	4.0	0	8.9
2	-0.8	0	0	0	-10.6	1.8
3	-1.3	5.0	0	0	0	3.1
4	-4.5	0	-4.0	0	-3.1	-0.1
net injection adjustments $\Delta p_k^* = \sum_{m \in \hat{M}} \Delta p_k^{*(m)}$	-8.0	8.0	-4.0	4.0	-13.7	13.7

Since none of the generators and loads that participate in transactions 1 and 2 participates in the adjustment auction, then transactions 1 and 2 remain unchanged in the IGO-determined schedule. The optimal solution in Table 5.10 also shows that the IGO cuts 13.7 MW from the generator at bus 2, and this reduction is balanced by reducing the load at bus 6 by exactly 13.7 MW. We may interpret these congestion relief actions of the IGO as, in effect, purchasing 13.7 MW injection from the load at bus 6 and selling the same quantity to the generator at bus 3. In this way the IGO acts as a middleman since he does not have any physical resources, neither generators nor loads. Since the generator at bus 2 and the load at bus 6 do not participate in any of the submitted preferred transactions, these "IGO-arranged" transactions are not in violation of the separation of markets constraints.

The optimal dual variables $r_l^{*(m)}$, m_l^*, and c_l^* are tabulated in Table 5.11. A dash in Table 5.1 indicates that $r_l^{*(m)}$ is not applicable to the transaction and the line. Line 1 is the line that joins buses 1 and 2. The results of c_l^* for all the other lines are zero. The flow limits of lines 1, 2, 3 and 8 are active at the optimum. Note that line 1 is not one of the overloaded lines in the preferred schedules. The usage prices for charging the flow contribution $f_l^{*(m)}$ associated with each transaction m in the lines at their limits in the IGO-determined schedule are given by c_l^* in cost per megawatt-hour. These are shown together with the total usage charges $C_l^{(m)}$ to each transaction m in Table 5.12. The negative usage change for a line means that the transaction is being paid by the IGO to use the line. The rationale for this payment is to induce, in effect, a counter flow in the line so as to be able to increase the dominant flow through the line.

Table 5.11 Values of $r_l^{*(m)}$, m_l^*, and c_l^* in \$/MWh for Test System G

Line l	$r_l^{*(m)}$ for transaction				m_l^*	c_l^*
	1	2	3	4		
2	0	0	-	-	98.4	98.4
3	37.8	37.8	37.8	-	6.9	44.7
18	-	0	-17.3	21.5	122.6	129.9
1	-	-	-	-	81.1	81.1

Table 5.12 The usage charges $C_l^{(m)}$ and total usage charges $C^{(m)}$ for Test System G

line l	c_l^* (\$/MWh)	$f_l^{*(m)}$ (MW) / $C_l^{(m)}$ (\$/h) for transaction			
		1	2	3	4
2	98.4	70.4 / 6933	34.2 / 3362	-2.8 / -275	-36.4 / -3585
3	44.7	58.0 / 2613	17.0 / 764	22.0 / 990	-73.2 / -3299
18	129.9	-24.7 / -3202	17.1 / 2112	10.3 / 1336	31.6 / 4127
1	81.1	74.7 / 6054	34.9 / 2827	9.8 / 798	-37.2 / -3015
total usage charges $C^{(m)}$ (\$/h)		12398	9164	2849	-5773

We illustrate the latitude that a transaction has under the proposed congestion management rule. The voluntary nature of the adjustment auction allows a transaction to decide on its own whether or not to participate. As discussed above, transactions 1 and 2 choose not to participate in the auction. In addition, a transaction that decides not to participate in the auction may determine how to construct its offer effectively so as to achieve the desired portfolio in the IGO-determined schedule. Consider, for example, transaction 4. The optimal results in Table 5.10 indicate that the transaction is modified in the IGO-determined schedule, i.e., 4.0 MW of generation is shifted from bus 8 to bus 9. Now let us suppose that transaction 4 desires to remain unchanged. To do so, the transaction raises the price c_9^+ offered by its generator at bus 9 from \$15/MWh to \$35/MWh. We resolve the LP problem with the increased c_9^+ and the optimal net injection adjustments $\Delta p_k^*,\ k \in K$ are shown in Table 5.13. The comparison between Table 5.13 and the corresponding last row of Table 5.10 shows that with c_9^+ raised to \$35/MWh, the generator at bus 9 becomes the most expensive unit in the auction and its bid is not chosen, i.e., Δp_9^* becomes zero. Due to the separation of mark**ets** constraint on transaction 4, it follows that Δp_8^* must also become zero. Therefore, transaction 4 remains unchanged. The decision by transaction 4 to raise c_9^+ also results in changes in the total usage charges $C_l^{(m)}$ for each transaction m. The modified usage payments are shown in Table 5.14. The comparison between Table 5.14 with the corresponding last row of Table 5.12 shows that while the IGO's usage payment to transaction 4 decreases by \$3343/h, the usage charges for the other transactions decrease.

The auction-based congestion management provides transactions with considerable flexibility. A transaction may decide whether or not to participate in the auction. If the transaction chooses to take part in the auction, it may construct its offer effectively to attempt to keep its preferred schedule intact. On the other hand, however, the transaction and all the other transactions would bear the consequences of this transaction's decision in terms of the usage charges determined by the auction.

Table 5.13. Values of $\Delta p_k^*,\ k \in K$ in MW of the LP with c_9^+ = \$35/MWh for Test System G

Offerer at bus k	Participants in transaction 3		participants in transaction 4		other participants	
	3	12	8	9	2	6
net injection adjustments $\Delta p_k^* = \sum_{m \in \hat{M}} \Delta p_k^{*(m)}$	-8.6	8.6	0	0	-13.6	13.6

Table 5.14. The total usage charge $C_l^{(m)}$ with c_9^+ = $35/MWh for Test System G

transaction	1	2	3	4
Total usage charges $C_l^{(m)}$ ($/h)	8504	7694	1769	-2430

While the auction-based congestion management mechanism allows transactions a great deal of freedom, the decision made by a market participant according to its own interests may disadvantage the system or even result in the failure of the IGO's congestion relief. For example, suppose that the generator at bus 8 of transaction 4 decides not to participate in the adjustment auction. Consequently, Δp_8 stops being a decision variable for the LP problem. We assume that all the other offerers and their offers remain unchanged. Without the presence of Δp_8 as a decision variable for the IGO, however, the LP problem does not even have a feasible solution. It means that the overloads cannot be removed from the network in this case with the set of offers received. Consequently, in order to ensure that the indispensable relief services from the generator at bus 8 are available, the IGO must designate it as a *reliability must run* unit and pay for its relief service at the contractually negotiated price.

We examine the capability of the congestion management allocation scheme to respond to variations in the amount of a transaction. Let us focus on transaction 2. While all other transactions are kept fixed, we vary the amount of transaction 2 using a scaling factor 0.95 $\leq g \leq$ 1.05, where $g = 1$ corresponds to the reference case. Now, the flow associated with transaction 2 contributes to the dominant flow in the overloaded line 3, so that the overload Δf_3 increases as the amount of transaction 2 increases, as shown in Figure 5.2. The overload $D f_3^{(2)}$ allocated to transaction 2 also increases over this range and is also shown in Figure 5.2. Since the overloads and the allocation of the overloads to transaction 2 on lines 2 and 18 increase as the amount of transaction 2 increases and results in the removal of the larger overloads as the amount of the transaction increases, both the total congestion costs Z^* and the usage price c_3^* associated with line 3 increase as a function of the scaling factor g. This is shown in Figure 5.3. As illustrated in Figures 5.2 and 5.3, the congestion management results and the corresponding transmission usage charges given by the proposed scheme behave in an appropriate manner when the amount of a transaction varies.

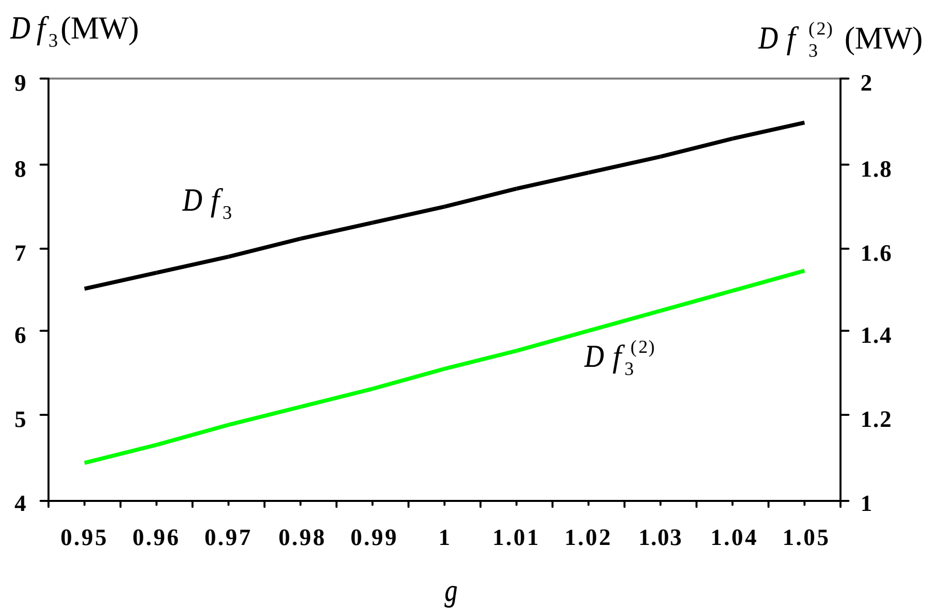

Figure 5.2 The variations of Δf_3 and $\Delta f_3^{(2)}$ as a function of the scaling factor g for Test System G

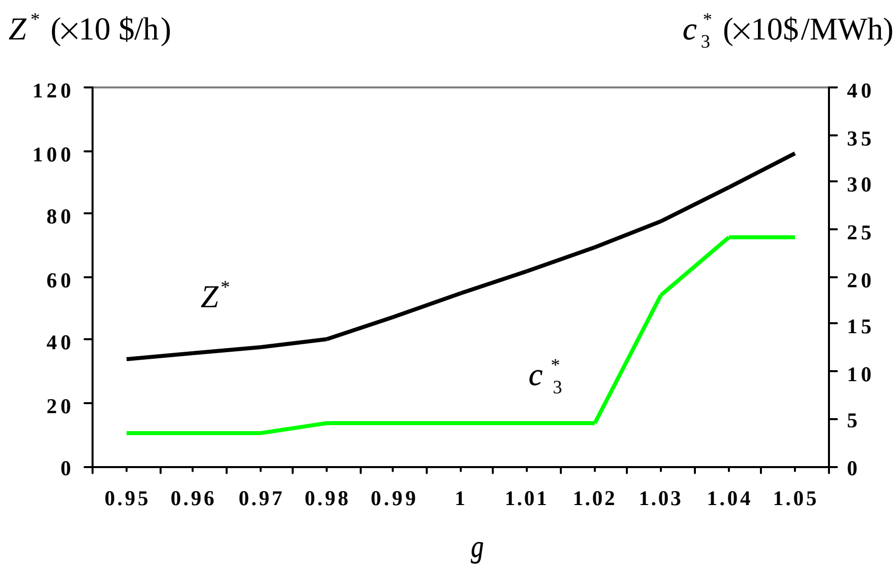

Figure 5.3 The variations of the total congestion management costs Z^* and the usage price c_3^* on line 3 as a function of the scaling factor g for Test System G

We also study the sensitivity of the LP optimal solution to uncertainty: we examine the change in the optimum with respect to one of the line flow limits, f_2^{max} of line 2. Since we represent all the transmission constraints in terms of the line flow limits, the values specified for f_2^{max} may involve considerable uncertainty. We investigate the behavior of the scheme with the uncertainty in such a parameter being explicitly represented. Let the range of the uncertainty of f_2^{max} be in an interval of $\pm 25\%$ around its reference value. We illustrate the effects of this uncertainty on the optimal solution with respect to the overloads and the resulting congestion management costs/charges. As the limit f_2^{max} decreases, the overload Δf_2 on line 2 increases. In turn, the allocations $\Delta f_2^{(1)}$ and $\Delta f_2^{(2)}$ to transactions 1 and 2 correspondingly increase. Consequently, the IGO needs to remove the larger overloads attributed to transactions 1 and 2, and still ensure that no overloads occur in any of the lines of the system. The impacts on the allocated overloads $\Delta f_2^{(1)}$ and $\Delta f_2^{(2)}$ are shown in Figure 5.4. The changes increase the congestion relief costs Z^*. The usage price c_2^* on line 2 remains constant or increases as the line limit decreases as well. Figure 5.5 displays the impacts of the uncertainty on Z^* and c_2^* as a function of the f_2^{max} values. It follows from this example that the proposed congestion management allocation scheme has the ability to study the impact of parameter uncertainty involved in the modeling of the transmission constraints.

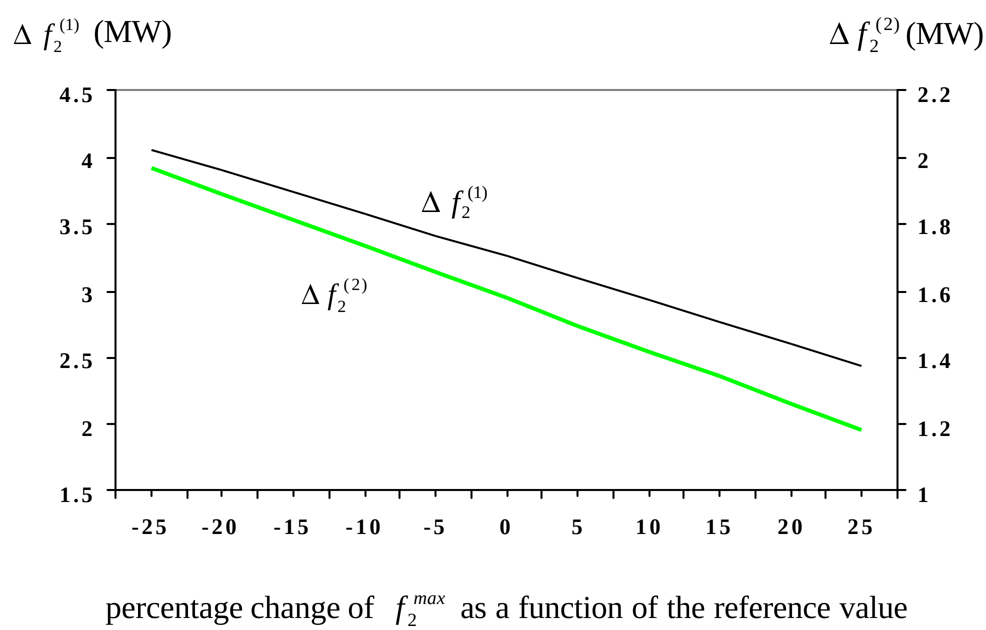

Figure 5.4 Plot of $\Delta f_2^{(1)}$ and $\Delta f_2^{(2)}$ as a function of uncertainty in the value of f_2^{max} for Test System G

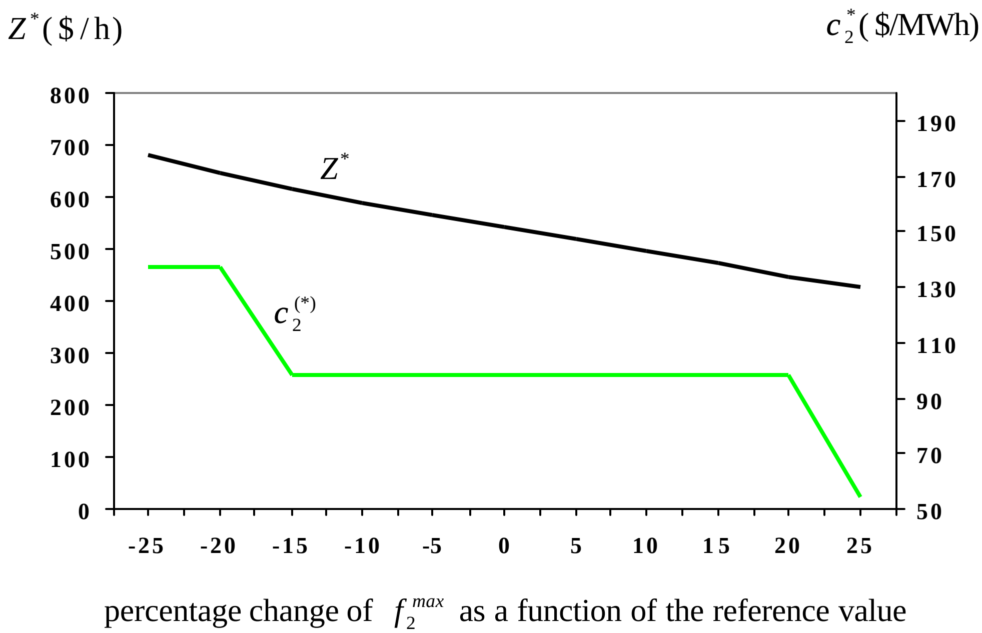

Figure 5.5 Plot of Z^* and c_2^* as a function of uncertainty in the value of f_2^{max} for Test System G

In this section, we have examined and studied the capability of the proposed congestion management allocation scheme in Test Systems F and G. Various sensitivity analyses of the scheme have been performed. As borne out by the extensive testing and the representative results shown here, the proposed scheme provides the IGO with an appropriate and useful tool to allocate transmission congestion on its network among the transactions. The scheme determines the overload allocation on a physical flow basis, provides a congestion relief mechanism that removes the overload attributed to each transaction in the least-cost manner to the IGO, and determines the appropriate transmission charges to each transaction for its usage of the system.

5.5 Conclusions

We have developed a congestion management allocation scheme for multiple-transaction networks in this chapter. We have used the multiple-transaction network framework constructed in Chapter 2 to characterize transmission congestion and then determine the contribution to congestion attributable to each transaction on the physical-flow basis. We have presented an adjustment auction-based congestion relief mechanism that enables the IGO to acquire relief services to remove the overload congestion attributed to each transaction from the network in the most economic manner. This congestion relief scheme also determines the appropriate transmission charges to each transaction for its usage of the network. We have discussed the policy implications of the congestion relief formulation. Finally, we examined and studied the capability of the proposed congestion management allocation scheme in Test Systems F and G