

## The Integration of *PHEV* Aggregations into a Power System with Wind Resources

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### Abstract

The rising energy independence and environmental concerns are key drivers in the growing popularity of plug-in hybrid cars (*PHEVs*). Studies indicate that for 90 % of the Americans who use their cars to get to work every day, the daily commute distance is less than 50 km – or 30 miles – and, on the average, the commuter car remains parked is about 22 hours per day. All the *PHEVs* have in common the batteries, which provide storage capability that can be effectively harnessed when the vehicles are integrated into the grid. Such storage capability can be used to effectively integrate wind power into the grid. By nature, wind power is intermittent which raises many challenges for the grid operator. The utilization of the storage from the *PHEVs* enables the power system operator to smooth out the output of the wind farms by storing energy when the wind power output is too high and releasing such energy when the power output is too low. We develop a probabilistic model to take into account the effect of the variability in the *PHEV* owner behavior and the fact that the *PHEVs* are not always plugged into the grid. The numerical studies show the positive levelization impact *PHEVs* can have on wind power operations when grouped in aggregations of large size.

### I - Introduction

There are growing concerns around the world about energy independence and global warming issues. In the USA, energy independence is a major political issue due to the fact that the nation imports two thirds of the oil it consumes – virtually, all the fuel consumed for transportation purposes [1]. The strong dependence on foreign sources to satisfy this so-called “oil addiction”, together with the growing awareness of global warming impacts that CO<sub>2</sub> emissions produce, are key drivers for the development of new transportation technologies. Such technologies aim to drastically reduce the need for oil by making the vehicles more fuel-efficient and by turning to alternate sources of energy. In particular, the development of the plug-in hybrid vehicles (*PHEVs*) is directly addressing these issues. A common characteristic of all *PHEVs* is the requirement for a battery, which is the source of part of the energy required for propulsion. Car manufacturers have heeded the call for the generation of

new vehicles and are currently designing new products. The tremendous success of the *Toyota Prius* has been a motivating factor for car manufacturers in pushing out the development of *PHEVs*. The various activities underway are likely to lead to a massive deployment of *PHEVs* over the next few years. The growth in *PHEVs* creates both a new load class for charging the batteries and a new resource that can be harnessed from the effective integration of *PHEVs* into today’s grid. The nation’s top energy regulator, *FERC*, recently communicated on the essential need to integrate plug-in hybrid vehicles into the national power grid [2].

The fast *PHEV* developments are accompanied by the rapid implementation of wind farms which are being integrated into the power grid. A key challenge in the integration of wind farms is the management of the high variability and intermittency of wind power output in light of the volatility of wind speed. Typically, power system operations raise the system reserves levels to handle the variability and intermittency effects. Certainly, energy storage can mitigate these effects but the limited amount of existing storage and the high costs of new storage devices are major deterrents to the wider deployment of storage. However, with the deeper integration of *PHEVs* into the grid, a new source of storage will be available for use by the power system operator. Indeed, given the nature and physical characteristics of *PHEVs*, their integration into the grid is performed at the distribution voltage level. Such an interconnection allows each *PHEV* to be plugged into the grid to obtain the energy to charge the battery. The *PHEVs*, when aggregated in sizeable numbers, constitute a new load that the electricity system must supply. On the other hand, a *PHEV* can be much more than just a simple load given that bidirectional power flows are possible once the integration is made. Indeed, the integration of *PHEVs* allows their deployment as a storage device during certain periods of time. As each individual *PHEV* represents only “noise” to the power system, the *PHEVs* need to be aggregated into large collections whose combined impacts are tangible for the grid. Such aggregations can range from a few thousands to a few hundreds of thousand *PHEVs* that are grouped together by an aggregator. We discuss in [3] in considerable detail the nature and role of such aggregations and their impacts. In this paper, we investigate the deployment of *PHEV* aggregations to

facilitate the integration of wind power into the grid. We show that the integration of a *PHEV* aggregation into the grid with an interconnected wind farm can be effectively harnessed to levelize the wind farm output. The flattened power output of a wind farm becomes much easier to manage in system operations as the variability and the uncertainty in the wind farm power output is considerably reduced.

The role that *PHEVs* can play in the smoothing of the wind farm output has been evoked earlier [4] and the impacts of the *PHEV* storage have been assessed in [5]. The studies cited provide results under several possible scenarios without accounting for either the uncertainty inherent in wind speeds or in the variability in the *PHEV* owner behavior and the vehicle usage. Indeed, the various results are derived under rather unrealistic assumptions such as the *PHEVs* being always plugged into the grid and that the around-the-clock ability of the *PHEV* batteries to absorb or release power as needed. The quantification of the potential benefits of *PHEVs* in the studies does provide some insights but the evaluation under more realistic conditions is needed to better assess the synergies between *PHEVs* and wind resources.

This paper directly assesses such a need by explicitly representing the various sources of uncertainty in the deployment of *PHEVs*. We take into account the variability in the behavior of *PHEV* owners by developing a probabilistic framework to model the times at which the *PHEVs* are plugged into the grid as well as the charge status of each battery in the aggregation. We also develop a procedure to levelize the output of a given wind farm using *PHEV* aggregations while ensuring that the individual *PHEV* battery characteristics are explicitly considered. We make use of our procedure to show that the power output of a wind farm can be levelized over a longer period through the use of the *PHEVs*. We illustrate with a realistic case study that uses historical wind data and a large *PHEV* aggregation. The results indicate that effective *PHEV* integration can harness the synergism with wind resources and bring about economic and environmental benefits.

This paper consists of four additional sections. We develop a model to represent the behavior of the *PHEV* owners as well as the state of charge of the *PHEV* batteries. Then, we described the proposed approach for the deployment of *PHEV* aggregations in a grid with integrated wind resources. We present our numerical studies and results in section IV and summarize the thrust of the paper in section V.

## II – The *PHEV* aggregation and its behavior

We must keep in mind that the principal utility of a *PHEV* is to provide clean and economic transportation to its owner rather than to facilitate electricity grid operations. As a result, the *PHEVs* in an aggregation may not always be plugged into the grid. Since each *PHEV* may travel different distances every day, its battery energy storage may differ from that of the other *PHEVs* at the time it interconnects with the distribution network. The time required for commute travel impacts the participation level at which each *PHEV* contributes to the services provided to the grid. We analyze the nature and impacts of the sources of uncertainty and construct an appropriate model of the *PHEV* aggregation under a set of reasonable assumptions.

Specifically, we include the following sources of uncertainty:

- the duration of the periods during which each *PHEV* in the aggregation is connected to the grid;
- the commute distances of each *PHEV*; and,
- the energy storage in each *PHEV* battery at each point in time.

Rather than using the energy stored in a *PHEV* battery as a variable, we use the *s.o.c.* – ratio of the energy stored to the maximum storage capability of the battery – as the figure of merit.

To simplify our analysis, we introduce the following assumptions:

- losses in the *PHEV* batteries are negligibly small;
- the storage capability of each *PHEV* battery remains unchanged during the study period;
- *PHEVs* are used for commuting purposes only;
- each *PHEV* owner goes to work every day and parks the vehicle for some period of time with the vehicle connected to the electricity system; and,
- *PHEV* owners do not change their *PHEVs* during the time of the study.

These assumptions are reasonable for the studies undertaken on the *PHEVs* that are expected to be made available within the next few years. Typically, the losses due to conversion efficiency are small – less than 10%. Since *PHEVs* cannot be used for long distances, we consider that their principal use is for commuting.

For the analysis of uncertainty, we introduce the following assumptions to allow its characterization:

- each *PHEV* in the aggregation has the efficiency  $\eta$ ;
- the behavior of each *PHEV* owner is independent of that of any other *PHEV* owner; and,
- each battery is independent of every other battery in the aggregation.

The duration of the study period is of the order of a few days. We adopt a resolution commensurate with the needs of the study. For those purposes, a granularity of one minute is adequate. We assume that each *PHEV* battery can provide its maximum output instantaneously.

We next consider a collection of  $B$  *PHEVs* and use the set  $\mathcal{B} \triangleq \{i : i = 1, 2, \dots, B\}$  to denote the index set of the aggregation. A study period has  $J$  days and we denote by the set  $\mathcal{G} = \{j : j = 1, \dots, J\}$  the index set of the days in the study. This set may consist of nonconsecutive days as we limit our consideration to commute days. The commute and departure times and the commute distances of each *PHEV* in the aggregation may vary from day to day in the study period. To model the uncertainty, we introduce random variables or *r.v.s*.

We now discuss the *r.v.s* defined on the set of days of the study period. We introduce the *r.v.*  $\left(\Delta^i\right)_{i \in \mathcal{B}}$  we use to represent the distance traveled by *PHEV*  $i$ . The distances driven in the morning and in the evening may be different since they correspond to two distinct realizations of the *r.v.* as *PHEV* owners may have different trajectories for their forward and return commutes. We denote the *r.v.* corresponding to the time at which *PHEV*  $i$  starts its forward commute by  $\left(T^i\right)_{i \in \mathcal{B}}^{\sim f,d}$ . We also define the commuting time *r.v.*  $\left(C^i\right)_{i \in \mathcal{B}}$  for *PHEV*  $i$ . Then, we introduce *r.v.*  $\left(G^i\right)_{i \in \mathcal{B}}$  corresponding to the duration of the period during which *PHEV*  $i$  remains parked during time away from home. Under our assumptions, the *r.v.s*  $\left(T^i\right)_{i \in \mathcal{B}}^{\sim f,d}$ ,  $\left(C^i\right)_{i \in \mathcal{B}}$  and  $\left(G^i\right)_{i \in \mathcal{B}}$  are independent *r.v.s*. We can compute the *r.v.*  $\left(T^i\right)_{i \in \mathcal{B}}^{\sim f,a}$  corresponding to the time at which *PHEV*  $i$  completes its forward commute, by summing the time at which the *PHEV* departs for its morning commute and of its commuting time:

$$T^i_{\sim f,a} = T^i_{\sim f,d} + C^i, i \in \mathcal{B}.$$

We can also compute the *r.v.*  $\left(T^i\right)_{i \in \mathcal{B}}^{\sim r,d}$ , corresponding to the time at which *PHEV*  $i$  starts its return commute by

$$T^i_{\sim r,d} = T^i_{\sim f,a} + G^i, i \in \mathcal{B}.$$

The time of completion of the return commute is given by the *r.v.*  $\left(T^i\right)_{i \in \mathcal{B}}^{\sim r,a}$  defined by

$$T^i_{\sim r,a} = T^i_{\sim r,d} + C^i, i \in \mathcal{B}.$$

To take into account the limits on the values that the *r.v.s* can take on, we make use of the so-called truncated normal *r.v.s*. Such *r.v.s* are said to be characterized by the truncated normal distribution obtained from the normal distribution by accounting for the *r.v.* being bounded either below, or above, or both. The derivation of the probability density function (p.d.f) and cumulative distribution function (c.d.f) of a truncated normal distribution is summarized in the Appendix. We develop the truncated normal distribution for the *r.v.s*  $T^i_{\sim f,d}$ ,  $G^i$ ,  $C^i$  and  $\Delta^i$ .

Furthermore, because we consider a large aggregation of *PHEVs*, we can apply the Central Limit Theorem to compute the c.d.f. of the storage of the aggregation as a normal *r.v.* whose parameters are determined from the first two moments of the storage of each *PHEV*, which we represent as the *r.v.*  $\tilde{K}$  [6]. Again, we compute for the aggregation storage *r.v.* the truncated normal *r.v.* c.d.f. under the assumption that each storage capability is in the 0 – 30 *kWh* range.

We define the *s.o.c.* *r.v.*  $\left(S^i\right)_{i \in \mathcal{B}, t}$  for *PHEV*  $i$  to characterize the ratio of the amount of energy stored in the *PHEV* battery at any point in time  $t$  to the storage capability of the battery. Indeed, the *s.o.c.* of each *PHEV* battery is uncertain as it depends on the distances traveled by the *PHEV* and on the use made of the *PHEV* battery as a load or a generation/storage device. The *s.o.c.* decreases when the *PHEV* is being driven or when energy is withdrawn by the Aggregator. It increases when the *PHEV* is being charged. We compute such *r.v.* using the energy withdrawn from each *PHEV* battery to undertake its trips as well as the energy that gets absorbed by the battery when the *PHEV* is charged.

We discuss in the next section how to integrate *PHEV* aggregation with wind resources.

### III – The deployment of a *PHEV* aggregation in a system with integrated wind resources

The output of a wind plant, unlike that of conventional power plants, cannot be controlled. The output depends directly on the wind speed at the location. A wind turbine generates power as long as the wind speed is higher than the cut-in wind speed but lower than the cut-out wind speed. We give in Fig. 1 the shape of a typical wind power curve for a given turbine.

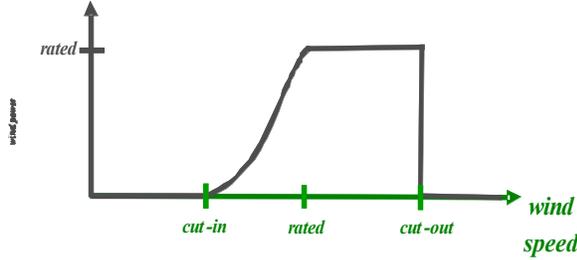


Fig. 1. Typical wind power curve

When the wind does not blow, no power is produced. Zero output represents a major challenge to the power system operator whenever the system needs power and no other plant is available to supply the demand.

We develop a simple adaptive procedure to levelize the power output of a wind farm over a period of several hours to several days through the effective deployment of *PHEV* aggregations. For this procedure, we need the monitored values of the wind farm output, the set  $\mathcal{B}$  of aggregated *PHEVs* and their monitored status and *s.o.c.* values. The desired power output level of the wind farm  $P^s$  is specified. The procedure samples the monitored variables at each time step  $t$  using a uniform step size  $\Delta t$  between two consecutive samples. The basic idea is to use the *PHEV* aggregation as either a resource or a load depending on the relationship of the monitored  $P_t$  wind output at time  $t$  vis-à-vis the  $P^s$  value. Whenever  $P_t > P^s$ , we identify the subset of *PHEVs* in  $\mathcal{B}$  characterized by an *s.o.c.*  $< 0.9$ . The charging continues until the *s.o.c.* reaches 1.0 or the excess output over the desired level reduces to 0 within the specified tolerance. However, when each *PHEV* battery in  $\mathcal{B}$  has *s.o.c.*  $> 0.9$ , the ability to provide load by the aggregation becomes highly limited. We stop the charging action and increase the desired wind output level to a reduced value  $P^{s'} > P^s$ .

Analogously, whenever  $P_t < P^s$ , we identify the *PHEVs* which have a *s.o.c.* high enough to allow for

energy to be taken from the battery and still provide the ability to undertake the return commute trip after partial discharge of their batteries. The discharge continues until the *s.o.c.* falls below the specified value or the need for output reduces to 0 within the specified tolerance. However, when no *PHEV* has an adequate *s.o.c.*, we stop the discharge action and decrease the desired wind output level to a lower value  $P^{s''} < P^s$ .

This simple procedure has direct application in both planning and operational studies. The determination of the highest levelized output from a given wind farm attainable with a given collection of *PHEVs* is particularly useful in wind farm investment analysis of a wind farm for assessing the synergies from the integration of a *PHEV* fleet. Such a result can help power system planning tremendously by assessing the capacity of the system to integrate wind farms. In operations, the procedure provides the basis for the aggregator to control the charging/discharging of the *PHEV* batteries in a synergistic manner with the wind power production so as to levelize the power output.

We provide in the next section the simulation results of the deployment of this procedure to demonstrate the effective deployment of a *PHEV* aggregation to provide synergism to a grid with the integrated wind power resources.

### IV – Simulation results

We illustrate how to effectively harness the synergism in the deployment of a *PHEV* aggregation and a power system into which a wind farm is integrated. To make the illustration realistic, we use historical wind speed data for the power production and make use of relevant data in the public domain for the *PHEV* representation. Specifically, we consider a single parking lot where the aggregation of *PHEVs* is connected to the distribution network. We make use of historical data for office parking from the city of Livermore, CA [7]. We use the truncated normal representation and compare to the actual data. We note the remarkably close fit of the model output to the historical data, as shown in Fig. 2.

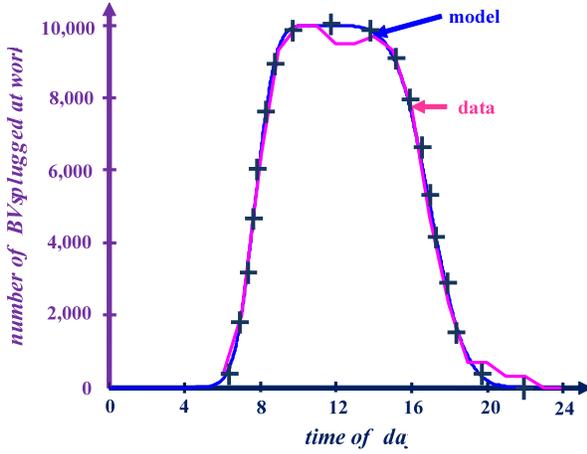


Fig. 2. Number of useable *PHEVs* in a parking lot as a function of time

We observe that the model fits the actual data well, except maybe for the lunch break during which some *PHEV* owners use their cars to go to lunch. Indeed, lunch breaks have not been taken into account in the formulation of our model. Taking such events into account can be done by specifying distinct subperiods during which the *PHEV* is plugged in. However, for the sake of simplicity, such a modification was not performed in this study.

Typically, only a small fraction of the aggregation is used for transportation at any point in time. We represent in Fig. 3 the total number of *PHEVs* plugged into the grid at any point in time for  $B = 10,000$ .

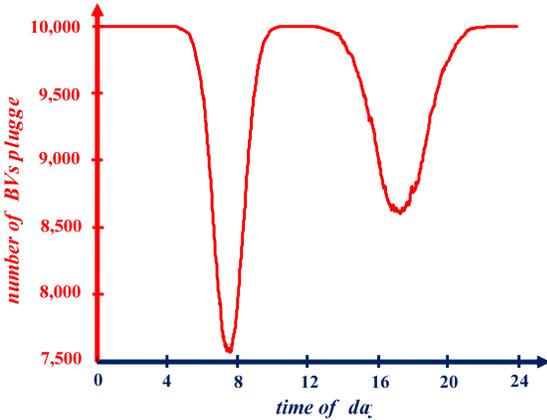


Fig. 3. Total number of useable *PHEVs* for  $B = 10,000$  for a typical day

The total number of useable *PHEVs* is higher in the evening than in the morning because people leave for work over a wide interval of times. Additionally, the total

number of useable *PHEVs* remains pretty high even during the typical commuting times. The explanation for such a phenomenon is that people have relatively short commuting times on average – around 30 min – and leave at different starting times in the morning. This is a key finding as it shows that there are always more than 50 % of the *PHEVs* in the aggregation plugged in at any point in time

We can quantify the number of *PHEVs* plugged at any point in time. Though the goal of the paper is not to determine this quantification, we bring to the attention of the reader that for a *PHEV* aggregation, the particular behavior of an individual *PHEV* owner has no effect on the overall group as heterogeneity impacts are smoothed in the aggregation. Indeed, this important effect is key to understanding the synergistic relationship between a *PHEV* aggregation and the power output of a wind farm. In the examples discussed below, we make explicit use of the *PHEVs* as both a resource and a load. Moreover, we take advantage of the fact that, typically, only a fraction of the aggregated *PHEVs* contributes to the resource- and demand- side services at any point in time.

For the study, we consider a system into which a 56 MW-wind farm is interconnected and the aggregated set  $\mathcal{B}$  of *PHEVs*. The wind farm has three types of turbines whose characteristics are given in Table 1.

Table 1. Turbine characteristics of the 56-MW wind farm of the study system

turbine	cut-in speed (m/s)	cut-out speed (m/s)	rated wind speed (m/s)	number
GE 1.5 MW	3.5	20	12.5	15
GE 2.5 MW	3.5	25	12.5	10
Vestas 850 kW	4	25	16	10

We do not model the uncertainty in wind output in this paper; rather, we use historical data for the winter 2009 period at a location in Aberdeen, ID [8]. For the examples discussed here, we use a 5-day period of wind data, as shown in Fig. 4.

We note that the wind speed is highly variable - from nearly 0 to nearly 25 m/s - during the selected week. The wind output for the 5-day period is shown in Fig. 5. The highly intermittent power output is characterized by the presence of subperiods with the power at its rated 56-MW output and of subperiods during which the output power is much smaller, including 0 MW.

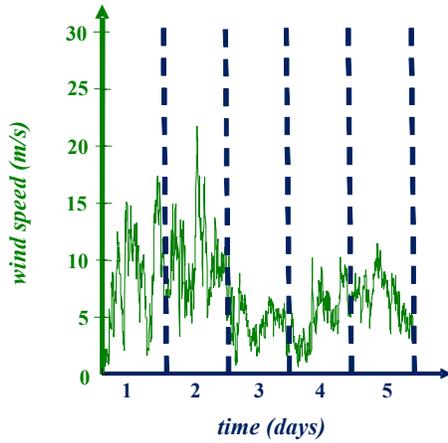


Fig. 4. Wind speed for a 7 day period in Aberdeen, ID

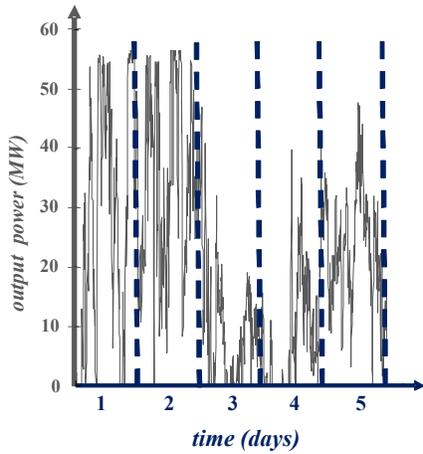


Fig. 5. The Aberdeen, ID wind power of the 56 MW wind farm for the selected 5-day period in the study

For the *PHEV* aggregation, the parameters of the *r.v.s* are given in Table 2.

Table 2. Values of the parameters of the probability distributions

<i>r.v.</i>	mean	standard deviation
	20 kWh	5 kWh
$\tau_{dep}^s$	9 h 20 min	70 min
	7:15 a.m.	50 min
	30 min	15 min
	25 km	12 km

We made judicious use of the available data in the published literature to construct the parameters of the distributions for the departure times, the commute durations and the parking lot durations. Specifically, we use the data collected by the city of Livermore in California [7]. In our simulation, we assume that, on average, a *PHEV* owner starts the commute in the morning at 7:15 a.m. with, on average, a 30 m duration. The *PHEV* remains parked for slightly over 9 h before the return commute to the home begins. The fact that the commute duration can be as large as twice the average duration under bad traffic conditions stems from the standard deviation of 15 m used in the study. For the simulation, we use an average commute distance of 25 km with a standard deviation of 12 km. The distribution reflects that the distances may vary for each *PHEV* and, for a particular *PHEV*, from day to day since some *PHEV* owners may take a different route each day to do some shopping or to avoid traffic jams. For the reference study, we set an aggregation with  $B = 44,000$  *PHEVs*.

We provide the results of a 7-day simulation that we performed on this system with the aim to maximize the leveled power output attainable for the study period and the minimum number of *PHEVs* in the set.

For the wind signal given before, we use a desired power output level of  $P^s = 7$  MW. The simulation indicates that the attainment of such a leveled power output for the wind farm is feasible. The value of the leveled power output is much lower than the maximum capacity of the wind farm for two reasons. The wind power output is highly intermittent, so the wind farm produces electricity at its rated power a smaller percentage of the time as is visible in Fig. 5. In addition, the *PHEVs* in the aggregation use the electricity produced by the wind turbines for their commuting needs and thus the leveled power output is above that used for charging the *PHEVs*. By leveling the wind power output of the wind farm, the *PHEV* aggregation decreases the variability in the “net” output of the wind farm. The importance of the result in this study is the fact that the procedure is dependable even for those periods with wind speeds resulting in zero power output.

We next discuss various sensitivity studies we performed with changes in a number of *PHEV* aggregation parameters. With a smaller *PHEV* aggregation, the amount of storage capability useable by the grid decreases but the total energy needed for the propulsion of the *PHEVs* is also smaller. The effect of decreasing the aggregation by 2,000 *PHEVs* is minimal because of the size of the change with respect to that of the aggregation. However, we can see the effect of the size of the aggregation by considering what happens when the aggregator holds only 20 000 *PHEVs*. The results for

the wind farm power output are depicted in Fig. 6. In this case, we observe two distinct phenomena. For some periods, the output power is higher than the specified output power. The reason is that there is insufficient storage capability from the *PHEV* batteries plugged into the grid to absorb the excess in energy. Either the *PHEV* batteries are full or the aggregated *PHEVs* are not plugged into the grid. At some points in time, the output is below the desired power level of 7 MW. The reason is the insufficient energy stored to provide the ability to maintain the levelized specified output. This lack of storage capability makes the reduced aggregation unable to provide the required “smoothing” service.

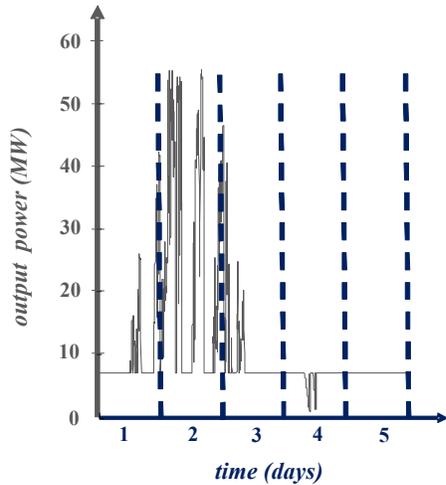


Fig. 6. The lack of storage in the aggregation of 20,000 *PHEVs* is responsible for the inability to levelize the output of the wind farm at 7 MW

We also conduct sensitivity studies on the specification of desired power output. We study a case in which the specified levelized output is higher with the same aggregation of 44,000 *PHEVs* used in the reference case study. An example is provided in Fig. 7 which shows the output for a specified power level of 10 MW. During the period of low wind speeds in day 4, there is insufficient energy stored and insufficient capacity to attain the specified power level. We also note that simply increasing the number of *PHEVs* cannot address this problem. In this case, an increase in the number of *PHEVs* in those hours results in the inability to charge the additional batteries since the energy produced by the wind farm is too small. Therefore, the only feasible solution is to lower the desired power output.

Our simulation studies indicate that the effects of wind power variability can be effectively managed through the deployment of the appropriately sized *PHEV* aggregation. The deeper the penetration of *PHEVs* in the system, the more effective the aggregation is in providing the storage

capability to facilitate the integration of the intermittent energy sources. However, caution needs to be exercised to not construct an overly large aggregation in relation to the rated power output of the wind farm. Another key finding from the simulations is that the variability inherent in the behavior of individual *PHEV* owners has virtually no impact on the overall effectiveness of the aggregation’s ability to smooth the wind farm production volatility. The *PHEV* batteries are very effective in providing load following capability due to the fast response times of their storage devices. The provision of such reserves is an important contribution of *PHEV* aggregations. Furthermore, the batteries of such aggregations can get charged during the night by the wind energy generated using similar control strategies to levelize the off-peak loads on the grid.

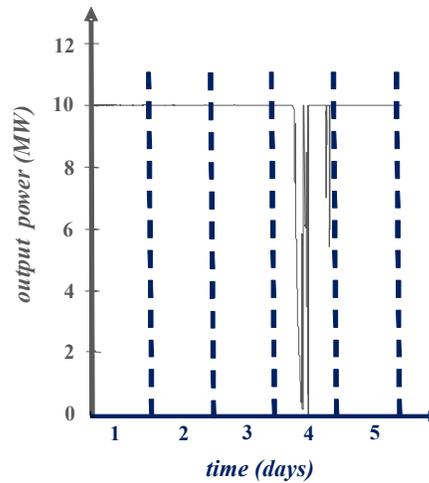


Fig. 7. Wind power output for a 10 MW level and 44,000 *PHEVs*

## V – Concluding remarks

In this paper, we explicitly show the synergism between the integration of *PHEVs* into the grid and wind resources. The deeper penetration of *PHEVs* will greatly benefit the grid because the storage capability can help to effectively manage the problems of wind intermittency by levelizing the output of the wind farms. Indeed, *PHEV* aggregations can provide storage to the grid which enables the power system to smooth out the power output of a wind farm despite the variability inherent in the behavior of the *PHEV* owners, the *PHEV* characteristics and the wind speed. The simple levelization procedure making the flattened output possible takes into account both the characteristics of the wind power generation and that of the *PHEV* owner behavior. A key finding of the paper comes from the modeling of the behavior of the

*PHEV* owners. The studies conducted demonstrated that in a *PHEV* aggregation, the effect of each individual *PHEV* owner cannot impact the support services the overall aggregation provides. Such storage support will also have synergism with other renewable resources, such as solar PV generation.

## VI – References

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## VII – Appendix

Definition: Let  $\tilde{X}$  be a normal variable with mean  $\mu$  and variance  $\sigma^2$

$$\tilde{X} \sim \mathcal{N}(\mu, \sigma^2)$$

which lies within the interval  $(a, b)$

$$\tilde{X} \in (a, b), -\infty < a < b < \infty.$$

Then the random variable  $\tilde{Y}$  defined as  $\tilde{X}$  conditional on  $a < \tilde{X} < b$  has a truncated normal distribution with probability density function

$$f_{\tilde{Y}}(y; \mu, \sigma; (a, b)) = \begin{cases} \frac{\frac{1}{\sigma} \phi\left(\frac{y-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} & \text{if } y \in (a, b) \\ 0 & \text{if } y > b \text{ or } y < a \end{cases}$$

and cumulative distribution function

$$F_{\tilde{Y}}(y; \mu, \sigma; (a, b)) = \begin{cases} 0 & \text{if } x < a \\ \frac{\Phi\left(\frac{y-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} & \text{if } a < x < b \\ 1 & \text{if } x > b \end{cases}$$

where  $\phi$  is the probability density function of the standard normal distribution  $\mathcal{N}(0, 1)$  and  $\Phi$  its standard cumulative distribution function.

The expression of the cumulative distribution function of a truncated normal variable comes from the use of conditional probabilities.

We have for every  $y \in \mathbb{R}$

$$F_{\tilde{Y}}(y) = F_{\tilde{Y}}(y; \mu, \sigma, a, b) = P\{\tilde{Y} \leq y\} = P\{\tilde{X} \leq y \mid \tilde{X} \in (a, b)\}$$

Thus, we can write

$$F_{\tilde{Y}}(y) = \frac{P\{\tilde{X} \leq y \cap \tilde{X} \in (a, b)\}}{P\{\tilde{X} \in (a, b)\}}$$

And we know that

$$P\{\tilde{X} \in (a, b)\} = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

and we can easily get the expression of the probability of the event

$$\{\tilde{X} \leq x \cap \tilde{X} \in (a, b)\}$$

$$P\{\underline{X} \leq y \cap \underline{X} \in (a,b)\} = \begin{cases} 0 & \text{if } y < a \\ \Phi\left(\frac{y-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right) & \text{if } a < y < b \\ \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right) & \text{if } y > b \end{cases}$$

## VIII – Biographies



**Christophe Guille** received his B.S degree in Electrical and Computer Engineering from Supelec, Paris, France in 2007 and his M.S. degree in Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign as well as his M.S. degree in Electrical and Computer Engineering from Supelec in 2009. His research interests include power system

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In the 1999-2000 academic year, he was invited to be Visiting Professor at a number of Italian institutions including University of Pavia, Politecnico di Milano and Politecnico di Torino.

His undergraduate work was completed at McGill University, and he earned his graduate degrees from the University of California, Berkeley. He was previously employed by Pacific Gas and Electric Company in various technical, policy and management positions. Dr. Gross won the Franz Edelman Management Science Award in 1985 and is the recipient of several prize paper awards. He has consulted widely to organizations in North and South America, Europe and Asia