CHAPTER 4

ALLOCATION OF REACTIVE SUPPORT REQUIREMENTS

4.1 Introduction

We propose in this chapter a physical-flow-based scheme to allocate the generator-provided reactive support requirements to each individual transaction in the system. We make detailed use of the multiple-transaction network framework developed in Chapter 2. We assume that for the proposed transactions in $\mathcal{M}$, the IGO determines the voltage setting point V_k^s of each bus $k \in \mathcal{Q}$ and the voltage profile for the buses $j \in \mathcal{Q}^c$. Recall that $\mathcal{Q}$ is the set of buses at which generators that may provide reactive support are located.

We focus on a normal steady-state condition of the multiple-transaction system with the set $\mathcal{M}$ of proposed transactions. We assume that the various generators in $\mathcal{Q}$ are capable of providing the required reactive support for these transactions so that all bus voltages meet the specified voltage profile requirements. Consequently, in this chapter we ignore reactive power limits of the generators. In addition, we assume that all other constraints are satisfied, i.e., there is no congestion.

In Section 4.2, the reactive power output of the generator is formulated as the sum of the intrinsic reactive support requirements and the transaction-based reactive support requirements. Section 4.3 presents a physical-flow-based allocation mechanism to determine the amount of reactive support required by each transaction from the various generators providing the service. This reactive support requirement allocation scheme was implemented and tested on a number of test systems. Representative numerical results on the systems tested are presented in Section 4.3.

4.2 Reactive Support Requirements

In this section, we evaluate the reactive support requirements under certain simplifying assumptions. The focus is on the system-wide requirements and so we assume that any local reactive load is locally met at the bus. In other words, we assume that at each load bus the power factor is 1. We assume for each transmission line or transformer between any pair of buses *i* and *j* that $q_i - q_j$ is small. The assumption has the following implications:

$$\sin(q_i - q_j) \approx q_i - q_j \tag{4.1}$$

$$\cos(q_i - q_j) \approx 1 - \frac{(q_i - q_j)^2}{2} \tag{4.2}$$

We define the reactive support requirements due to the presence of the transactions with respect to a reference case. The reference is the reactive power injected into the lines connected to the generator at each bus $k \in Q$ for the case of no transactions on the system. We use the superscript 0 to denote the case. Since for $k \in Q$, the voltage magnitude $V_k^0 = V_k^s$. Then Equation (4.3) becomes[1]

$$Q_{kj}^0 = -\frac{b_{kj}}{2}(V_k^s)^2 + \frac{x_{kj}}{r_{kj}^2 + x_{kj}^2} V_k^s [V_k^s - V_j^0 + \frac{V_j^0}{2}(q_k^0 - q_j^0)^2] - \frac{r_{kj}}{r_{kj}^2 + x_{kj}^2} V_k^s V_j^0 (q_k^0 - q_j^0) \tag{4.3}$$

[1] If $r_{ij} << x_{ij}$ for all the lines of the network, the no-transaction network is essentially purely reactive. In such case, the angles $q_i^0 - q_i^0 \approx 0$ for $i, j = 0, 1, 2, \ldots, N$ and $Q_{kj}^0 = -b_{kj}(V_k^s)^2/2 + x_{kj}/(r_{kj}^2 + x_{kj}^2)[(V_k^s)^2 - V_k^s V_j^0]$.

Then

$$Q_k^{g,0} = \sum_{j \in H_k} Q_{kj}^0 \tag{4.4}$$

is the no-transaction reactive power generation at bus $k \in Q$. Equation (4.4) expresses the *intrinsic* reactive support provided by the generator to maintain the specified voltage profile in the absence of any transaction. This quantity is an intrinsic part of the provision of transmission services and as such is not allocated to any of the transactions that may be undertaken.

We next consider the case in which the set M of the proposed transactions is presented. In this case, we write the voltage magnitude and angle at each bus *i* as

$$V_i = V_i^0 + \Delta V_i \tag{4.5}$$

and

$$q_i = q_i^0 + \Delta q_i \tag{4.6}$$

where DV_i and Dq_i are the changes in the voltage magnitude and angle at bus i from the reference case, respectively, and are due to the presence of the transactions. Note that for $i = k \in Q$, $DV_k = 0$. We rewrite Equation (2.10) by using Equations (4.1), (4.2), and (4.5):

$$\begin{aligned} Q_{kj} = & -\frac{b_{kj}}{2}(V_k^s)^2 - \frac{r_{kj}}{r_{kj}^2 + x_{kj}^2} V_k^s V_j (q_k - q_j) + \frac{x_{kj}}{r_{kj}^2 + x_{kj}^2} V_k^s [V_k^s - V_j^0 + \frac{V_j^0}{2}(q_k - q_j)^2] \\ & - \frac{V_k^s}{r_{kj}^2 + x_{kj}^2} \left[r_{kj}(q_k - q_j) + x_{kj} - \frac{x_{kj}}{2}(q_k - q_j)^2 \right] DV_j \end{aligned} \tag{4.7}$$

We further substitute for $q_k - q_j$ in the second and the third terms of Equation (4.7) by using Equation (4.6) so that

$$Q_{kj} = -\frac{b_{kj}}{2}(V_k^s)^2 + \frac{x_{kj}}{r_{kj}^2 + x_{kj}^2}(V_k^s)^2 - \frac{r_{kj}}{r_{kj}^2 + x_{kj}^2} V_k^s V_j^0 \left[(q_k^0 - q_j^0) + (Dq_k - Dq_j) \right]$$

$$- \frac{x_{kj}}{r_{kj}^2 + x_{kj}^2} V_k^s V_j^0 \left[1 - \frac{(q_k^0 - q_j^0)^2}{2} - \frac{(Dq_k^0 - Dq_j^0)^2}{2} - (q_k^0 - q_j^0)(Dq_k^0 - Dq_j^0) \right] \qquad (4.8)$$

$$- \frac{V_k^s}{r_{kj}^2 + x_{kj}^2} \left[r_{kj}(q_k - q_j) + x_{kj} - \frac{x_{kj}}{2}(q_k^0 - q_j^0)^2 \right] DV_j$$

Let

$$Q_{kj}^{Dq} = \frac{V_k^s V_j^0}{r_{kj}^2 + x_{kj}^2} \left[-r_{kj}(Dq_k^0 - Dq_j^0) + x_{kj} \frac{(Dq_k^0 - Dq_j^0)^2}{2} + x_{kj}(q_k^0 - q_j^0)(Dq_k^0 - Dq_j^0) \right] \qquad (4.9)$$

and

$$Q_{kj}^{DV} = -\frac{V_k^s}{r_{kj}^2 + x_{kj}^2} \left[r_{kj}(q_k - q_j) + x_{kj} - \frac{x_{kj}}{2}(q_k - q_j)^2 \right] DV_j \qquad (4.10)$$

and recall the definition of Q_{kj}^0 from Equation (4.3). We may therefore rewrite Equation (4.8) as

$$Q_{kj} = Q_{kj}^0 + Q_{kj}^{Dq} + Q_{kj}^{DV} \qquad (4.11)$$

Consequently, we may also decompose Q_k^g into three components:

$$Q_k^g = Q_k^{g,0} + Q_k^{g,Dq} + Q_k^{g,DV} \qquad (4.12)$$

where

$$Q_k^{g,Dq} = \sum_{j \in H_k} Q_{kj}^{Dq} \qquad (4.13)$$

and

$$Q_k^{g,DV} = \sum_{j \in H_k} Q_{kj}^{DV} \qquad (4.14)$$

We call $Q_k^{g,Dq}$ the *voltage angle variation component* and $Q_k^{g,DV}$ the *voltage magnitude variation component* of the reactive power output of the generator at bus $k \in Q$. These components are in addition to the intrinsic reactive support requirement $Q_k^{g,0}$ of the network. The additional reactive support requirements as expressed by these two components result in added reactive power support from the generators. We next develop the physical-flow-based allocation schemes of these additional requirements.

4.3 Allocation of Reactive Support Requirements

Let us first consider the component $Q_k^{g,Dq}$. In order to express $Q_k^{g,Dq}$ explicitly in terms of the transactions, we impose the assumption $r_{kj} << x_{kj}$. Then the voltage angle changes $Dq_k - Dq_j$ with respect to the reference case may be approximated by the DC power flow results. Recall that $\underline{\hat{q}} = [\hat{q}_1, \hat{q}_2, ..., \hat{q}_N]^T$ is the vector of the nodal voltage angles computed by the DC power flows in Equation (2.14) in Chapter 2. Then we may approximate $Q_k^{g,Dq}$ in Equation (4.13) by

$$\tilde{Q}_k^{g,Dq} = \sum_{j \in H_k} \frac{V_k^s V_j^0}{r_{kj}^2 + x_{kj}^2} \left\{ \frac{x_{kj}}{2}(q_k - q_j) + \frac{x_{kj}}{2}(q_k^0 - q_j^0) - r_{kj} \right\} (\hat{q}_k - \hat{q}_j) \qquad (4.15)$$

Then, using Equation (2.19) we may express

$$\tilde{Q}_k^{g,Dq} = \sum_{m \in M} V_k^{(m)} t^{(m)} \qquad (4.16)$$

where

$$V_k^{(m)} = \sum_{j \in H_k} \frac{V_k^s V_j^0}{r_{kj}^2 + x_{kj}^2} \left[\frac{x_{kj}}{2}(q_k - q_j) + \frac{x_{kj}}{2}(q_k^0 - q_j^0) - r_{kj} \right] p_{kj}^{(m)} \qquad (4.17)$$

Next, in line with our assumption that the component $Q_k^{g,\Delta V}$ is entirely due to the transactions, we attribute the voltage variations ΔV_j at buses $j \in Q^c$ to the presence of the transactions in the system. Due to lack of the explicit analytical expression for ΔV_j in terms of $t^{(m)}$, $m \in M$, we introduce the impacts of the transactions in an indirect way. We rewrite ΔV_j by multiplying and dividing it by the weighed sum of the transaction amounts where we explicitly weigh each $t^{(m)}$ by its sensitivity $x_j^{(m)}$, where

$$x_j^{(m)} \triangleq \frac{\partial V_j}{\partial t^{(m)}} \tag{4.18}$$

$x_j^{(m)}$ is evaluated at $\underline{t} = [t^{(1)}, t^{(2)}, ..., t^{(|M|)}]^T$ and the analytical derivation of $x_j^{(m)}$ is given in detail in Appendix D. In this way, we explicitly represent the local nature of reactive power and the fact that the generator that is electrically remote from the transaction is expected to have a considerably smaller impact on the reactive support required by the transaction than one that is electrically near. Hence we use $\sum_{m \in M} x_j^{(m)} t^{(m)}$ as the factor by which we multiply and divide ΔV_j:

$$\Delta V_j = \frac{\sum_{m \in M} x_j^{(m)} t^{(m)}}{\sum_{m \in M} x_j^{(m)} t^{(m)}} \Delta V_j = \sum_{m \in M} \frac{x_j^{(m)} t^{(m)}}{\sum_{m' \in M} x_j^{(m')} t^{(m')}} \Delta V_j \tag{4.19}$$

In this way, we may interpret $[x_j^{(m)} t^{(m)} / \sum_{m' \in M} x_j^{(m')} t^{(m')}] \Delta V_j$ to be the voltage variation component allocated to transaction m. Note that we assume $\sum_{m \in M} x_j^{(m)} t^{(m)}$ is not equal to 0.

It follows from Equations (4.14) and (4.19) that

$$Q_k^{g,DV} = \sum_{m \in M} v_k^{(m)} t^{(m)} \tag{4.20}$$

where

$$v_k^{(m)} = \sum_{j \in H_k} \left\{ \frac{V_k^s}{r_{kj}^2 + x_{kj}^2} \left[r_{kj} (q_k - q_j) + x_{kj} - \frac{x_{kj}}{2} (q_k - q_j)^2 \right] \frac{x_j^{(m)}}{\sum_{m' \in M} x_j^{(m')} t^{(m')}} \right\} \tag{4.21}$$

The sum of the entities in Equations (4.16) and (4.20) gives the total amount of reactive support requirement of each transaction $m \in M$ from each generator $k \in Q$:

$$Q_{k,a}^{(m)} = g_k^{(m)} t^{(m)} \tag{4.22}$$

where

$$g_k^{(m)} = V_k^{(m)} + v_k^{(m)} \tag{4.23}$$

A special case of this allocation scheme is that the seller and the buyer of a transaction *m* are located at the same bus. In this case, since this transaction has no impact on the power flows of the transmission network, then both $p_{kj}^{(m)}$ and $x_j^{(m)}$ are zero. Consequently, $g_k^{(m)} = V_k^{(m)} = v_k^{(m)} = 0$ and $Q_{k,a}^{(m)} = 0$, $k \in Q$. This result is physically intuitive in that the transaction is considered to be not *using* the network so it does not require reactive support from any generator.

Note that the sum of the allocated amounts $Q_{k,a}^{(m)}$ is an approximation of the voltage magnitude and angle variation components defined in Equations (4.16) and (4.20) and so we have that Q_k^g may be approximated by

$$\tilde{Q}_k^g = Q_k^{g,0} + Q_{k,a}^g \tag{4.24}$$

where

$$Q_{k,a}^g = \sum_{m \in M} Q_{k,a}^{(m)} \tag{4.25}$$

4.4 Numerical Results

We have tested the proposed allocation scheme on a number of different test systems. We present a representative sample of the results on Test Systems D and E, which are based on the IEEE 57-bus and the IEEE 30-bus systems, respectively. The test systems are described in detail in Appendix E. The results show that the scheme allocates among the transactions the reactive support provided by the various generators in a physically appropriate manner.

We discuss first the results of our investigations of the overall performance of the reactive power output approximations under different operating conditions. We present typical results using Test System D. We vary uniformly and simultaneously the amount of each transaction using a scaling factor $0.85 \leq h \leq 1.15$. The case with $h = 1$ is the base case. Figure 4.1 illustrates the performance of the $\tilde{Q}_k^g$ approximation of Q_k^g for the generator at bus 1. For the selected range of h, the error magnitude of the approximation

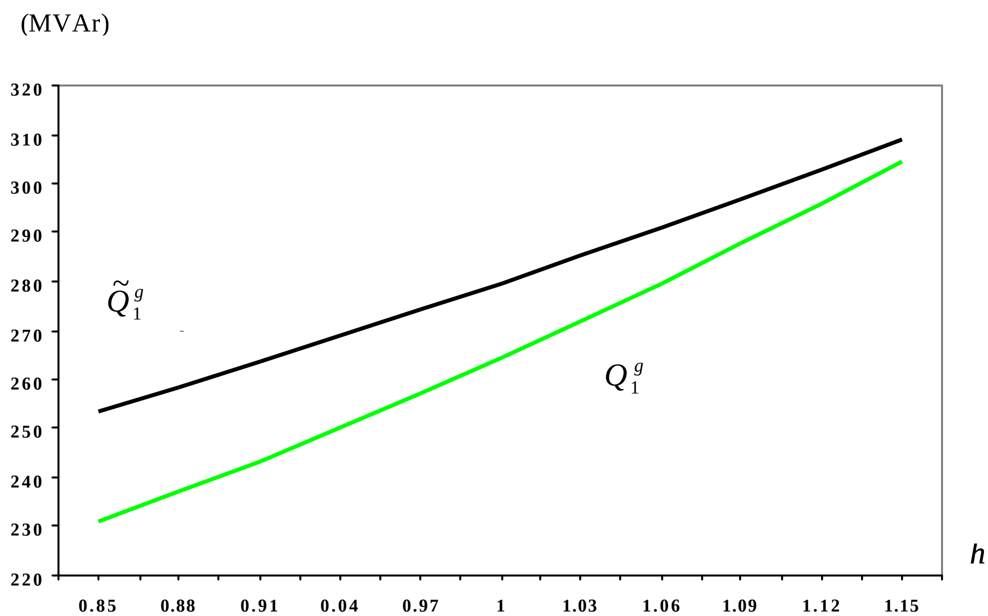


Figure 4.1 Comparison of the approximation $\tilde{Q}_1^g$ with the AC power flow value Q_1^g for Test System D

is under 10%. The approximation $\tilde{Q}_1^g$ tracks closely Q_1^g over the range. Such close tracking was observed in all the generators in all the cases that were investigated.

We next examine in some detail the actual allocation results for a specific case. We use Test System E for this purpose. There are five transactions imposed on the test system. There are six generators in the network that provide reactive support service with $\mathcal{Q} = \{1,2,13,22,23,27\}$. The IGO-determined voltage setting point V_k^s of each of the six generators is 1.0 p.u., and the IGO-required voltage profile at the load buses $j \in \mathcal{Q}^c$ is in the range [0.93, 1.05]. The values of Q_k^g, $Q_k^{g,0}$, $\tilde{Q}_k^{g,Dq}$, and $Q_k^{g,DV}$ of the generator at bus $k \in \mathcal{Q}$ are shown in Table 4.1. The results indicate that the approximations of Q_k^g, $k \in \mathcal{Q}$ used in the decomposition and allocation work well. Values for $Q_k^{g,0}$ in Table 4.1 show that for the reference case in which no transaction is present on the network, each of the six generators provides the intrinsic reactive support by absorbing the reactive power injected by the transmission line charging capacitors so as to maintain the specified voltage profile.

Table 4.1 The MVAr value of components of the reactive output of the generator at $k \in \mathcal{Q}$ for Test System E

generator bus k	1	2	13	22	23	27
V_k^s (p.u.)	1.0	1.0	1.0	1.0	1.0	1.0
Q_k^g	29.6	106.2	102.7	51.0	38.4	52.7
$Q_k^{g,0}$	-3.8	-9.5	-1.2	-0.9	-0.2	-0.5
$\tilde{Q}_k^{g,Dq}$	11.4	40.2	54.7	3.4	7.9	19.8
$Q_k^{g,DV}$	21.5	74.2	47.7	48.8	30.7	33.8
$(\tilde{Q}_k^g - Q_k^g)/Q_k^g$ (%)	-1.7	-1.2	-1.5	0.6	0	0.8

The allocation results $Q_{k,a}^{(m)}$, $k \in Q$, $m \in M$ are summarized in Table 4.2. The allocation results in Table 4.2 strongly indicate that for each transaction the required reactive support comes from the generators participating in the transaction and the generators that are electrically in the vicinity of the loads participating in the transaction. We use transaction 4 as an example. The generator at bus 23, which participates in transaction 4, provides a considerably large part of the reactive support requirements for this transaction. The generator at bus 27, while not participating in transaction 4, is allocated 21.34 MVAr for transaction 4. This is because the loads at buses 29 and 30 that participate in transaction 4 are connected only to the generator at bus 27. Due to the local nature of reactive support, the maintenance of the voltage profile at buses 29 and 30 depends on the reactive support from the generator at bus 27. Next we consider transaction 5 as another example. The generators at buses 1 and 2 and the loads at buses 3, 4, 7, and 8 participate in this transaction. Since the generators are adjacent to the loads in the transaction, 79% and 81% of the reactive support provided by the generators at buses 1 and 2, respectively, are allocated to meet the requirements of transaction 5. On the other hand, only 9% of the reactive support given by the generator at bus 23 is

Table 4.2 The allocation results $Q_{k,a}^{(m)}$, $m \in M$, $k \in Q$ in MVAr for Test System E

generator bus k / transaction m	$Q_{k,a}^{(m)}$ (MVAr)					
	27	22	13	23	1	2
1	24.56	14.67	4.99	6.90	2.08	6.16
2	0.20	4.90	0.27	0.02	0.11	0.44
3	0	0	0	0	0	0
4	21.34	19.84	90.76	28.35	4.62	15.42
5	7.58	12.89	6.38	3.35	26.05	92.60
participation in transaction(s)	1	2, 3	4	4	5	5

devoted for transaction 5 since it is electrically remote from the loads that participate in transaction 5. Thus, the allocation results are physically meaningful; it would be unconvincing to impute an allocation to a transaction for a significant amount of reactive support from a remote generator since physically it does not make sense to maintain the specified voltage profile by using the reactive support provided at a distant location.

We may also use the examples of transactions 1 and 2 to illustrate the considerable impacts of the network location of the participants in a transaction on the amount of the reactive support required by that transaction. While, the amounts of the transactions 1 and 2 are of comparable magnitude, the amount of reactive support required by transaction 1 from each generator is much larger than that required by transaction 2. This is due mainly to the fact that wile the selling and buying entities participating in transaction 2 are directly connected, those entities participating in transaction 1 are electrically remote from each other. Consequently, the reactive losses entailed by the power transfer in transaction 1 are much larger than those in transaction 2 and so the reactive support requirements of transaction 1 are correspondingly larger than those of transaction 2. For the special case of transaction 3 whose participants are at the same bus 2, the fact that the *transaction does not use the transmission system* implies that

Table 4.3 The allocation results $Q_{k,a}^{(m)}$, $m \in \mathcal{M}$, $k \in \mathcal{Q}$ in MVAr for Test System D

generator bus k / transaction m	$Q_{k,a}^{(m)}$			
	1	3	8	12
1	-5.81	-0.13	-0.34	-13.55
2	82.84	55.26	8.23	49.39
3	52.22	9.65	55.84	28.36
participation in transaction(s)	2, 3	2	3	1, 3

no reactive losses are incurred. Hence, there are no reactive support requirements. Our scheme allocates zero MVAr from each generator to the transaction. We conclude that the proposed allocation scheme behaves in a physically reasonable way for the various transactions in the network. This is representative of the results on various systems under different conditions.

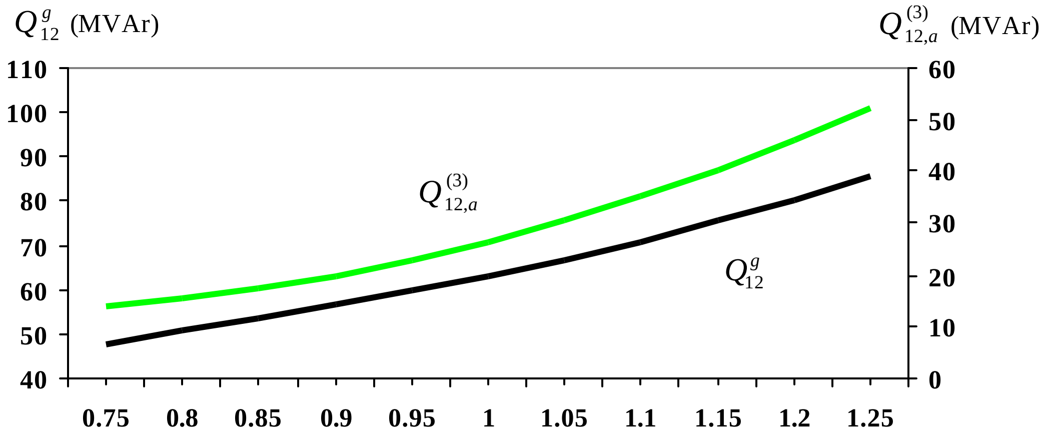
Next, we present a physical interpretation of the allocation of reactive support requirements by investigating the behavior of the scheme in response to the variation of a transaction amount. Consider Test System D in which three transactions are constructed and the generators that provide reactive support services are located at buses 1, 3, 8, and 12, i.e., $\mathcal{Q} = \{1, 3, 8, 12\}$. The allocation results $Q_{k,a}^{(m)}$, $m \in \mathcal{M}$, $k \in \mathcal{Q}$ are summarized in Table 4.3. Note that the allocation results show that the individual support requirements allocation $Q_{k,a}^{(m)}$ at bus k for transaction m may be either a reactive power injection or absorption.

An interpretation of $Q_{k,a}^{(m)}$ from Equation (4.22) is obtained by considering the impact on the allocation of the variation of a particular transaction amount. We focus on the allocations of the reactive support of the generator at bus 12 to two particular transactions, 1 and 3. We first investigate the impact on Q_{12}^{g} and $Q_{12,a}^{(3)}$ when we vary $t^{(3)}$ with a scaling factor over the range [0.75, 1.25] and hold all the other transactions constant. The scaling factor of 1 corresponds to the base case $t^{(3)} = 1441\,\text{MW}$. Figure 4.2(a) shows that as the amount of transaction 3 increases, the reactive support requirements Q_{12}^{g} from the generator at bus 12 and the allocation $Q_{12,a}^{(3)}$ to transaction 3 increase. As these plots indicate, the resulting change $Q_{12,a}^{(3)}$ gives a satisfactory approximation of the change in the generator reactive output Q_{12}^{g}. In other words, the *sensitivities* $DQ_{12}^{g}/Dt^{(3)}$ and $DQ_{12,a}^{(3)}/Dt^{(3)}$ are considerably close. We may interpret this observation in the following way: as we vary the amount of transaction 3 say in 1 MW step, with all other transactions fixed, the changes in Q_{12}^{g} and $Q_{12,a}^{(3)}$ are virtually equal. Given the close tracking of Q_{12}^{g} by $Q_{12,a}^{(3)}$, it follows that the sensitivities $DQ_{12}^{g}/Dt^{(3)}$ and $DQ_{12,a}^{(3)}/Dt^{(3)}$ are in close agreement over the range of variation of the scaling factor.

Next, we vary the amo unt of transaction 1, as all other transactions are kept fixed. As the plots in Figure 4.2(b) show, Q_{12}^{g} and $Q_{12,a}^{(1)}$ decrease as the amount of transaction 1 increases. In other words, increasing transaction 1 actually lessens the reactive support requirements from the generator at bus 12. Consequently, the reactive support requirement allocated to transaction 1 consists of an absorption. The sensitivity $DQ_{12}^{g}/Dt^{(1)}$ is closely tracked by the sensitivity $DQ_{12,a}^{(1)}/Dt^{(1)}$ over the entire range of the scaling factor. We conclude that the allocation scheme appropriately captures the impact of the transactions and correctly determines the allocation of the reactive support requirement among the transactions.

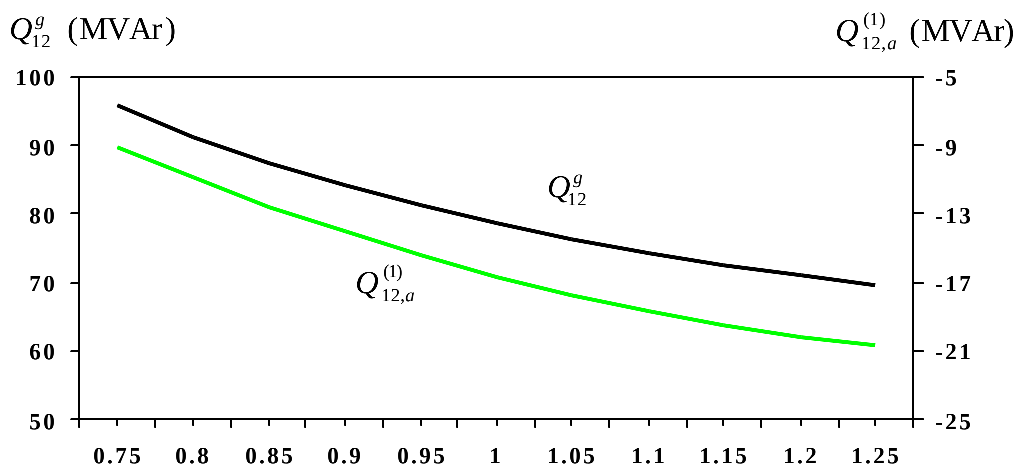
4.5 Conclusions

This chapter presents a new physical-flow-based mechanism for allocating the reactive power support requirements provided by generators in multiple-transaction networks. The allocatable reactive support requirements are defined with respect to the



scaling factor of the amount of transaction 3 (all other transactions are kept fixed)

(a) Transaction 3



scaling factor of the amount of transaction 1 (all other transactions are kept fixed)

(b) Transaction 1

Figure 4.2 The variation of Q_{12}^{g} and its allocations in response to variations in the transaction amounts for Test System D

support required for the network with no transactions in place. The requirements in the presence of the proposed transactions are formulated as the sum of two specific components - the voltage magnitude variation component and the voltage angle variation component. The formulation utilizes the multiple-transaction framework developed earlier and makes use of certain simplifying approximations. The formulation leads to a natural allocation as a function of the amount of each transaction. The physical interpretation of each allocation as a sensitivity of the reactive output of a generator is discussed. The extensive testing indicates that the allocation scheme approximates with good fidelity the actual net reactive power outflow from the generator buses. The numerical results also indicate that the proposed scheme behaves in a physically reasonable and intuitive way.