CHAPTER 2

THE MULTIPLE-TRANSACTION NETWORK FRAMEWORK

2.1 Introduction

In this chapter we develop the general multiple-transaction network framework which we use as the basis for the formulation of the various problems treated in the thesis. This framework explicitly represents the multiple transactions undertaken simultaneously on the transmission grid.

We consider an electric network in which the sellers and buyers of electricity independently arrange bilateral power transactions with each other according to their own interests. We term the schedule resulting from the proposed transactions as the *preferred schedule*. We construct a comprehensive framework for analysis of the multiple transaction systems. We define formally a real power transaction and recast the steady-state AC power flows in the transmission system explicitly in terms of transactions. In addition, we state explicitly the assumptions under which we formulate the approximations used in the subsequent analysis. The resulting framework is then the mathematical basis for the analytical development of the subsequent chapters.

2.2 A Multiple-Transaction Network

We consider a system of $N+1$ buses with bus 0 being designated as the slack bus. Each load acts as a buyer to get its demands met through transactions with one or more sellers. Similarly, each generator acts as a seller and undertakes transactions with one or more buyers. Let M denote the set of the proposed transactions. A bilateral transaction is characterized by specifying the seller, the buyer, and the amount of real power. Formally, we define a bilateral transaction as a set of selling buses (injection buses) supplying a specified amount of real power to a set of buying buses (withdrawal buses). We adopt the following notation for a transaction $m \in M$:

$$T^{(m)} = \left\{ t^{(m)}, S^{(m)}, B^{(m)} \right\} \tag{2.1}$$

The elements of this triplet are $t^{(m)}$, the transaction amount in megawatts and $S^{(m)}$ $(B^{(m)})$, the selling (buying) entities. The set $S^{(m)}$ is the collection of 2-tuples

$$S^{(m)} = \left\{ (s_i^{(m)}, \, s_i^{(m)}), \, i = 1,2,...N_s^{(m)} \right\} \tag{2.2}$$

with the selling bus $s_i^{(m)}$ supplying $s_i^{(m)} t^{(m)}$ MW of the transaction amount. The fraction $s_i^{(m)}$ must satisfy the conditions $\sum_{i=1}^{N_s^{(m)}} s_i^{(m)} = 1$ with $s_i^{(m)} \in [\,0, 1\,]$, $i = 1,2,...N_s^{(m)}$. Similarly, $B^{(m)}$ is the collection of 2-tuples

$$B^{(m)} = \left\{ (b_j^{(m)}, \, b_j^{(m)}), \; j = 1,2,...N_b^{(m)} \right\} \tag{2.3}$$

where the buying bus $b_j^{(m)}$ receives $b_j^{(m)} t^{(m)}$ MW of the transaction amount. The fraction $b_j^{(m)}$ must satisfy the conditions $\sum_{j=1}^{N_b^{(m)}} b_j^{(m)} = 1$ with $b_j^{(m)} \in [0,1]$, $j = 1,2,...N_b^{(m)}$.

In our definition of a bilateral transaction m, $t^{(m)}$ MW is injected at the $N_s^{(m)}$ selling buses and $t^{(m)}$ MW is withdrawn at the $N_b^{(m)}$ buying buses. This formulation is very general and includes the situation in which a transaction may involve a marketer who does not own any generation or any load. For instance, a marketer at bus j may buy t MW in one or more transactions and sell the t MW in one or more other transaction(s).

The net real power injection at bus j caused by all the buy and sell transactions is zero. A transaction may be also considered as the outcome of the market run by an entity such as a power exchange (PX) whose role is to determine a set of buyers and sellers in forward electricity markets. In this case, $t^{(m)}$, $S^{(m)}$ and $B^{(m)}$ are determined by the aggregate supply and demand curves bid by the participants in the PX [49]. We assume that the losses associated with each transaction are compensated at the system designated slack bus.

For each transaction m, we construct an injection vector $\underline{p}^{(m)}$ with components

$$p_n^{(m)} = d_n^{(m)} t^{(m)},\ n = 0,1,2,\ldots N \tag{2.4}$$

where, the components of $\underline{d}^{(m)}$ are

$$d_n^{(m)} = \begin{cases} s_i^{(m)} & \text{if } n = s_i^{(m)},\ i = 1,2,\ldots N_s^{(m)} \\ -b_j^{(m)} & \text{if } n = b_j^{(m)},\ j = 1,2,\ldots N_b^{(m)} \\ 0 & \text{otherwise} \end{cases} \tag{2.5}$$

Then, for the system, the net real power injection at bus $n = 1,2,\ldots N$ is given by the nth component of the N-dimensional vector $\underline{p}^{net}$ with

$$p_n^{net} = \sum_{m \in M} p_n^{(m)} = \sum_{m \in M} d_n^{(m)} t^{(m)} \tag{2.6}$$

Note that by definition $\underline{p}^{net}$ does not have a component at bus 0 even though the slack bus 0 may be involved in transactions as a selling bus.

We consider the p -model for any transmission line between buses i and j. We characterize the line with the series impedance $r_{ij} + jx_{ij}$. We denote by b_{ij} the total shunt susceptance. The matrix $\underline{G} + j\underline{B}$ is the $(N+1)\times(N+1)$ bus admittance matrix with elements $G_{ij} + jB_{ij}$.

The real power flow equations at each bus except bus 0 are stated explicitly in terms of $t^{(m)}, m \in M$:

$$\sum_{m \in M} d_n^{(m)} t^{(m)} = G_{nn} V_n^2 + V_n \sum_{k \in H_n} V_k [G_{nk} \cos(q_n - q_k) + B_{nk} \sin(q_n - q_k)], \quad n = 1,2,...N \quad (2.7)$$

where q_n and V_n are the angle and voltage magnitude at bus n, $n = 1,2,...N$. At the slack bus 0, q_0 and V_0 are set to their specified value. The set H_n is the set of buses that are directly connected to bus n, $n = 0,1,2,...N$.

Note that Equation (2.6) expresses a linear transformation from the transaction space to the space of net bus injections. Since multiple combinations of transactions may result in identical net injections, the transformation is not one-to-one and so may not be invertible.

Typically, transactions involve only real power. A load bus may, however, have both real and reactive loads. While a load gets its real loads met by participating in one or more transactions, its reactive loads are served in other ways which need not be directly related to the transactions in place. Under our convention, there is no need for area control since, by definition, the net physical flow on the tie lines is equal to the net interchange specified for the inter-area transactions in effect.

In order to allow us to focus on the reactive power support by the generators for the transactions on the system, we adopt the assumption that there exist sufficient local reactive power resources to supply the reactive loads. Consequently, we assume that each load in the network has a unity power factor.

We denote by Q the set of buses at which there are generators that can provide reactive support. Note that this set Q may include the slack bus. We assume that for the proposed transactions in M, the IGO determines that the voltage setting point V_k^s of the generator at bus $k \in Q$ and the voltage profiles for buses $j \in Q^c$. The generator at bus k is responsible for supplying the necessary reactive support to maintain the specified voltage profile. We complete the formulation of the framework by stating the reactive power balance equations from which we derive the expression for the reactive power support requirements of the transmission system. Under the assumption of no reactive loads, the reactive power equation at bus $j \in Q^c$

$$V_j \sum_{i \in H_j} V_i [G_{ji} \sin(q_j - q_i) - B_{ji} \cos(q_j - q_i)] - B_{jj} V_j^2 = 0, \quad j \in Q^c \tag{2.8}$$

The unknown components of the system state $q_n, n = 1,2,...,N$ and $V_n, n \in Q^c$ are determined from solving Equations (2.7) and (2.8). The reactive support is considered to be provided by the generators. At each generator bus $k \in Q$, the net reactive power outflow is the sum of the reactive power injected into the lines connected to bus k:

$$Q_k^g = \sum_{j \in H_k} Q_{kj} \tag{2.9}$$

where

$$Q_{kj} = -\frac{b_{kj}}{2}(V_k^s)^2 + \frac{1}{r_{kj}^2 + x_{kj}^2}\left\{x_{kj}[(V_k^s)^2 - V_k^s V_j \cos(q_k - q_j)] - r_{kj} V_k^s V_j \sin(q_k - q_j)\right\} \tag{2.10}$$

Special Case: The definition of the bilateral transaction reduces to a simple node pair transaction when the transaction is between a single selling bus and a single buying bus. Then, for transaction m, $S^{(m)} = \{ s^{(m)}, 1 \}$ and $B^{(m)} = \{ b^{(m)}, 1 \}$, and Equation (2.1) is written more simply as

$$T^{(m)} = \left\{ t^{(m)}, s^{(m)}, b^{(m)} \right\} \tag{2.11}$$

It follows that the injection vector components $p_n^{(m)} = d_n^{(m)} t^{(m)}$, $n = 0, 1, 2, ..., N$ are

$$d_n^{(m)} = \begin{cases} 1 & \text{if } n = s^{(m)} \\ -1 & \text{if } n = b^{(m)} \\ 0 & \text{otherwise} \end{cases} \tag{2.12}$$

Whenever the seller and the buyer are at the same bus, i.e., $s^{(m)} = b^{(m)}$, $\underline{d}^{(m)} = \underline{0}$. Such a transaction is assumed to have no impact on and not be impacted by the transmission system.

2.3 DC Power Flow Approximations

In this section, we formulate the assumptions under which the DC power flow approximations used in the subsequent analysis are performed and set up the basis for the mathematical development of the future chapters.

In the development of the subsequent chapters, we introduce a number of approximations of the real power flows. The main assumptions under which these approximations are performed are that the DC power flow conditions hold:

- Reactive power flows maintain bus voltage magnitude close to 1.0 p,u., i.e.,

  $V_n \approx 1.0,\ n = 1,2,...,N.$ (2.13)

- The bus voltage angle difference across any branch is small so that

  $\left| q_i - q_j \right| \approx 0,\ i, j = 0,1,2,...,N, i \neq j.$ (2.14)

These assumptions are generally acceptable for the interconnected bulk transmission networks under the normal operating conditions.

The DC power flow results that are derived within the multiple-transaction framework under these assumptions in Equations (2.12) and (2.13) provide us with the basis for the mathematical development in the approximations. We denote by $\underline{\hat{q}} = [\hat{q}_1, \hat{q}_2, ..., \hat{q}_N]^T$ the voltage angle vector computed by the DC power flow. Let $\underline{\hat{B}}$ be the $N \times N$ submatrix of the $(N+1)$ node network susceptance matrix $\underline{B}$ [50]. Without any loss of generality we may set $\hat{q}_0$ at the slack bus to be 0. The DC power flow formulation of the transaction-based network then is

$$\underline{\hat{B}}\,\underline{\hat{q}} = -\underline{P}^{net} = -\sum_{m \in M} \underline{d}^{(m)} t^{(m)} \tag{2.15}$$

Let

$$\underline{D} = [d_{ij}] = \underline{\hat{B}}^{-1} \tag{2.16}$$

Then,

$$\hat{q}_n = -\sum_{m \in M} \sum_{k=1}^{N} d_{nk} d_k^{(m)} t^{(m)},\ n = 1,2,...N \tag{2.17}$$

Let us rewrite Equation (2.16) as

$$\hat{q}_n = \sum_{m \in M} m_n^{(m)} t^{(m)},\ n = 1,2,...N \tag{2.18}$$

where we define, for $n = 1,2,...N$,

$$m_n^{(m)} = -\sum_{k=1}^{N} d_{nk} d_k^{(m)} = \sum_{j=1}^{N_b^{(m)}} d_{nb_j^{(m)}} b_j^{(m)} - \sum_{i=1}^{N_s^{(m)}} d_{ns_i^{(m)}} s_i^{(m)} \tag{2.19}$$

For completeness, we define $m_0^{(m)} = 0,\ m \in M$, $d_{n0} = 0,\ n = 1,2,...N$. So we can write

$$\hat{q}_i - \hat{q}_j = \sum_{m \in M} p_{ij}^{(m)} t^{(m)},\ i, j = 0, 1, 2, ..., N,\ i \neq j. \tag{2.20}$$

with

$$p_{ij}^{(m)} = m_i^{(m)} - m_j^{(m)},\ m \in M,\ i,\ j = 0, 1, \ldots, N \qquad (2.21)$$

2.4 Conclusions

In this chapter, we have introduced the definition of a transaction and developed the comprehensive framework for networks in which multiple transactions are presented simultaneously. The framework recasts real power flows explicitly in terms of transactions and formulates reactive power flows under the assumption of no local reactive power loads. We have also formulated the DC power flow approximations. These approximation results are used in the analytic development of the allocation and management schemes of various unbundled services treated in the thesis.