CHAPTER 3

REAL LOSS ALLOCATION AND COMPENSATION

3.1 Introduction

We use the general multiple-transaction network framework constructed in Chapter 2 to consider the problems of allocating and compensating real transmission losses in multiple-transaction networks. We first develop a physical-flow-based mechanism to allocate the total system losses to each transaction in the network in Section 3.2. This scheme allocates losses in an appropriate way that is physically reasonable and, in particular, deals effectively with the issue of losses associated with counter flows. In Section 3.3, we make use of this allocation scheme to develop flexible and efficient loss compensation procedures in a multiple transaction network. The numerical results of the extensive testing that we performed with the proposed allocation and compensation schemes are given in Section 3.4.

3.2 Real Loss Allocation

We consider a system with $N + 1$ buses and the set $\mathcal{M}$ of transactions. Bus 0 is designated as the slack bus. Using the multiple-transaction network framework constructed in Chapter 2, we recast the power flows in terms of the transactions in Equation (2.7). We assume that a transmission line between buses i and j is represented by its line impedance $r_{ij} + jx_{ij}$. It follows that the off-diagonal element $i, j, i \neq j$ of the bus admittance matrix is $G_{ij} + jB_{ij} = -(r_{ij} + jx_{ij})^{-1}$ and shunt elements are negligibly small. Under these conditions, the real power loss l_{ij} on a line connecting buses *i* and *j* is

$$l_{ij} = \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}[V_i^2 + V_j^2 - 2V_iV_j \cos(q_i - q_j)] \tag{3.1}$$

Note that since $l_{ij} = l_{ji}$, the total system losses are given by

$$l = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in H_i}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}[V_i^2 + V_j^2 - 2V_iV_j \cos(q_i - q_j)] \tag{3.2}$$

where the division by 2 is introduced so as not to count the losses on each line twice. We next assume the DC power flow conditions specified in Equations (2.10) and (2.11) hold. Then the total losses may be approximately represented by $\tilde{l}$, where

$$\tilde{l} = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in H_i}\left\{\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j)(q_i - q_j)\right\} \tag{3.3}$$

In order to express the approximation of the system losses explicitly in terms of the set M of the transactions, we further approximate $\tilde{l}$ in Equation (3.3) by $\hat{l}$, where

$$\begin{aligned}\tilde{l} &\approx \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in H_i}\left\{\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j)(\hat{q}_i - \hat{q}_j)\right\} = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in H_i}\left\{\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j)\sum_{m=1}^{M} p_{ij}^{(m)} t^{(m)}\right\} \\ &= \sum_{m\in M}\left\{\frac{1}{2}\sum_{i=0}^{N}\sum_{j\in H_i}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j)p_{ij}^{(m)}\right\} t^{(m)} \triangleq \hat{l}\end{aligned} \tag{3.4}$$

where $\hat{q}_i, i = 0, 1, 2, ..., N$ are the voltage angles computed by the DC power flow and are determined by Equation (2.12), and $p_{ij}^{(m)}$, $m \in M$, is given by Equation (2.18). We define for $m \in M$

$$l^{(m)} = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in H_i}\left\{\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(q_i - q_j)p_{ij}^{(m)}\right\} \tag{3.5}$$

Then, $\hat{l}$ is explicitly written in terms of $t^{(m)}$ as

$$\hat{l} = \sum_{m \in M} l^{(m)} t^{(m)} \tag{3.6}$$

Using the analogous development above with the assumption that the DC conditions in Equations (2.10) and (2.11) hold, we may approximate $l^{(m)}$ by a linear function of $\hat{\underline{q}}$ as

$$l^{(m)} \approx \frac{1}{2} \sum_{i=0}^{N} \sum_{j \in H_i} \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} (\hat{q}_i - \hat{q}_j) p_{ij}^{(m)} = \frac{1}{2} \sum_{k \in M} g_k^{(m)} t^{(k)} \tag{3.7}$$

where

$$g_k^{(m)} = \sum_{i=0}^{N} \sum_{j \in H_i} \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} p_{ij}^{(m)} p_{ij}^{(k)} \tag{3.8}$$

Note that in Equation (3.7) $l^{(m)}$ appears as a linear function of $t^{(k)}, k \in M$. The expression for $\hat{l}$ in Equation (3.6) is quadratic in $t^{(k)}, k \in M$.

It is convenient to use the form of the terms summed in Equation (3.6) to define the quantity

$$\hat{l}_a^{(m)} = l^{(m)} t^{(m)}, \; m \in M. \tag{3.9}$$

so that the expression in Equation (3.6) is rewritten as

$$\hat{l} = \sum_{m \in M} \hat{l}_a^{(m)} \tag{3.10}$$

and we may interpret $\hat{l}_a^{(m)}$ to be an expression for the "contribution"[1] made by transaction

[1] The expression for $\hat{l}_a^{(m)}$ may be given a straightforward physical interpretation. The $|I|^2 r$ losses in a line joining buses i and j due to the line current resulting from transaction m are $V_{r_{ij}} I_{ij}^{(m)} \approx \frac{r_{ij}}{x_{ij}} (q_i - q_j) \frac{p_{ij}^{(m)} t^{(m)}}{x_{ij}} \triangleq \hat{l}_{aij}^{(m)}$. Here, $V_{r_{ij}}$ is the voltage drop across r_{ij} that results from all the flows due to all the M transactions in the system, and $I_{ij}^{(m)}$ is the current flowing between buses i and j as a result

m to the total system losses. In certain cases, however, $l^{(m)}$ may be negative so that one may conclude that transaction *m* has a negative loss allocation. Such a conclusion would imply, however, that by using the transmission network, transaction *m* may in effect "inject" additional power into the system. Such an implication is, of course, physically incorrect and the allocation scheme must address this issue.

Physically, whether a particular transaction increases or lowers the system losses depends on the system state and the impact of the particular transaction on the system. Through the use of the DC power flow assumptions, we derive an approximation of the system losses as a linear expression of the transactions of the system. Some transactions cause flow in the same direction as the net flow, while others cause flow in the opposite direction. The flow in the same direction as the net flow is called a *dominant flow*, while the flow in the opposite direction is a *counter flow*. Dominant flows increase the total system losses, while counter flows lower the total system losses as the amount of the corresponding transaction is increased.

Absent the dominant flow, the counter flow cannot exist. If the dominant flow disappears, the counter flow itself becomes the dominant flow. The counter flow helps reduce the losses only in the presence of the dominant flow. Thus, the reduction in the system losses is not due to a particular transaction but is an attribute of the system state. As such it should be shared by all the transactions on the system.

Therefore, we modify the loss allocation by replacing $l^{(m)}$ with its absolute value of $\left|l^{(m)}\right|$. However, this modification would lead to allocating more losses than are

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of transaction *m*. Since $V_{r_{ij}}$ and $I_{ij}^{(m)}$ can have different signs, $\hat{l}_{a,ij}^{(m)}$ may be negative. It follows that $\hat{l}_a^{(m)}$, the algebraic sum of all line losses due to transaction *m*, may also be negative.

actually incurred. Consequently, we allow all transactions to benefit from the “negative” losses and will normalize the allocations to ensure that the sum of the allocated losses equals $\hat{l}$. Thus, the loss allocated to transaction m is

$$l_a^{(m)} = \frac{\left| l^{(m)} \right| t^{(m)}}{\sum_{k \in M} \left| l^{(k)} \right| t^{(k)}} \hat{l} \tag{3.11}$$

Consequently, a positive loss is associated with each transaction. Additional motivation for the normalization in the allocation formula is given in Appendix A of this chapter. This motivation is based on sensitivity information. The sensitivity $\partial \hat{l} / \partial t^{(m)}$ of the system losses with respect to a transaction $t^{(m)}$ may be approximated by a linear function of all transactions $t^{(k)}, k \in M$. Additional aspects of this approximation are also discussed in more detail.

An attractive property of the allocation scheme is that the allocation may be evaluated for any subset of the transactions. We use Equation (3.5) to evaluate $l^{(m)}$ for each transaction m in the specified subset. For the unspecified transactions, an equivalent transaction representing the effect of all the unspecified transactions is constructed. Its corresponding $t^{(eq)}$ term is determined by subtracting from the total system load the sum of the specified transactions. Its $l^{(eq)}$ term is evaluated by subtracting from $\hat{l}$ the sum of the loss allocations to the specified transactions. The allocation scheme allows the computation of the allocation for as few transactions as desired without requiring information on all the unspecified transactions. An application of this useful property is discussed in the next section.

Another physically attractive property of the allocation scheme is the treatment of the losses for a transaction which is the equivalent of two or more equivalent transactions. Consider a transaction m with $t^{(m)} = t$, $s^{(m)} = i$, and $b^{(m)} = j$, so that $\mathcal{T}^{(m)} = \{ t^{(m)}, i, j \}$. Let k be an arbitrary bus $k \neq i$, $k \neq j$. Consider the transactions $\mathcal{T}^{(m')} = \{ t', i, k \}$ and $\mathcal{T}^{(m'')} = \{ t'', k, j \}$ each with the identical transaction amount t. Note that the net effect of these two independent transactions is identical to that of transaction m. Under our definitions, the net injections into the network for the transaction $\mathcal{T}^{(m)}$ and the two independent transactions $\mathcal{T}^{(m')}$ and $\mathcal{T}^{(m'')}$ are identical. Correspondingly, the states of the system under the transaction $\mathcal{T}^{(m)}$ and under the transactions $\mathcal{T}^{(m')}$ and $\mathcal{T}^{(m'')}$ are identical. The proposed allocation scheme provides allocation $l_a^{(m)}$ for the system under the transaction $\mathcal{T}^{(m)}$ and allocations $l_a^{(m')}$ and $l_a^{(m'')}$ under the transactions $\mathcal{T}^{(m')}$ and $\mathcal{T}^{(m'')}$, respectively. It is shown in Appendix B that these allocations have the property that

$$l_a^{(m')} \leq l_a^{(m')} + l_a^{(m'')} \qquad (3.12)$$

The equality holds only in the case when all of them result in dominant or counter flows at the same time. This property may be generalized by deduction for any number of independent transactions that are equivalent to a single transaction. It follows that the loss allocation for a single transaction given by the allocation scheme provides a lower bound to the sum of the loss allocations of the set of independent transactions equivalent to that transaction, with each transaction in the set having the identical transaction amount.

Implementation issue: the DC power flow assumptions may result in an error in the estimate for the sum of the losses since l need not equal $\hat{l}$. To account for this error, we can adjust the allocation results $l_a^{(m)}$ after the fact by scaling $l_a^{(m)}$ by the ratio of the actual losses to the estimated value. A balancing fund can be set up to handle the resulting over and under charges.

3.3 Real Loss Compensation

The loss allocation scheme in Section 3.2 was developed under the assumption that the losses are covered by the supplemental generation at the single bus designated as the slack bus. The grid operator of the interconnected network may be the entity that receives reimbursement from each transaction for providing the loss compensation $l_a^{(m)}$ and pays this money to the slack bus generator. However, a transaction may wish to exercise choice by self-acquiring loss compensation from a third party at buses other than the slack bus. Then, the problem arises of how to compute such loss compensation. This problem is addressed in this section.

3.3.1 Equivalent loss compensation

The net power injection at each bus $n = 1,2,...N$ under the set M of transactions is specified in Equation (2.7). For convenience, we drop the superscript *net* and use p_n to denote the net injection at bus *n*. At bus 0 additional power is produced to cover the total system losses *l* so that the total injection at bus 0 is p_0, where

$$p_0 = l + \sum_{m \in M} d_0^{(m)} t^{(m)} \tag{3.13}$$

We define for an arbitrary bus k the equivalent loss compensation for Δp_0 MW generated at the slack bus to be the injection of real power Δp_k with the property that such injection results in Δp_0 MW decrease in the injection at the slack bus with all transactions on the system remaining unchanged. At a load bus the real power injection is equivalent to decreasing the load by the same amount of injection.

The power balance equation before injection of Δp_k is

$$p_0 + \sum_{n=1, n\neq k}^{N} p_n + p_k = l \tag{3.14}$$

With the additional amount of power Δp_k injected at bus k into the system to offload the loss compensation at the slack bus 0, the power balance equation becomes

$$p_0 - \Delta p_0 + \sum_{n=1, n\neq k}^{N} p_i + p_k + \Delta p_k = l + \Delta l \tag{3.15}$$

where Δl is the change in the total system losses due to the additional injection Δp_k at bus k. Thus, using the equality in Equation (3.14)

$$\Delta l = \Delta p_k - \Delta p_0 \tag{3.16}$$

Typically, the transmission losses are a small percentage of the total generation in a system [14]. Since $\Delta l < l$, it follows that Δl is indeed a small quantity. We consequently use linear analysis for the *small* signal model for Δl. In Equations (3.13) and (3.14) we may view l as a function of $p_1, p_2, ..., p_N$ and since the only change in these variables is Δp_k, it follows that

$$\Delta l \approx \frac{\partial l}{\partial p_k} \Delta p_k \tag{3.17}$$

From Equations (3.16) and (3.17), we have

$$\Delta p_k \approx \Delta p_0 \left[1 - \frac{\partial l}{\partial p_k}\right]^{-1} \tag{3.18}$$

We introduce further approximation by replacing $\frac{\partial l}{\partial p_k}$ by the value Ω_k obtained under the DC power flow conditions. The approximation is

$$\frac{\partial l}{\partial p_k} \approx \frac{\partial \hat{l}}{\partial p_k} = \sum_{i=o}^{N} \sum_{j \in H_i} \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} [(q_i^0 - q_j^0)(d_{jk} - d_{ik})] \triangleq \Omega_k \tag{3.19}$$

Here, the angles q_i^0 are computed using the AC power flow and correspond to the case before the additional injection Δp_k. We call Ω_k the DC loss sensitivity factor at bus k. This quantity Ω_k is already computed in the loss allocation stage. The derivation of the approximation of $\partial l / \partial p_k$ by Ω_k is shown in Appendix B. For the small signal analysis, we drop the approximation and replace by equality. This leads to the expression

$$\Delta p_k = z_k \Delta p_0 \tag{3.20}$$

where,

$$z_k = [1 - \Omega_k]^{-1} \tag{3.21}$$

We call z_k the bus k loss compensation multiplier. The physical interpretation of z_k is that z_k MW injected at bus k is equivalent to reducing loss compensation by 1 MW at the slack bus. In fact, z_k has the form of the penalty factor of the generator at bus k in the classical economic dispatch problem formulation [50]. Note that at the slack bus, $z_0 = 1$.

The linearity of the small signal model in Equation (3.20) allows us to extend it to the more general case where a set of generation buses is used to compensate the loss coverage by the slack bus. Let the set of compensating buses be R. Let the generator (load) at bus $k \in R$ inject Δp_k to provide equivalent loss compensation for $a_k \Delta p_0$ MW, with $0 \le a_k \le 1$ and $\sum_{k \in R} a_k = 1$. Since superposition applies to the small signal model,

$$\sum_{k \in R} \frac{\Delta p_k}{z_k} = \sum_{k \in R} a_k \Delta p_0 = \Delta p_0 \tag{3.22}$$

Next, let us consider the special case where $\Delta p_0 = l_a^{(m)}$. The general problem becomes: if transaction m wants to self-acquire at bus k the equivalent compensation for the losses $l_a^{(m)}$ allocated to it, how many MW of power does it need to inject at bus k? Since $l_a^{(m)}$ is a small quantity compared with the total generation on the system, the small signal analysis of Equation (3.17) holds for this case. The equivalent loss compensation at bus k is

$$\Delta p_k = z_k l_a^{(m)} \tag{3.23}$$

If a compensating bus k is selected from the set $S^{(m)}$, the seller of transaction m needs to inject an additional Δp_k MW above the amount $s_k^{(m)} t^{(m)}$. If the selected compensating bus k is from the set $B^{(m)}$, then Δp_k is the effective reduction in the delivered power $b_k^{(m)} t^{(m)}$ at the buying bus k. This particular choice is, in fact, the compensation scheme proposed for the multilateral trade structure in [13]. Thus, that scheme is a specific case of the general compensation framework developed here.

More generally, the compensation can be exercised at a set of buses R. Suppose that transaction m selects to compensate $a_k l_a^{(m)}$ by the generator at bus k in the set. From Equation (3.23) we have

$$\Delta p_k = z_k \quad a_k \quad l_a^{(m)}, \;\; k \in R \tag{3.24}$$

Note the choice of a_k is under the complete control of each transaction.

The self-acquisition of loss compensation is flexible and can be tailored to satisfy each individual transaction's needs. However, some transactions might lack the capability to self-acquire real power injection, or the compensation from the third-party might not be realized due to some constraints such as those caused by transmission that are not under the control of the transaction. On the other hand, the IGO with sufficient information about the system may be in a better position to provide the loss compensation service to those transactions. We consider this option in the following subsection.

3.3.2 IGO-acquired loss compensation

We define a subset $\tilde{M} \subset M$ to consist of all transactions that choose to purchase the loss compensation services from the IGO. Then the transactions $m \in \tilde{M}^c$ choose to acquire loss compensation on their own. The self-acquisition of loss compensation may be carried out along the lines presented above. The IGO may, however, provide loss compensation as a value-added service to the transactions. To do so, the IGO may solicit generation supplies or load reductions from any player capable and willing to do so. A player who has generation at bus *k* may offer an incremental price of c_k \$/MWh for each MW increase in output for an hour. A player who has a demand at bus k' may offer a decremental price $c_{k'}$ \$/MWh for each MW decrease in the load for an hour. Let G be the set of players that offer to provide the incremental or decremental power to the IGO. Let the maximum capacity that the player at bus k is willing/capable to provide be Δp_k^{max} MW. Without any loss of generality, the incremental/decremental cost is assumed constant for the interval $[0, \Delta p_k^{max}]$. The IGO uses the bids of the players to determine the least-price loss compensation acquisition for the transactions of the subset $\tilde{M}$. Under the assumptions introduced, the determination of the optimal compensation levels at each bus $k \in G$ for the transactions in $\tilde{M}$ is established from the solution of a linear programming (LP) problem. The least-price loss compensation acquisition is the one in which the total costs incurred by the IGO for acquiring the compensation service are minimized while all the necessary physical limits are satisfied.

The decision variables are $\Delta p_k^{(m)}$, $k \in G$, $m \in \tilde{M}$, representing the amount of compensation acquired from the player at bus k for transaction m. The objective function is

$$\min \sum_{k \in G} c_k \sum_{m \in \tilde{M}} \Delta p_k^{(m)} \tag{3.25}$$

The constraints we are considering include the following:

- **Loss coverage allocation**: For each transaction $m \in \tilde{M}$, the sum of the compensation acquired from the buses $k \in G$ covers the losses allocated to it, $l_a^{(m)}$; from Equation (3.28) we have

$$\sum_{k \in G} \frac{\Delta p_k^{(m)}}{z_k} = l_a^{(m)}, \; m \in \tilde{M} \tag{3.26}$$

- **Compensation limit**: The sum of the compensation supplied by the player at bus k for the transactions of $\tilde{M}$ cannot be larger than the maximum capacity that it is willing/able to provide

$$\sum_{m \in \tilde{M}} \Delta p_k^{(m)} \le \Delta p_k^{max}, \; k \in G \tag{3.27}$$

- **Transmission line limits**: The additional injections or load reductions for loss compensation cannot violate the transmission line limits:

$$\underline{F}\,\Delta\underline{p} \le \underline{f}^{max} \tag{3.28}$$

where $\Delta\underline{p}$ is the $|\mathcal{G}|$-dimensional vector with components

$$\Delta p_k = \sum_{m \in \tilde{\mathcal{M}}} \Delta p_k^{(m)},\ \ k \in \mathcal{G} \tag{3.29}$$

Let H be the total number of lines so that $\underline{F}$ is the $H \times |\mathcal{G}|$ matrix of the generation shift factors [50]. The component F_{hk} represents the sensitivity of the line flow on the transmission line h with respect to the injection at bus k, and $\underline{f}^{max}$ is the H-vector with components f_h^{max}, where the latter is the unused line capacity of transmission line h above the line flows caused by the existing transactions and the loss compensation of transactions $m \in \tilde{\mathcal{M}}^{\,c}$.

- **Nonnegativity compensation**: By definition, the loss compensation is nonnegative:

$$\Delta p_k^{(m)} \ge 0,\ \ k \in \mathcal{G},\, m \in \tilde{\mathcal{M}} \tag{3.30}$$

The LP formulation of the least-price loss compensation problem is given then by

$$\min\ Z = \sum_{k \in R} c_k \left[\sum_{m \in \tilde{\mathcal{M}}} \Delta p_k^{(m)} \right]$$

$$\begin{cases} \sum_{k \in R} \dfrac{\Delta p_k^{(m)}}{z_k} = l_a^{(m)},\ m \in \tilde{\mathcal{M}} & \leftrightarrow r^{(m)} \\ \sum_{m \in \tilde{\mathcal{M}}} \Delta p_k^{(m)} \le \Delta p_k^{max},\ k \in \mathcal{G} & \leftrightarrow t_k \\ \underline{F}\,\Delta\underline{p} \le \underline{f}^{max} & \leftrightarrow h_h \\ \Delta p_k^{(m)} \ge 0,\quad k \in \mathcal{G}, m \in \tilde{\mathcal{M}} & \end{cases} \tag{3.31}$$

Let us denote by $r^{(m)}, t_k \geq 0,$ and $h_h \geq 0$ the dual variables associated with constraints in (3.26), (3.27), and (3.28), respectively. Let $\Delta p_k^{*(m)}$ be the optimal solution of the LP problem and let the optimal dual variables be $r^{*(m)}$, t_k^*, and h_h^*. Then, $r^{*(m)}$ has an important economic interpretation: it is the sensitivity of the total costs of loss compensation paid by the IGO with respect to the losses allocated to transaction m. This sensitivity information can be very useful in that it provides to the IGO a basis for pricing the compensation service supplied to transaction m. It follows that the IGO's costs for providing the loss compensation service to transaction m are $r^{*(m)} l_a^{(m)}$. From the complementary slackness condition at the optimal solution, we can obtain

$$\left[\frac{r^{*(m)}}{z_k} + t_k^* + \sum_{h=1}^{H} f_{hk} \quad h_h^* - c_k\right] \Delta p_k^{*(m)} = 0, \ \forall \ k \in G, \ \forall \ m \in \tilde{M} \qquad (3.32)$$

For each bus $k \in G$, we define a set $M_k \triangleq \{m : \Delta p_k^{*(m)} > 0\} \subset \tilde{M}$, i.e., the subset of transactions for which loss compensation is acquired from bus k. Then, it follows from Equation (3.31) that

$$r^{*(m)} = \left[c_k - t_k^* - \sum_{h=1}^{H} f_{hk} \quad h_h^*\right] z_k, \ m \in M_k \qquad (3.33)$$

The right-hand side of Equation (3.33) is independent of m so that $r^{*(m)} = r^*$ for each $m \in M_k$. In other words, whenever a player provides loss compensation for more than one transaction, the sensitivity of the total charges for loss compensation with respect to each of those transactions is identical. For the important special case when there exists a certain bus k such that $\Delta p_k^{*(m)} > 0, \ m \in \tilde{M}$, then all the sensitivities are identical, i.e.,

$$r^{*(m)} = r^*, \;\; m \in \tilde{M} \tag{3.34}$$

This is a physically intuitive result because the problem we are solving involves the compensation of losses at a single bus: the slack bus. The impact of a 1-MW increase in the losses allocated to any one transaction on the total compensation payment is clearly independent of the transaction. Therefore, the IGO incurs a payment of $r^{*(m)} = r^*$ (\$/MWh) which is identical for all the transactions m.

The IGO can provide this least-price loss compensation service to the interested transactions at cost or at slightly above cost. This option may allow certain transactions to cover their allocated losses at a lower cost than they otherwise would incur.

The loss compensation on a self-acquisition basis and that provided by the IGO can easily coexist. The transactions may undertake self-acquisition, or use the IGO least-price compensation acquisition, or any mixture of the two.

3.4 Numerical Results

We have tested the proposed loss allocation and compensation schemes extensively on a number of different systems including the IEEE 57-, 118-, 300-bus systems. For each system, we use the generation/load data of the system to construct a set of transactions. The transactions of these three systems are given in Appendix E. Our numerical work indicates not only that the scheme is effective in providing an allocation of the losses, but also that the allocation behaves in a physically meaningful and appropriate manner in all cases studied. The numerical results indicate that the self-acquisition and the IGO-provided compensation mechanisms developed in our work are effective and provide good flexibility in arranging for the loss compensation service in a multiple-transaction framework.

3.4.1 Real loss allocation

We provide below a representative sample of the studies carried out to test the allocation scheme. We discuss first the results of our investigations of the overall performance of the estimate given by the approximation formula in Equation (3.10) of the total system losses under different operating conditions. For each of Test Systems A, B, and C, we vary uniformly and simultaneously the amount of each transaction using a scaling factor $0.75 \le h \le 1.25$. The case with $h = 1$ is the reference case. The performance of the $\hat{l}$ approximation for Test System B is shown in Figure 3.1. For the selected range of h, the error magnitude of the approximation is under 16%. It is interesting to note that in this range of h, the total system losses l may be approximated by a linear function of the total volume of the transactions. The approximation $\hat{l}$ tracks

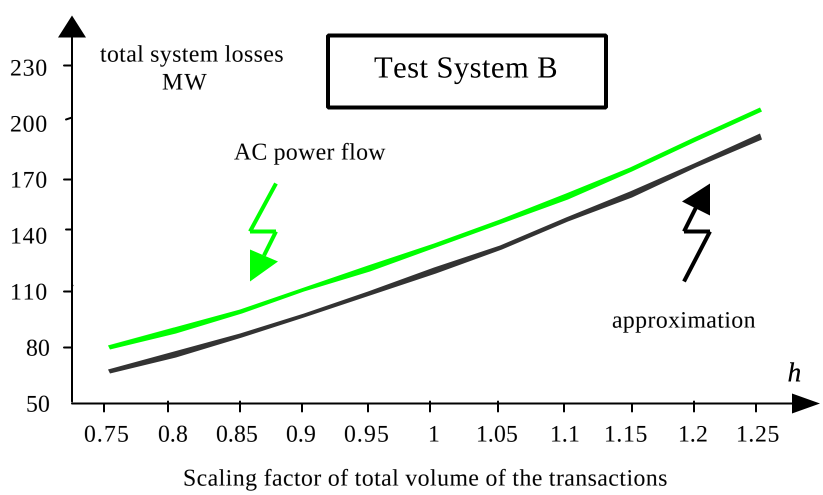


Figure 3.1. Comparison of the total system losses computed by AC power flow and the approximation formula in Equation (3.10) for Test System B

closely l over the range. Such close tracking was observed in all cases studied for the various test systems.

We next illustrate how the allocation scheme evaluates losses associated with specified transactions. We give as an example Test System A with four transactions. We consider the case where no counter flow results from the four specified transactions in the system. The allocation mechanism produces allocations that behave in a physically meaningful way. As the amount of each transaction increases with all other transactions remaining fixed, the corresponding loss allocation also increases. Figure 3.2 provides a plot of the behavior of the total system losses approximation and the loss allocation as a function of the amount of one of the four transactions. The amount of transaction 3 is varied around its base case value within the range of $\pm 25\%$ of the base case value. The base case corresponds to the value 0. The result shown in Figure 3.2 is representative of the behavior of the allocation mechanism on systems where transactions produce no counter flows.

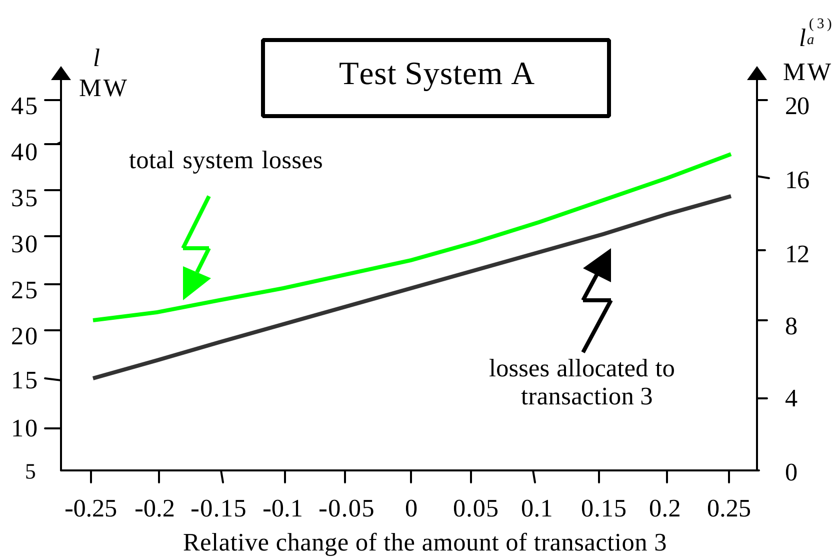


Figure 3.2 Loss allocation with no counter flow in Test System A

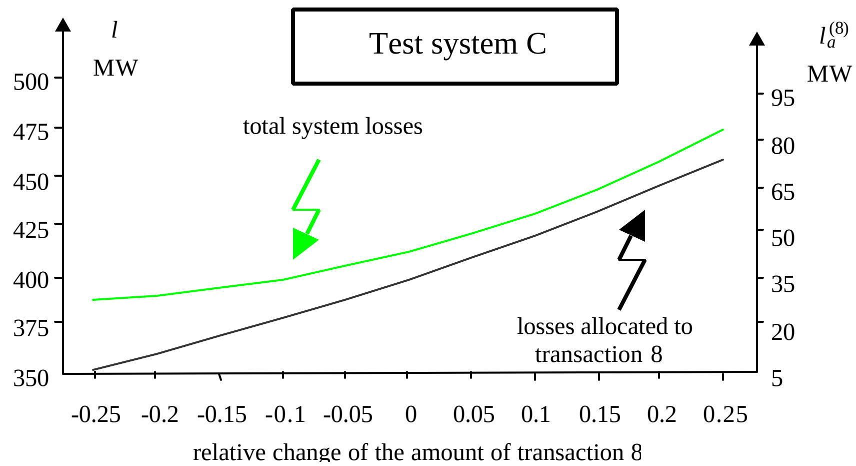
We next illustrate the capability of the allocation scheme to evaluate losses associated with transactions in the presence of counter flow. Consider Test System C with the given 13 transactions. Each transaction may involve multiple generation sources and multiple load centers. Table 3.1 summarizes the transaction profiles and the loss allocation results. The total amount of the thirteen transactions is 23,247 MW. The total system losses evaluated by A.C. power flow are 411 MW, and the approximation $\hat{l}$ gives a value of 367 MW.

Table 3.1 Transaction profile and allocation results in MW for Test System C

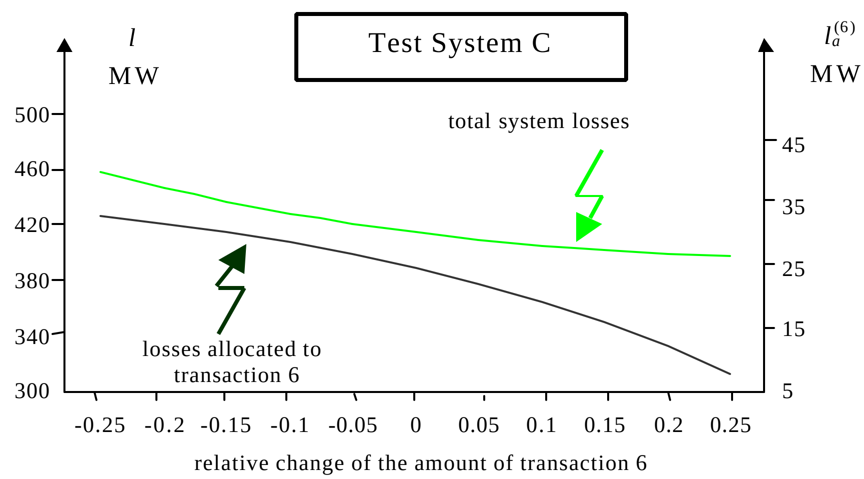
m	1	2	3	4	5	6	7	8	9	10	11	12	13
$t^{(m)}$	2130	1850	2271	1142	1973	1000	623	1200	983	925	4935	3353	862
$\hat{l}_a^{(m)}$	10.69	43.72	121.6	2.74	89.75	-49.2	7.35	90.38	62.11	17.25	-111	57.93	23.38
$l_a^{(m)}$	5.70	23.33	64.89	1.46	47.89	26.23	3.92	48.23	33.14	9.20	59.28	30.91	12.47

We focus on transactions 8 and 6 to illustrate the allocation mechanism when a particular transaction results in counter flow. The amount of transaction 8 is changed around its base value with all the other transactions being kept fixed. We investigate the impact on the total system losses. Figure 3.3(a) shows that as the amount of transaction 8 increases, the total system losses also increase. The same is true for $l_a^{(8)}$, the losses allocated to transaction 8. If, on the other hand, we vary the amount of transaction 6 as all other transactions are kept fixed, we obtain the plots in Figure 3.3(b). These plots show that both the system losses and $l_a^{(6)}$, the losses allocated to transaction 6, actually decrease as the amount of transaction 6 increases. The results indicate that transaction 8 produces a dominant flow while transaction 6 results in a counter flow. The plots in

Figure 3.3 show that while the total system losses move in the opposite direction as the amounts of transactions 6 and 8 increase, the losses allocated to each transaction capture appropriately the impacts of the transactions on the system. In both cases, the scheme gives a physically reasonable loss allocation.



(a) dominant flow



(b) counter flow

Figure 3.3 Variation of the total system losses and loss allocation as a function of the transaction amount for Test System C

Moreover, let us consider the case when transaction 8 is canceled. With the absence of transaction 8, we vary the amount of transaction 6. Figure 3.4 shows that in the absence of the dominant flow created by transaction 8, the previous counter flow caused by transaction 6 becomes a dominant flow. Correspondingly, the system losses increase as transaction 6 increases. The allocation scheme once again captures the appropriate movement of the system losses as a function of the amount of the transaction and gives an appropriate loss allocation to transaction 6.

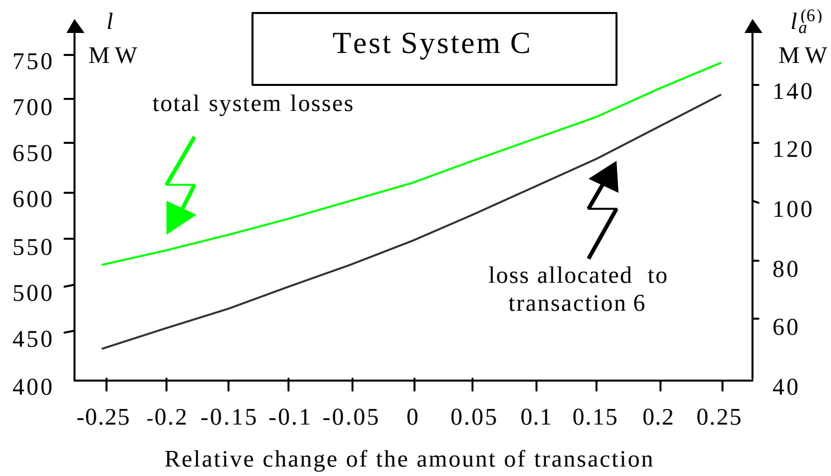


Figure 3.4 Plots of the total system losses and loss allocation as a function of the amount of transaction 6 for Test System C

We next use an example of Test System A to illustrate the property in Equation (3.12). Suppose that a broker is considering arranging a trade that buys 40 MW of the power produced at bus 3 and sells to the load at bus 16. For some reason, the broker is unable to arrange a direct transaction from the selling bus 3 to the buying bus 16. Instead, he sets up two transactions using the intermediate bus 12. He thus replaces the single transaction $\{40,3,16\}$ by the two transactions $\{40,3,12\}$ and $\{40,12,16\}$. Since these two transactions involve different entities, they are considered as two independent transactions. According to our allocation scheme, the sum of losses allocated to these two transactions is always no less than the losses allocated to the direct transaction from bus

3 to bus 16. The losses allocated to the single transaction $\{40, 3, 16\}$ are 0.34 MW; the losses allocated to the two transactions $\{40, 3, 12\}$ and $\{40, 12, 16\}$ are 0.62 MW and 0.25 MW, respectively. Clearly, the sum of the losses allocated to the transactions $\{40, 3, 12\}$ and $\{40, 12, 16\}$ is larger than the transaction $\{40, 3, 16\}$.

Finally, we discuss the property of the allocation scheme that allows the evaluation of loss allocation for a specific subset of transactions. Suppose that in the example of Test System C discussed above, we are interested in the evaluation of loss allocation for the transactions 1, 2, 3, 4, 5 and 6. The use of Equations (3.10) and (3.11) provides results identical to those in Table 3.1 without the use of information on the unspecified transactions.

3.4.2 Real loss compensation

We have tested the compensation scheme on all the test systems for which we developed loss allocation. For illustrative purposes, we limit our attention here to Test Systems B and C.

We first focus on the loss allocation results summarized in Table 3.2. This table also reports the data for loss compensation. Transactions 2 and 3 use self-acquisition for their loss compensation. The other four transactions make use of the least-price loss compensation service acquired by the IGO. The locations, compensating capacities, and prices of the players that bid to provide loss compensation services are shown in the table.

The self-acquisition option is exercised in a straightforward manner by a transaction as shown in the table. The IGO-acquired least-price loss compensation scheme for the remaining four transactions is the solution of the LP problem formulated with the specified data. Since at the optimal solution, the IGO acquires loss compensation for each of the transactions from bus 46, the sensitivity of the total costs to the IGO to a change in the loss allocated to one transaction, i.e., the marginal cost r^* to the IGO of the compensation it acquires, is uniform and is equal to \$21.8/MWh. If, on the other hand, these four transactions were able to obtain their loss compensation at the designated slack bus, bus 69, their costs would be \$25/MWh. This rate is over 14% higher than the costs under the IGO acquisition. In fact, however, compensation at the slack bus is not available since it is willing and/or able to provide no more than 45 MW for the purposes of loss compensation while 89 MW are required to be injected into the slack bus. Note that at the optimal solution, none of the line limits is active.

Next, we investigate the use of the IGO-acquisition by all six of the transactions, including transactions 2 and 3 for Test System B. The comparison of the IGO's least-

Table 3.2 The loss compensation of Test System B

Tran. m	$l_a^{(m)}$ (MW)	Compen-sation Choice	Compensating bus (MW)								
			54	87	46	89	66	69 (Slack)	49	61	Total Costs (\$)
2	19.28	SELF			(40%) 7.7		(60%) 11.6				535.1
3	10.14	SELF								(100%) 11.6	294
1	43.43	IGO	19.33	7.97	9.40	4.73					946.77
4	11.25	IGO	0	4.36	5.41	1.31					245.25
5	12.29	IGO	0	4.70	5.79	1.64					267.92
6	22.92	IGO	0.67	7.97	9.40	4.73					499.66
IGO's Total Compensation			20	25	20	12.41					1959.6
Compensating Bus Data		z_k	0.91	1.03	0.93	1.09	0.99	1	0.95	0.99	
		c_k (\$/MWh)	12.5	15	19	20	22.5	25	27	30	
		$\Delta p_k^{\max}$ (MW)	20	25	30	60	35	45	50	50	

price loss compensation service, obtained from the solution of the new LP problem with the participation of the two additional transactions, and the self-acquisition results is shown in Table 3.3. The solution of the LP is such that r^* remains unchanged, leading to significant savings for each transaction.

For Test System C, we consider the IGO-acquired loss compensation for all but transactions 1, 4, 7, and 13, which acquire the loss compensation service on their own. The focus of our study is on the effect of the transmission constraints of the IGO-acquired least-price compensation. At the optimal solution of the LP with given values for the line limits, no limit is binding. The results are summarized in Table 3.4 with the results being the first entries in the upper part of the table. Since bus 7166 provides nonzero compensation for all the participating transactions, r^* is uniform and its value is \$22.8/MWh. We next resolve the LP with a 10% reduction on the line flow capacity of the line between buses 7049 and 49. This line limit becomes binding, and the results are shown as the second entries in the upper part of Table 3.4. We compare the IGO-acquired compensation solutions in this case with the reference case (no decrease). The marginal compensation cost r^* increases from \$22.8 to \$23.3/MWh. The results also indicate that, because of the active line limit, the compensation from bus 147 has to be replaced by the more expensive compensation at bus 7071, resulting in a 2% increase in the total compensation costs.

Table 3.3 Comparison of self-acquisition and least-price loss compensation in Test System C

Transaction	Self-acquisition	IGO-acquisition	% above IGO-acquisition
2	535.1	420.3	27.3
3	294	221	33.0

Table 3.4 The loss compensation of Test System C

Tran. m	$l_a^{(m)}$ (MW)	Compensating bus (MW) (without/ with 10% reduction in the line limit)									
		125	7002	7166	176	147	7049 (Slack)	7071	185	227	Total Cost ($)
2	23.3	0/0.2	0.2/0.5	19.5/19.3	4.6/4.3						533/543
3	64.9	0/0.9	0/1.1	15.2/19.9	0/4.9	50/16.3		0.7/19.1			1481/1511
5	47.9	1.0/4.2	1.3/4.0	20.5/22.7	5.6/7.7	20.7/0		0/8.5			1093/1152
6	26.2	0.7/1.0	0.9/1.2	20.3/19.9	5.3/5.0						599/611
8	48.2	6.8/4.2	6.4/4.0	25.6/22.7	10.5/7.7			0/8.8			1100/1123
9	33.1	2.6/2.9	2.7/2.9	21.9/21.6	6.9/6.6						756/772
10	9.2					9.3/9.3					210/214
11	59.3	6.8/4.2	6.4/4.0	25.6/22.7	10.6/7.7			9.0/17.9			1353/1380
12	30.9	2.0/2.3	2.1/2.3	21.4/21.1	6.4/6.1						706/720
IGO's Total Compensation		20/20	20/20	170/170	50/50	80/25.6		9.7/54.4			7989/7831
Compen-sating Bus Data	Z_k	0.91	1.02	1.04	1.06	1.01	1	0.83	0.95	0.87	
	c_k ($/MWh)	15	18	20	20	22.5	25	27.5	35	40	
	$\Delta p_k^{\max}$ (MW)	20	20	170	50	80	100	150	200	250	

A second study considered the impacts of four arbitrarily chosen lines whose limits are continuously reduced simultaneously. Figure 3.5 shows that the IGO's least-price acquisition becomes increasingly more expensive when the line limits of these four lines become restrictive and the system becomes increasingly more constrained. This illustrates the capability of the IGO-acquired procedure to adapt effectively to the physical constraints of the network.

3.5 Conclusions

We have used the comprehensive multiple-transaction network framework constructed in Chapter 2 as the basis to develop allocation and compensation mechanisms

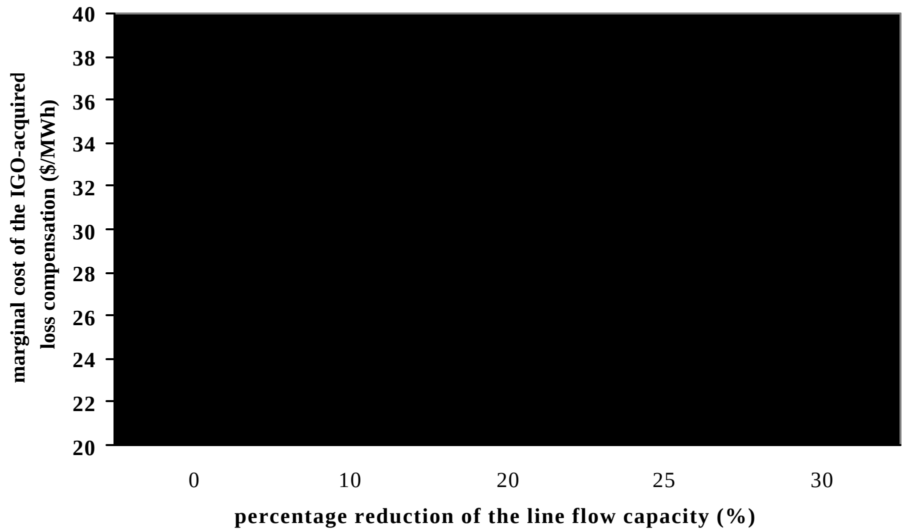


Figure 3.5 The impact of the line flow capacity on the least-price IGO-acquired loss compensation for Test System C

for losses in multiple-transaction networks in this chapter. The physical-flow-based allocation scheme attributes the total system losses to each transaction in the network in a manner that is physically plausible. In particular, the scheme is capable of appropriately dealing with the issue of counter flows. We have used the loss allocation results to develop the equivalent loss compensation concept and applied it to construct flexible and efficient loss compensation procedures for compensating losses in a multiple-transaction network. The proposed procedures provide transactions with the choice of selecting self-acquisition of loss compensation at designated bus(es) or purchasing the loss compensation service from the IGO. The IGO provides loss compensation as a value-added service to transaction by acquiring the service in the least-cost manner. IGO-acquisition of loss compensation uses a linear program formulation in which network constraints are explicitly represented. The self-acquisition service may coexist side-by-side with the IGO-acquisition, and any physically feasible combination of these acquisition schemes is possible. The capabilities and effectiveness of the loss allocation and compensation schemes for multiple transaction networks have been extensively tested and illustrated on a number of test systems.