

# Reactive Support and Voltage Control Service: Key Issues and Challenges

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**Abstract** — This paper reviews the nature and salient characteristics of reactive power support provision and voltage control issues in electricity markets. The major challenges in the provision of such services from the generation units under both normal and emergency conditions are highlighted. The key issue of assessing the value of the reactive support is addressed. Some key problems and challenges in the adoption of market mechanisms for reactive power support services are discussed. This paper serves as an overview of the challenges that need to be resolved in effectively managing and pricing reactive support and voltage control services.

**Keywords** – Competitive Electricity Markets, Reactive Power Support, Voltage Control, Security, Optimal Power Flow, Opportunity Costs, Generator Capability Curve.

## I. INTRODUCTION

Voltage control and reactive power support are essential services for enabling the delivery of power in the transmission grid and for providing adequate reserves margins and appropriate dynamic response capabilities. While various equipment is able to produce and/or absorb reactive power, we focus our attention on the reactive support provided by synchronous generators. Such support depends principally on the set points assigned to the primary and, in some cases, to the secondary and tertiary voltage control systems [1]. These settings are under the discretion of the Independent Grid Operator (IGO)<sup>1</sup> and are typically specified by criteria formulated in an optimal power flow (OPF) framework. In fact, the definition of ancillary services in the U.S. regulatory structure requires reactive support and voltage control to be provided by generators, or possibly, loads [2].

Availability of a sufficient amount of reactive power support depends on the operational limits specified by the capability curves of the generation units. The ability of a generation unit to provide reactive support at a specified bus of the system depends on the “static” reactive power margin at the given operating point, which is a function of the distance to the boundary of the unit capability curve and on the capability of transporting this reactive

power margin via the transmission network to the specified bus.

In the competitive electricity market framework, the acquisition of reactive power support could, in principle, be obtained using a market-based mechanism. However, the ability of setting up a workable reactive power market has yet to be proven. While a number of researchers have attempted to construct a conceptual market structure in which the IGO is the buyer and the independent owners of reactive power generation equipment are the sellers along the lines of the markets for the *MWh* commodity, there remain major unresolved issues concerning practical implementation of such a market, given the limited ability of reactive power to travel from a source to a bus requiring the support. Moreover, there is the additional challenge to mitigate the naturally arising opportunity to exercise market power in the provision of reactive support [3],[4].

The fact that a reactive power market is not set up in no way reduces the scope and difficulty of the problem of pricing reactive power and voltage control services [5],[6],[7]. The costing of reactive power is well understood [8],[9],[10]. However, this is far from true for reactive power pricing due to the numerous technical and economic challenges that arise [5]. The recent FERC Staff Report [11] points out that price signals and incentives to encourage investments for reactive support from market participants, including those on the demand side, are weak and ineffective and represent a major need. While some payment structures have already been adopted in certain jurisdictions [11],[12], major issues need to be addressed in the pricing and management of reactive support services.

This paper reviews the key issues and provides an assessment of the challenges faced in the area of reactive power services. We start out with a brief overview of the characteristics of the reactive support and voltage control service and discuss the basic costing and pricing issues in Section II. We focus on the value of reactive support in Section III. In Section IV, we provide a discussion of the principal challenges faced in the development reactive power markets. We close with some concluding remarks on the overall state of the art in the field.

<sup>1</sup> The IGO is the entity responsible for the reliable and secure operation of the power system, as well as for providing non-discriminatory transmission services to all transmission customers.

## II. REACTIVE POWER SUPPORT CHARACTERISTICS

We review in this section the most characteristic attributes of reactive power and discuss the economic considerations in the costing/pricing of the reactive support and voltage control services. We start out with the consideration of the physical attributes and constraints arising from the capability curve considerations and then turn to the economic assessment.

We consider the *capability curve* representing the ability of a generator to simultaneously produce real power and generate/absorb reactive power. The boundary of the feasible operating region of the generator is formed by the intersection of four physical limiting relationships: the minimum loading, the field current, the armature current and the under excitation of the generator [13]. A possible partitioning of the area contained by the generator capability curve into three regions to represent specific operation regimes of the generator has been proposed [14]. The *three-region* model consists of:

- the *obligation to serve* region within the capability curve area delimited by the regulatorily mandated constraints, such as lead/lag power factor or reactive power limits, under which service is provided;
- the *boundary* region specified by the capability curve with the operation of the generator at its reactive power limits;
- the *remaining* region in the area contained by the capability curve and not belonging to either of the two regions above.

Operation in the *obligation to serve* region is not eligible for any additional payment. The operation in the *boundary* region may receive payment to compensate for the reduction in the active power generation so as to allow the required change in the reactive power. Such a change incurs a loss of opportunity to generate real power and should be, therefore, eligible for opportunity cost payments for this loss [8]. The payment for operation at a point located in the remaining region compensates for the costs of additional losses incurred. However, the entire notion of defining the *obligation to serve* region brings about numerous difficulties in actual implementation. In the competitive environment, generators will find very challenging to satisfy the requirements of operating within a reactive power band defined in terms of the power factor limits specified by the regulator and to do so without compensation. Moreover, the geographic location within the network of the generators, which provide the required reactive support, may result in generator outputs that cannot maintain the reactive power within the *obligation to serve* region for each generator. Such a situation leads to unequal treatment of generators providing reactive support since those operating outside the *obligation to serve* region get compensated while the ones within the *obligation to serve* region receive no payment. As the control settings are at the discretion of the IGO, such situations can be highly problematic in the decentralized decision making environment in open access regimes.

A further complication arises from the need to consider the constraints of the step-up transformer associated with

the generator, any required compensation devices and the local transmission facilities. For this purpose, the capability curve is replaced by the *capability chart* of the entire local subsystem [15],[16]. The deployment of the capability chart has repercussions on the obligation to serve requirements. Similar issues also arise in the interconnection of new generation resources due to the lack of specificity in the interconnection standards since the side of the step-up transformer the power factor is measured is not necessarily specified [11].

We next turn to consider the cost components of reactive power provision. The two basic components are the fixed and the variable costs. The fixed or investment costs are independent of the quantity produced and are not addressed here since the decision of installing generation is outside the scope of the problems considered in this paper. Our focus is on the variable costs of reactive power generation/absorption. As long as a generator operates within the limits of its generation capability curve, the costs associated with the additional losses, due to the current arising from the *vars* provided, are negligibly small with respect to those for real power generation. Yet, once the reactive power limits are reached in actual operations, the only way to satisfy the reactive power support requirements is to curtail a portion of the active power generation, resulting in forgone profits representing the lost opportunity costs [8],[9]. These costs are determined from the value of the opportunity the generator forfeits to provide the reactive power support required from the system. Since the lost opportunity costs are of the same order of magnitude as the profits of the generator, they represent the dominant component of the cost structure [8]. Note, however, the calculation of the lost opportunity costs is based on the generator capability curves, that may be proprietary seller information and that the IGO may not have.

## III. VALUE OF THE REACTIVE SUPPORT

In the competitive market environment, the value of a service may have no direct relationship to its actual costs. Consequently, the assessment of the *value* of reactive support under normal or emergency conditions is a key necessity in the management of reactive support services. Since reactive power cannot travel far from its source, its value depends, to a great extent, on the proximity of the source to the location requiring reactive support. The value of reactive support under normal operating conditions was studied by constructing a set of “value curves” for comparing the *var* support from different sources [17] and by setting up a procedure for the calculation of the minimum amount of reactive power needed by a generator to transmit its own real power [18]. This evaluation takes into account the geographic unit location and its reactive power provision satisfies the concept of “minimum *var* needs” of a generator. If a generator produces more *vars* than this minimum, it receives remuneration, but in the converse condition it requires reactive support from the network and must pay for such support. The elements of this approach are consistent with the principle of rewarding the providers

of reactive power and charging those who benefit from the reactive power provision [11]. The operating security and economic impacts are important determinants of the value of reactive power delivered to a bus requiring reactive support. This value must reflect the worth of reactive resource available at a generation node and the ability of the network to effect its delivery to the bus in question. As long as a generator operates within the limits of its generation capability curve or in the case where, if a capability limit is attained, the network voltage profile satisfies the security considerations/constraints, there are two very significant indicators of the value of the reactive resource available at a generation node. The first, concerning the security of operation, can be determined from the unit capability to provide its reactive support under both normal (base case) and disturbance conditions. On the basis of this consideration, the value of the reactive power resource available at the generation bus depends not only on the *Mvar* distance from the actual operating point to the capability limit, but to a greater extent on its capability to ensure a secure operation under disturbances caused by load variations or line and generator outages. Among the possible solutions that satisfy security considerations, the IGO may pursue the objective of reducing as much as possible the transmission losses by means of adequate dispatch of the reactive resources. The assessment of the marginal value, from the point of view of the transmission losses ( $MW/Mvar$ ) and cost reduction ( $$/Mvarh$ ), of a reactive resource available in a network bus require some sensitivity information that can be determined, for a given loading and generation pattern, using a conventional *Optimal Reactive Power Flow (ORPF)*. This kind of sensitivity information is represented in the indices proposed in the hierarchical voltage control or *HVC* scheme with three control levels – primary, secondary and tertiary – that were used to investigate the impacts on the definition of regional reactive power support [19]. These indices provide measures of the impacts of the *HVC* on the reactive power quantities produced in each area and the marginal values of a reactive power injection at a load or generation bus [20]. Such indices can provide useful indications to the IGO for assessing the value of the reactive power support.

We use the small test system example shown in Fig. 1 to illustrate the dependence of the valuation of the security and economic impacts of the reactive support on the geographic location of a generator for different network loading levels. This longitudinal 10-bus network consists of three distinct and non-overlapping areas  $\mathcal{A}_1$ ,  $\mathcal{A}_2$  and  $\mathcal{A}_3$ .  $\mathcal{A}_1$  is an exporting area,  $\mathcal{A}_2$  is an area with demand-supply balance and  $\mathcal{A}_3$  is an importing area. The generators in the three areas have the same nominal voltage, rated power of 370 *MVA* and capability curve and are connected to the transmission network operating at the nominal voltage of 220 *kV* via step-up transformers with short circuit reactance of 0.12 *p.u.* At the base case operating point,  $\mathcal{A}_1$  has a load of 20 *MW* and  $\mathcal{A}_2$ 's load is 200 *MW*. These loads are supplied through the corresponding OLTC transformers. The two loads of  $\mathcal{A}_3$  are 200 *MW* each and are served at two different buses through OLTC

transformers. The two loads have the same power factor of 0.95 lagging. The generators  $G_2$  and  $G_3$  of  $\mathcal{A}_2$  and  $\mathcal{A}_3$ , respectively, are supplied by 200 *MW* each, while the generator  $G_1$  serves the local load and sells 200 *MW* to the  $\mathcal{A}_3$  loads. The reactive power dispatch and the tap setting of the OLTC transformers are performed by an *ORPF* program. The three overhead 220 *kV* transmission lines, each 100 *km* long, have inductance of 1 *mH/km*, resistance of 0.06  $\Omega/km$  and capacitance of 10 *nF/km*. The evaluation of the reactive support provided by the three generators is performed simulating a load ramp in the system, starting at the base case operating values. The loads are increased proportionally to their values at the base case while maintaining the same power factor. The loading level  $\chi$  is the unitless ratio of the total load in the modified state operation to the total base case load. In the test system example, we assume that the real power generations are assigned on the basis of the proportional share of the total load among the three units characterized by identical input-output and capability curves.

For different load and generation patterns, Table I gives the minimal losses in percentage of the served load and the benefits in loss reduction  $\beta$ , evaluated in  $$/Mvarh$ , obtainable by the injection of an additional *Mvar* at the nominal voltage of the network side of each generation bus;  $\beta$  is evaluated under the assumption of an average energy costs of 50  $$/MWh$ . A continuation power flow program adapted for the assessment of the voltage collapse distance provides the generator reaction  $\rho$  to the load ramping used in the system, expressed in *Mvar/MW*. At the base case point the benefit  $\beta$  due to  $G_3$  is almost twice that due to  $G_2$  and four times that due to  $G_1$ . Similar ratios are evident in the index  $\rho$ . This ratio increases with the system load ramping, that drives to the saturation of the generator  $G_3$  capability for a loading level  $\chi = 1.23$  and to the voltage collapse for  $\chi$  greater than 1.27. Clearly, a modification of the real power dispatch, consisting in unloading unit  $G_3$  before reaching the capability limit and changing more the outputs of units  $G_1$  and  $G_2$ , impacts the  $G_3$  reactive service support and delays the onset of voltage collapse. No rescheduling of the units, as it happens in most of the operating conditions, is considered in this example, so that the above results do not include any lost opportunity costs as are incurred in the example in [8].

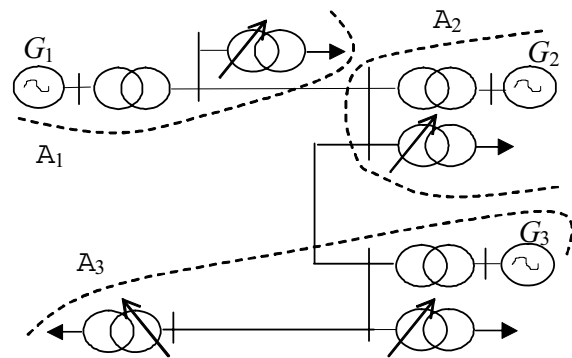


Fig. 1. The 10-bus test system.

TABLE I  
MARGINAL VALUES OF THE REACTIVE POWER SUPPORT

loading level $\chi$	losses (% load)	$\rho_{G_1}$ (Mvar/MW)	$\rho_{G_2}$ (Mvar/MW)	$\rho_{G_3}$ (Mvar/MW)	$\beta_{G_1}$ (\$/Mvarh)	$\beta_{G_2}$ (\$/Mvarh)	$\beta_{G_3}$ (\$/Mvarh)
1	2.91	0.114	0.286	0.513	0.115	0.208	0.401
1.05	3.09	0.121	0.300	0.542	0.131	0.238	0.471
1.11	3.27	0.130	0.317	0.579	0.147	0.274	0.555
1.15	3.53	0.140	0.338	0.622	0.168	0.314	0.656
1.21	3.75	0.152	0.362	0.677	0.191	0.363	0.782
1.23	4.00	0.522	1.547	0.000	0.323	0.860	3.062
1.27	4.99	2.280	7.172	0.000	0.486	1.611	7.268

A further way to address the impact of the reactive support has been described in [21] by defining a *reactive support share* that incorporates important aspects such as the geographic location of reactive power support sources, network topology and variations in the *var* support requirements. By varying the load and the generations along predefined directions in the respective parameter spaces, successive operating ranges  $r$  are defined, each of them including the system operating conditions for which the set of generators that do not reach their operating limits is the same.

Starting from an initial operating condition, let's vary the loads and the generations along a predefined direction, until one of the generators reaches its reactive power limit, thus defining the limit condition of the operating range. The reactive support share  $s_k^{(r)}$  of the generator  $k$  at the  $r^{\text{th}}$  operating range is defined as the per cent variation of the reactive power generation of the generator  $k$  from the initial condition to the limit condition, referred to the corresponding sum of variations of all the reactive power generations [21]. The reactive support share of the generator reaching its limit falls to zero. Application of this approach has shown that during the evolution of the system load the share *variations* of the individual generators could be non-monotonic, depending on the location of the generators successively reaching their reactive power capability limits.

The reactive support share concept is linked to the effectiveness of the reactive power support provision. For the test system in the above example, at the loading level  $\chi = 1.23$ , the generator  $G_3$  reaches its reactive power limits at 270 Mvar and the corresponding reactive power generations are shown in Table II. As long as  $G_3$  operates within limits, the three generators belong to the same operating range  $r = 0$ . Their reactive support share referred to the base case and to the specified direction of variation for the system loads and generations has been computed by using the reactive powers of Table II, obtaining  $s_{G_1}^{(0)} = 15.7\%$ ,  $s_{G_2}^{(0)} = 35.7\%$  and  $s_{G_3}^{(0)} = 48.6\%$ . The generator  $G_3$  share is the highest. As such, the generator  $G_3$  is the most effective in the specified conditions, since the provision of reactive support from the other generators (having identical reactive power limits but located at different nodes) to the most critical area is constrained by the transmission network.

TABLE II  
REACTIVE POWER GENERATION AT DIFFERENT LOADING LEVELS

loading level $\chi$	$Q_{G_1}$ (Mvar)	$Q_{G_2}$ (Mvar)	$Q_{G_3}$ (Mvar)
1.00	19.33	97.13	187.97
1.23	45.89	157.37	270.00

#### IV. REACTIVE POWER MARKET ISSUES AND CHALLENGES

We next discuss issues related to the feasibility and practicality of setting up a reactive power support market. A key limitation arises due to the very local nature of reactive power and its highly limited ability to travel in the network. This characteristic drives the need for local sources of *var* support either at a bus requiring such support or at a bus directly connected to it. The possibility of developing a reactive power market, in which the market participants submit their offers to the IGO, has been analyzed in some papers. A structure of the reactive power offers consistent with the three-region capability curve characterization has been formulated in [22], and a uniform auction model for competitively procuring the reactive power service has been presented in [14]. Within this model, reactive power suppliers submit reactive power offers to the system operator. After the collection of the offers, the IGO defines the amount of the reactive power requirement and the compensation due to each supplier. The notion here is to apply in a straightforward way the market structure for the MWh commodity to the Mvar "commodity", disregarding the distinct characteristics of the two commodities. Successively, by noting that using a uniform price in the three-region model for the whole system does not fit the local nature of the reactive power support, in [23] a location-dependent payment scheme has been proposed to recognize the different value of reactive power provided at various locations. An attempt is made to use the concept of *electrical distance* to establish the zones to define voltage control areas and determine different uniform reactive power price for each area.

Even without resolving the possibility of setting up a feasible market, the analyses of possible market power issues indicate that they are of such magnitude, due to the excessively localized markets, as to deter serious consideration of doing so [11]. The generators located in areas with critically acute reactive support requirements may use such opportunities to exercise market power.

Such situations might lead to the reaping of huge profits for support services. In the extreme case, some generators could lie within the compensation regions of their capability curve even close to or at null load, depending on the set points imposed by the IGO, so that they could make profits for providing support requested by the IGO for maintaining the desired voltage profile. This is in contrast to the definition of transmission service that requires the provision of reactive power support to maintain the specified voltage profile even in the absence of any transactions as an *intrinsic* part of the service. This concept has been exploited in [24] for multi-transaction systems by identifying the amounts of reactive power support not allocated to the market transactions. All the above aspects play a key role against the hypothesis of setting up a reactive power market.

#### V. CONCLUDING REMARKS

While considerable work in the area of reactive power support and voltage control in the competitive electricity market framework has taken place, there are still major fundamental problems that need to be effectively addressed to bring about some progress in this important area. While there is widespread agreement on the applicability of opportunity costs in the determination of the dominant cost component of reactive power support, the pricing and acquisition of the reactive power and voltage support services remains a major challenge. An important issue is the need to focus on the *value* as opposed to the costs of the services. In particular, we need to develop pricing that reflects that under emergency conditions, a reactive power source that may be relatively inexpensive is ineffective in providing support at electrically remote locations. Therefore, the development of pricing that explicitly recognizes the *value* of the reactive power support rather than the costs needs attention with the view of constructing schemes that provide a unified framework that address operating under both base case and emergency conditions.

There is no supportive evidence on whether establishing a reactive power market can be effective or even practical. The very nature of reactive support services, the myriad physical constraints and the possibility of widespread market power exercise – to name a representative sample – present formidable challenges for the organization of market structures in reactive services. Moreover, the wide discretion of the IGO in establishing the set points provides possibilities for discriminatory actions with respect to different generators. Such conditions may further exacerbate the potential for market power exercise.

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