Generalized Transmission Scheduling Problem: Scheduling of Nondiscriminatory Transmission Services in the Mixed Pool-Bilateral Systems

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***Abstract*— Nondiscriminatory transmission services are an essential requirement of open access and constitute a basic premise for the smooth functioning of competitive electricity markets. This requirement is particularly critical when there are bilateral transactions coexisting side-by-side with the centralized day-ahead electricity markets in which the so-called pool players participate as buyers and sellers. The nondiscriminatory allocation of transmission services to the pool players and the bilateral transactions is synonymous with the determination of the market outcomes and the provision of transmission services to the bilateral transactions. We propose the formulation of a generalized transmission scheduling problem (*GTSP*) to *simultaneously* accomplish this objective. The proposed *GTSP* formulation not only captures more appropriately the social welfare and market efficiency loss, but also results in more efficient market outcomes than the conventional transmission scheduling problem (*TSP*) formulations in the presence of transmission congestion. In addition, the *GTSP* solutions determine more appropriate locational marginal prices (*LMPs*) and, consequently, *LMP* differences for computing the congestion charges and the payoffs for the financial transmission rights (*FTR*). We assess the capabilities of the proposed *GTSP* formulation and quantify the improvements over conventional *TSP* solutions. These improvements become particularly important as the load served by the bilateral transactions constitute an increasingly large portion of the total load in the system. We illustrate quantifiable benefits of the *GTSP* solutions on a wide range of systems including the IEEE 118-bus network. The representative numerical results in the paper provide persuasive support for the effectiveness of the proposed formulation and the superiority of its solutions over those of the conventional *TSP*.**

***Index Terms*—day-ahead electricity markets, transmission service scheduling, locational marginal prices, congestion management, congestion charges, financial transmission right payoffs.**

I. Introduction

THE advent of open access transmission and the spread of competitive markets in electricity have resulted in the growing prominence of transmission congestion. Congestion occurs whenever the preferred generation/demand schedule requires the provision of transmission services beyond the capability of the transmission system. The presence of congestion introduces unavoidable losses in efficiency so that the benefits foreseen through restructuring may not be fully realized. There is a growing awareness that congestion is a major obstacle to vibrant competitive electricity markets. Therefore, the effective management of congestion is a critically important contributor to the smooth functioning of competitive electricity markets by minimizing impacts on the players and the market performance.

Various congestion management approaches from the *command-and-control* transmission loading relief (*TLR*) scheme used by *NERC* security coordinators [1] to the market-based methods using locational marginal prices (*LMPs*) and other market data [2]-[6] have been proposed. We consider a market-based scheme using *LMP*s and satisfying the basic guidelines in [3]. Congestion management is intimately involved with the centralized day-ahead pool markets run by the independent entity responsible for market operations and for operating and controlling the transmission network. We use the term *independent grid operator* (*IGO*) for this independent entity. The pool players sell [buy] energy directly to [from] the *IGO* by submitting sealed offers [bids] to the *IGO*. These offers [bids] specify the *MW* amounts of power the player is willing to sell [buy] and the per unit minimum [maximum] price it is willing to accept [pay]. The *IGO* determines the *successful* or winning bids and offers by maximizing the social welfare [7],[8]. The *IGO* is implicitly providing the transmission services necessary to deliver the energy traded in these markets. Coexisting with the centralized markets are the *bilateral transactions* [1] for trading energy directly between selling and buying entities. Whenever the two entities in a bilateral transaction are located at different nodes, the consummation of the transaction requires transmission services. One of these entities is responsible for procuring the transmission services from the *IGO* for the particular transaction.

The *IGO* must provide nondiscriminatory transmission services to both the pool players and the bilateral transactions [9]. The determination process of the transmission service provision is modeled by the so-called transmission scheduling problem (*TSP*) [10]-[13]. While the allocation of the transmission services without bilateral transactions has been investigated in the literature [10],[11], the nondiscriminatory trans-

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1 Multilateral transactions may be represented as the linear combination of the bilateral transactions. Therefore, without loss of generality, we assume all the transactions are bilateral in this paper.

mission provision to bilateral transactions and to pool players is not addressed. The consideration of the bilateral transactions entails, typically, the use of a three-step procedure. First, the *IGO* represents the impacts of the bilateral transactions by recalculating the real power flow limits of each line with all the bilateral transactions undertaken. The *IGO*, then, clears the pool market to maximize the total social welfare with the modified transmission constraints considered. As a by-product, the *LMP* at each node is determined. In the third step, the *IGO* computes the bilateral transaction *congestion charges* [2] as the product of the *MW* amount and the *LMP* difference between the withdrawal and the injection nodes of the transaction.

This conventional *TSP* formulation is workable as long as the total *MW* of injections [withdrawals] of the bilateral transactions is small compared to the total *MW* of injections [withdrawals] of the pool players. In other words, the total load served by bilateral transactions is much smaller than the load served in the pool market. In practical situations, however, this condition may not hold [14] and, consequently, the applicability of the conventional *TSP* formulation becomes questionable. Two key issues are of particular interest:

- the violation of the nondiscriminatory transmission service provision due to the sequential decision-making process which may result in assigning different priorities to the pool players than the bilateral transactions; and,
- the appropriateness of the *LMP*s and the resulting congestion charges given that they are determined without explicit representation of the bilateral transactions since the assumption used is that the bilateral transactions have unlimited willingness to pay for the congestion charges.

These issues become particularly acute as the fraction of total load served by bilateral transactions increases.

The objectives of this paper are to propose a general transmission scheduling problem (*GTSP*) formulation in which the transmission services are provided to the bilateral transactions and the pool players on a nondiscriminatory basis and to investigate the *GTSP* solutions. We consider the transmission services for the bilateral transactions as decision variables whose optimal values are determined *simultaneously* with the outcomes of the pool market in the *GTSP*. In this way, the solution of *GTSP* allocates the limited transmission resources to the transmission customers – both bilateral transactions and pool players – who value it most.

An inherent part of the *GTSP* formulation involves the explicit representation of the contribution to the social welfare by each bilateral transaction. By allowing each bilateral transaction to indicate its maximum willingness to pay for the congestion charges when submitting the transmission service requests, we demonstrate the nature of the relationships between the willingness to pay and the contribution to the social welfare of each transaction. The explicit representation of the contribution of each bilateral transaction to the social welfare allows a more appropriate formulation of the problem. The solutions of the *GTSP* have some distinct advantages over those of the conventional *TSP*. In particular, the market outcomes obtained from the *GTSP* results have larger social welfare and smaller market efficiency loss in the presence of congestion. In addition, the *GTSP* determines more appropriate *LMP*s and, consequently, *LMP* differences for computing the congestion charges and the payoffs for the so-called financial transmission rights (*FTR*) [2], [15]-[17] instrument. These aspects are discussed in detail and illustrated with numerical results.

This paper contains five additional sections. We devote section II to describe the proposed *GTSP* formulation. In section III, we explore the property of the *GTSP* solution and the resulting pricing information. In section IV, we provide an analytical assessment of the *GTSP* formulation and the nature of the schedule it generates. In section V, we demonstrate the appropriateness and effectiveness of the *GTSP* on a wide range of test systems including the IEEE 7-bus, 14-bus, 57-bus and 118-bus networks. Section VI provides a summary and suggestions for future work.

For simplicity, we use the lossless DC model to represent the power flows in the network. In addition, we represent all the transmission constraints by the real power line flow limits. To focus on the key characteristics of the formulation, we restrict our attention to only the base case conditions. Conceptually, however, we can generalize the proposed formulation to also consider a set of specified contingencies. Similarly, the extension of the approach to lossy networks with full AC representation and the modeling of additional constraints is, in principle, possible but may entail serious computational tractability issues.

II. General Transmission Scheduling Problem (*GTSP*) Formulation

We consider a power system with N+1 buses and L lines. We denote by $\mathcal{N} \triangleq \{0,1,2,\cdots,N\}$ the set of buses, with bus 0 being the slack bus, and by $\mathcal{L} \triangleq \{\ell_1,\ell_2,\cdots,\ell_L\}$ the set of transmission lines and transformers that connect the buses in the set $\mathcal{N}$. We associate with each element $\ell \in \mathcal{L}$ the *ordered* pair $\ell = (i, j)$ to denote the line ℓ with series admittance $g_\ell - jb_\ell$. The convention we use is that the direction of the flow in line ℓ is *from* node i *to* node j so that $f_\ell \geq 0$, where f_ℓ is the real power flow in line ℓ. We define $\underline{\boldsymbol{f}} \triangleq [\, f_1, f_2,\cdots, f_L\,]^T$. We denote by $\underline{\boldsymbol{B}}_{\boldsymbol{d}} \triangleq diag\{\, b_1,b_2,\cdots,b_L\,\}$ the diagonal $L \times L$ branch susceptance matrix and by $\underline{\boldsymbol{A}}$ the reduced incidence matrix [18]. $\underline{\boldsymbol{B}} \triangleq \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_{\boldsymbol{d}} \underline{\boldsymbol{A}}$ is then the reduced nodal susceptance matrix and $\underline{\boldsymbol{b}}_0$ is the column of the nodal susceptance matrix [18] corresponding to the slack bus.

Without loss of generality, we assume that, at each network node n, $n = 0,1,\cdots,N$, there is a single pool seller and a single pool buyer [2]. We denote by $\overline{p}_n^s$ [$\overline{p}_n^b$] the maximum amount of power the seller [buyer] at node n is willing to sell [buy]. The submitted marginal seller offer [buyer bid] is integrated and we

[2] The offers[bids] of multiple sellers[buyers] at a node may be combined into a composite offer[bid] function submitted by a single seller[buyer].

denote this integral by $\beta_n^s(p_n^s)$ [$\beta_n^b(p_n^b)$]. We assume $\beta_n^s(\bullet)$ [$\beta_n^b(\bullet)$] to be nonnegative, differentiable and convex [concave]. We define $\underline{\boldsymbol{p}}^s \triangleq [\, p_1^s, p_2^s, \cdots, p_N^s \,]^T$ and $\underline{\boldsymbol{p}}^b \triangleq [\, p_1^b, p_2^b, \cdots, p_N^b \,]^T$. The bilateral transactions are represented by $\mathcal{W} \triangleq \{\, \bar{\varpi}^1, \bar{\varpi}^2, \cdots, \bar{\varpi}^W \}$ where each element $\bar{\varpi}^w$ is an *ordered* triplet $\bar{\varpi}^w \triangleq \{\, m^w, n^w, \bar{t}^w \}$ representing a *basic* transaction with receipt point (*from* node) m^w, delivery point (*to* node) n^w and the *desired* transaction amount $\bar{t}^w$ *MW*. We define $\underline{\boldsymbol{t}} \triangleq [\,\bar{t}^1, \bar{t}^2, \cdots, \bar{t}^W \,]^T$. Each bilateral transaction $\bar{\varpi}^w$ provides a function $\alpha^w(t^w)$ in its transmission request to indicate the maximum congestion charges willing to be borne as a function of the delivered transaction amount t^w, $t^w \in [0, \bar{t}^w]$. We assume $\alpha^w(\bullet)$ to be nonnegative, concave and differentiable. A special case is the specification of a fixed per *MW* price $\bar{\alpha}^w$, i.e.,

$$\alpha^w(t^w) = \bar{\alpha}^w\, t^w, \qquad t^w \in [0, \bar{t}^w], \tag{1}$$

so that $\bar{\alpha}^w$ acts as a cap on transmission price for transaction $\bar{\varpi}^w$. In particular, when the customer has unlimited willingness to pay, then $\bar{\alpha}^w$ is a large number [3] $\bar{\bar{\alpha}}$ and

$$\alpha^w(t^w) = \bar{\bar{\alpha}}\, t^w. \tag{2}$$

In the conventional *TSP* formulation [11],[12], the *IGO* first recalculates the real power flow limits for each line with all the bilateral transactions in the set $\mathcal{W}$ undertaken. Then, the *IGO* determines the pool market outcomes by maximizing the social welfare subject to the recalculated line flow limits. The *social welfare* in the *TSP* is

$$\tilde{\mathscr{S}} \triangleq \sum_{n=0}^{N} \Big[\beta_n^b(p_n^b) - \beta_n^s(p_n^s) \Big]. \tag{3}$$

Clearly, the contributions of the bilateral transactions to the social welfare are not included in $\tilde{\mathscr{S}}$. However, for completeness, a term expressing such contributions is required.

We consider the bilateral transaction $\bar{\varpi}^w \triangleq \{\, m^w, n^w, \bar{t}^w \}$ which involves a seller at node m^w and a buyer at node n^w. We denote by $\mathscr{C}(t^w)$ [$\mathscr{B}(t^w)$] the seller's production costs [buyer's consumption benefits] as a function of the actually transmitted *MW* amount t^w of the transaction, $0 \le t^w \le \bar{t}^w$. If the transmission services are provided to deliver t^w *MW* of the transaction, the difference $\mathscr{B}(t^w) - \mathscr{C}(t^w)$ between the consumption benefits and the production costs of the transaction is its contribution to the social welfare. However, the actual costs and benefits information is confidential and known only to the bilateral transaction parties. Fortunately, the bilateral transactions' willingness to pay for the congestion charges is, in fact, a proxy for such term. In the Appendix, we prove

Theorem: *If the LMP difference $\mu^*_{n^w} - \mu^*_{m^w}$ is independent of the willingness-to-pay $\alpha^w(t^w)$ of the transaction $\bar{\varpi}^w = \{m^w, n^w, \bar{t}^w\}$, then, to maximize its profits, the transaction will set*

$$\alpha^w(t^w) = \mathscr{B}(t^w) - \mathscr{C}(t^w) \quad , \quad \forall \;\; t^w \in [0, \bar{t}^w]. \tag{4}$$

This result allows us to replace the conventional social welfare $\tilde{\mathscr{S}}$ by

$$\mathscr{S} \triangleq \sum_{n=0}^{N} \Big[\beta_n^b(p_n^b) - \beta_n^s(p_n^s) \Big] + \sum_{w=1}^{W} \alpha^w \left(t^w \right) \tag{5}$$

which includes the contributions of all the transactions. To avoid confusion, we refer to $\mathscr{S}$ in (5) as the *social welfare* and call the term $\tilde{\mathscr{S}}$ in (3) the *simplified social welfare*.

The *GTSP* determines simultaneously the day-ahead market outcomes and the amount of transmission services provided by the *IGO* to each bilateral transaction $\bar{\varpi} \in \mathcal{W}$ with the objective to maximize the social welfare $\mathscr{S}$. The formulation is

$$\left\{ \begin{array}{llll} \max \mathscr{S}(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}) = \sum_{n=0}^{N}\left(\beta_n^b(p_n^b) - \beta_n^s(p_n^s)\right) + \sum_{w=1}^{W} \alpha^w\left(t^w\right) & & & \\ s.t. \qquad p_0^s - p_0^b + p_0^\tau = \underline{\boldsymbol{b}}_0^T \underline{\theta} & \leftrightarrow & \mu_0 & \\ \qquad \underline{\boldsymbol{p}}^s - \underline{\boldsymbol{p}}^b + \underline{\boldsymbol{p}}^\tau = \underline{\boldsymbol{B}}\underline{\theta} & \leftrightarrow & \underline{\boldsymbol{\mu}} & \\ \qquad \underline{\boldsymbol{B}}_{\boldsymbol{b}} \underline{\boldsymbol{A}} \underline{\theta} \le \underline{\boldsymbol{f}}^{max} & & & (6) \\ \qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{t}} \le \underline{\bar{\boldsymbol{t}}} & & & \\ \qquad 0 \le p_n^s \le \bar{p}_n^s & & n = 0,1,\cdots,N & \\ \qquad 0 \le p_n^b \le \bar{p}_n^b & & n = 0,1,\cdots,N & \end{array} \right.$$

with the vector of net injections at each node due to the bilateral transactions undertaken given by

$$\underline{\boldsymbol{p}}^\tau \triangleq [\, p_1^\tau, p_2^\tau, \cdots, p_N^\tau \,]^T \tag{7}$$

where

$$p_n^\tau \triangleq \sum_{\substack{w=1 \\ \ni m^w = n}}^{W} t^w - \sum_{\substack{w=1 \\ \ni n^w = n}}^{W} t^w, \quad n = 0,1,\cdots,N \tag{8}$$

We refer to the system (6)-(8) as the statement of the *GTSP*. In the *TSP* formulation, $\mathscr{S}$ is replaced by $\tilde{\mathscr{S}}$ and the $\underline{\boldsymbol{f}}^{max}$ is modified to indicate the effect of all the transactions on the network, as given in [19] using similar notation.

III. Characterization of the *GTSP* Solution and Determination of the Pricing Information

Under the concavity assumptions, the objective function is concave and so we can show that the optimal solution ($p_0^{s*}, p_0^{b*}, \underline{\boldsymbol{p}}^{s*}, \underline{\boldsymbol{p}}^{b*}, \underline{\boldsymbol{t}}^*$) of the *GTSP* exists. This solution determines the pool market outcomes and the transmission services provided to the bilateral transactions. For a pool player at node *n*, the quantity of the energy sold by the seller to [purchased by the buyer from] the *IGO* is specified by p_n^{s*} [p_n^{b*}]. The amount of transmission services that the *IGO* allocates to the bilateral transaction $\bar{\varpi}^w = \{m^w, n^w, \bar{t}^w\}$ is given by t^{w*} and so the transaction undertaken is $\omega^w = \{\, m^w, n^w, t^{w*} \}$.

The optimal values μ_n^*, $n = 0,1,\cdots,N$, of the dual variables

[3] This number is specified by the *IGO* so as to ensure that the bilateral transaction obtains the requested transmission services.

associated with the nodal real power balance constraints determine the nodal prices or *LMPs*. The seller [buyer] at node *n* is paid [pays] the *LMP* μ_n^* by [to] the *IGO* for each *MWh* sold [bought] in the pool for the specific hour. The bilateral transaction pays the congestion charges of

$$\xi^w = (\mu_{n^w}^* - \mu_{m^w}^*)\ t^{w*} \tag{9}$$

to the *IGO* [2]. In principle, both the pool customers and the bilateral transactions are required to pay congestion charges since the injections/withdrawals of the two classes of transmission customers contribute to congestion in the network. In fact, the congestion charges paid in the pool are *implicitly* included in the *LMP*s of the pool players [19].

The function $\alpha^w(\bullet)$ specifies the bilateral customer's willingness to pay for the congestion charges assessed from the bilateral transaction ϖ^w. The *GTSP* solution ensures

$$\xi^w = (\mu_{n^w}^* - \mu_{m^w}^*)\ t^{w*} \le \alpha^w(t^{w*}), \qquad w = 1, 2, \cdots W \tag{10}$$

because of the necessary condition of optimality of (6)-(8) as shown in the Appendix.

An interesting characteristic of the *GTSP* solution is the nature of the t^{w*}, $w = 1, 2, \cdots W$. The complimentary slackness conditions imply that the *GTSP* solution satisfies

- if $\mu_{n^w}^* - \mu_{m^w}^* > \dfrac{\partial \alpha^w}{\partial t^w}\left(t^{w*}\right)$, then $t^{w*} = 0$;
- if $\mu_{n^w}^* - \mu_{m^w}^* < \dfrac{\partial \alpha^w}{\partial t^w}\left(t^{w*}\right)$, then $t^{w*} = \overline{t}^w$.

For a transaction *w* that specifies its willingness-to-pay function by the cap price $\overline{\alpha}^w$ as in (1) so that

$$\frac{\partial \alpha^w}{\partial t^w}\left(t^{w*}\right) \equiv \overline{\alpha}^w, \tag{11}$$

whenever $\overline{\alpha}^w < \mu_{n^w}^* - \mu_{m^w}^*$, no transmission service is provided to the transaction while the transaction will be delivered at the desired level $\overline{t}^w$ if $\overline{\alpha}^w > \mu_{n^w}^* - \mu_{m^w}^*$. Such a "nothing-or-all" result characterizes the discrete nature of the transmission service provision in case a cap is specified. In the special case of $\overline{\alpha}^w = \overline{\overline{\alpha}}$ as in (2), the result ensures that transmission service is provided at the full desired level. However, when $\overline{\alpha} = \mu_{n^w}^* - \mu_{m^w}^*$, i.e., the transaction's willingness to pay is exactly equal to the *LMP* difference, then, t^{w*} may have any value in $[\ 0, \overline{t}^w]$ and so the solution becomes a continuum.

IV. Assessment of the *GTSP* solution

In the proposed *GTSP* formulation, the transmission services for the bilateral transactions become explicit decision variables. Their values are determined *simultaneously* with the pool player market outcomes. As such, the bilateral transactions are not passive price takers but active market participants competing on the same footing with the pool players for the *IGO*'s limited transmission resources. It is important to note that the incorporation of each bilateral transaction's willingness-to-pay function allows the *IGO* to ensure that the transmission services are provided to the bilateral transactions and the pool participants in a nondiscriminatory manner. In this way, the optimization in the *GTSP* formulation allocates the limited transmission services to customers – be they pool players or transactions – who value them most. Consequently, the *GTSP* solution may lead to larger social welfare than the *TSP* result when the expression in (5) is used for the social welfare. Clearly, the optimality of the *GTSP* solution ensures that

$$\mathscr{S}\big|_{GTSP} \ \ge\ \mathscr{S}\big|_{TSP}. \tag{12}$$

The interpretation of (12) is that, under congestion, those bilateral transactions that make small contributions to the social welfare may be curtailed in order to provide the transmission to the pool players whose contributions to the social welfare are higher. When there is no congestion, however, the *GTSP* and *TSP* solutions are equal and so the equality in (12) holds.

We next consider the *market efficiency loss* [20],[21]

$$\mathscr{E} \triangleq \mathscr{S}\big|_u - \mathscr{S}\big|_c \tag{13}$$

where $\mathscr{S}\big|_u$ and $\mathscr{S}\big|_c$ are the values of the social welfare of the transmission unconstrained market and that of the transmission constrained market, respectively. $\mathscr{E}$ is an effective measure of the impacts of the congestion since it quantifies the social benefits that cannot be realized due to transmission constraints. It follows from the discussion above that

$$\mathscr{E}\big|_{GTSP} \ \le\ \mathscr{E}\big|_{TSP}. \tag{14}$$

In other words, the *GTSP* solution reflects the more efficient use of the constrained transmission resources than that given by the conventional *TSP*. The ability to appropriately capture this important measure constitutes a key advantage of the *GTSP*.

The *GTSP* determination of the *LMP*s, the values of the dual variables of the nodal balance equations, is made with the explicit presence of the bilateral transactions. The ability to represent the willingness to pay of each bilateral transaction with that of each pool buyer provides a realistic basis for the computation of these key economic signals. On the other hand, in the *TSP*, the only representation of the transactions is through their impacts on the real power line flow limits. This aspect of the *GTSP* results in more realistic values for the *LMP*s and, consequently, in more appropriate economic signals for congestion management, electricity forward markets and *FTR* applications. In particular, the *FTR* payoffs are explicit functions of the *LMP* differences. With the more appropriate *LMP*s generated by the *GTSP*, the information for *FTR* payoff evaluation is better.

It is worth noting that the conventional *TSP* may be considered to be a special case of the proposed *GTSP* formulation. If we assume that every bilateral transaction has an infinite willingness to pay, then

$$\alpha^w(t^w) = \overline{\overline{\alpha}}\, t^w, \quad \forall \quad w \in \mathscr{W}, \tag{15}$$

and the solution of the *GTSP* is identical to that of the conventional *TSP*. A side-by-side comparison between the *GTSP* and *TSP* formulation and solutions is given in Table I. The analytic comparison clearly demonstrates the superiority of the *GTSP* solutions over that of the *TSP*.

TABLE I
TSP AND *GTSP* COMPARISON

formu-lation	objective function	transmission service provision	the *LMP*s	market efficiency loss
TSP	simplified social welfare $\tilde{\mathscr{S}}$	determined sequentially	approximate	approximate upper bound
GTSP	social welfare $\mathscr{S}$	nondiscriminatory; determined simultaneously	appropriate	useful measure

We note another useful application of the *GTSP* is to the modular framework in [19] where the replacement of the *TSP* by the *GTSP* formulation provides improved results.

V. SIMULATION RESULTS

To investigate the appropriateness of the proposed *GTSP* formulation and the effectiveness of its solutions, we performed an extensive set of simulations on various test systems, including the IEEE 7-bus, 14-bus, 57-bus and 118-bus systems [21]. For each system, we compare the results of the *GTSP* and *TSP* solutions as the volume of the bilateral transactions is varied. We perform the comparison in terms of the pool market outcomes, the transmission services provided to the bilateral transactions, the resulting social welfare $\mathscr{S}$ and the *LMP*s. We also briefly illustrate the impacts on *FTR* payoff values. In this section, we report the representative results for these investigations. A key metric for the comparison is the absolute relative error between the *TSP* and *GTSP* results

$$\varepsilon_q \triangleq \left| \frac{q_{TSP} - q_{GTSP}}{q_{GTSP}} \right| \tag{16}$$

for each various variable q of interest.

We first compare the *GTSP* and *TSP* solution for a fixed volume of the bilateral transactions. We design the comparison for the IEEE-14 bus with the market data as given in Tables II-V. The *TSP* and *GTSP* results are plotted in Figs. 1 to 4.

TABLE II
THE SELLERS' OFFERS IN THE POOL MARKET

seller	located at node	$\bar{p}_n^s$ (*MW*)	*a*	*b*
S_1	1	550	3.0	0.03
S_2	2	750	4.0	0.04
S_3	14	650	6.0	0.06

Note: $\beta_n^s(p_n^s) = a\,p_n^s + b(p_n^s)^2$.

TABLE III
THE BUYERS' BIDS IN THE POOL MARKET

buyer	located at node	$\bar{p}_n^b$ (*MW*)	*a*	*b*
B_1	2	150	15.0	-0.012
B_2	3	100	20.0	-0.018
B_3	4	350	20.0	-0.018
B_4	5	200	22.0	-0.015
B_5	6	270	18.0	-0.015
B_6	8	130	14.0	-0.017
B_7	9	250	15.0	-0.018
B_8	10	250	13.0	-0.020
B_9	11	310	14.0	-0.018
B_{10}	12	100	15.0	-0.010
B_{11}	13	250	18.0	-0.015
B_{12}	14	100	22.0	-0.022

Note: $\beta_n^b(p_n^b) = a\,p_n^b + b\,(p_n^b)^2$.

The stark contrast of the *TSP* and *GTSP* solutions is seen in the plots of the transaction amounts for which transmission services are provided under the two formulations. The ability to explicitly have interaction between the willingness to pay and the amount of transmission services provided is clearly illustrated for transactions 2 and 3 which are completely curtailed in the *GTSP* solution as opposed to receiving all the requested transmission services under the *TSP* solution. The curtailments in the *GTSP* solution reflect the decision of the *IGO* to provide the limited transmission services to the pool players so as to result in higher $\mathscr{S}$ values.

TABLE IV
THE TRANSMISSION REQUESTS OF THE BILATERAL TRANSACTIONS

transaction	m^w	n^w	$\bar{t}^w$	$\bar{\alpha}^w$
ω^1	1	10	180	15.0
ω^2	2	7	130	5.0
ω^3	2	8	150	6.0
ω^4	14	4	120	7.0
ω^5	14	5	100	15.0

Note: $\alpha^w(t^w) = \bar{\alpha}^w t^w$.

TABLE V
FINANCIAL TRANSMISSION RIGHTS (*FTR*) HOLDINGS

FTR	*from* node	*to* node	amount (*MW*)
Γ^1	1	2	500
Γ^2	1	4	300
Γ^3	2	5	300
Γ^4	2	10	500
Γ^5	1	14	800

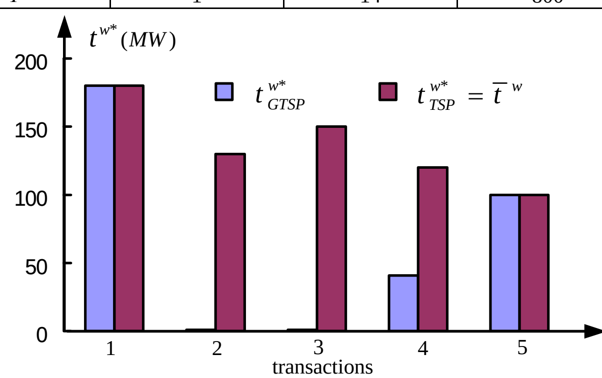

Fig. 1: Amount of delivered transactions under the *GTSP* and *TSP*

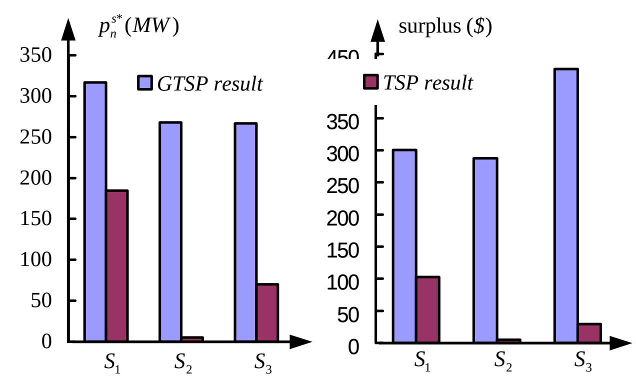

Fig. 2: Sellers' outcomes and corresponding surpluses under *TSP* and *GTSP*

The two formulations impact the sellers' sales amounts and their corresponding *surpluses* [20],[21], as shown in Fig. 2. The two solutions also lead to large differences in the social welfare and the efficiency loss with $\mathscr{S}\big|_{GTSP} = \$14,467$, $\mathscr{S}\big|_{TSP} = \$9,537$, $\mathscr{E}\big|_{GTSP} = \$3,409$ and $\mathscr{E}\big|_{TSP} = \$8,339$, reflecting the improved market efficiency provided by the *GTSP* solution.

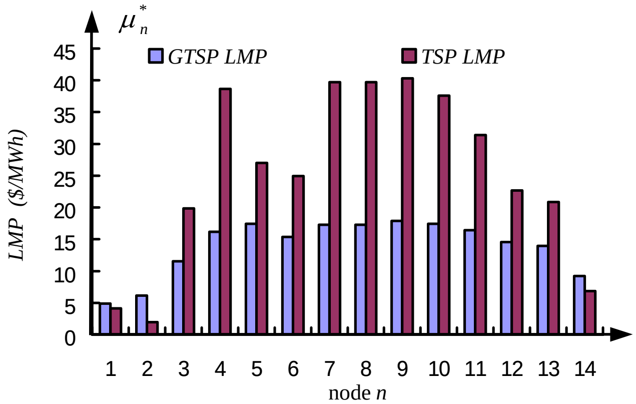

Fig. 3. Comparison of the *LMP*s computed from *GTSP* and *TSP*

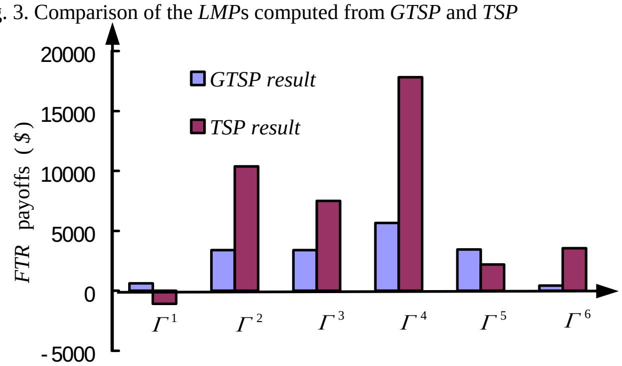

Fig. 4. Comparison of the *FTR* payoffs for *GTSP* and *TSP* results

The most distinct differences arise in the *LMP*s computed by the two solutions as shown in Fig. 3. The absolute relative errors range in magnitude from 16% to 139%. These differences in the *LMP*s not only impact the market-based settlements for the purchases/sales but also have major repercussions in the congestion rents for the congested system. In the *GTSP* solution, the *congestion rents* [21] assessed are \$ 8,613 versus \$ 22,265 in the *TSP* solution. Clearly, such differences are also reflected in the *FTR* calculations. For the set of *FTR* holdings given in Table V, the comparison of *FTR* payoffs is depicted in Fig. 4. The absolute errors range from 54% to 158%. In the extensive simulations performed, we found absolute errors to be as large as 500%. Such errors become particularly marked as the fraction of the total load served by the transactions increases.

We next illustrate some representative results of the impacts of varying the volume of the bilateral transactions. We examine this effect as we scale uniformly the desired transaction amounts $\bar{t}^w$ of every $\varpi^w = \{m^w, n^w, \bar{t}^w\}$, $w = 1, 2, \cdots, W$, while we keep the pool players' bids and offers unchanged. In this way, the percentage of the load served by the bilateral transactions increases from 0% to some value at which the *TSP* has no solution. For each value of the scaling factor, we compare the *TSP* and *GTSP* solutions as well as the corresponding *LMP*s and social welfare values. We compute the absolute relative errors for these variables. Typical plots for the test systems are presented in Figs. 5-7.

For each of the variables selected, the absolute relative error increases with the scaling factor. For example, the plots in Fig. 5 illustrate the increasing nature of the differences between the amounts sold by each pool seller in the IEEE 7-bus test system as the scaling factor increases. The 100% relative error corresponds to the outcome that the seller S_1 ceases to sell in the pool under the *TSP* since all the limited transmission services are provided to the bilateral transactions. In contrast, under the *GTSP*, S_1 is still able to sell since some transactions are curtailed due to the willingness to pay characteristics.

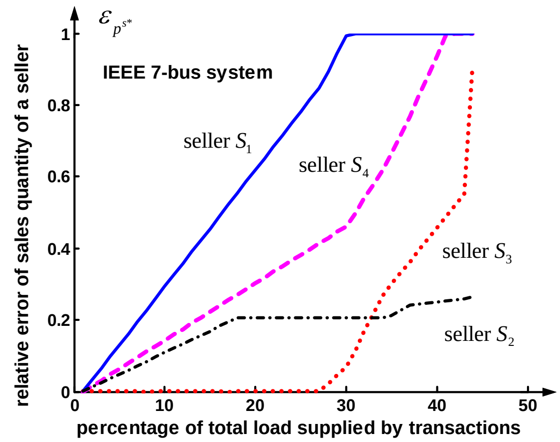

Fig. 5. The relative error on pool sellers' sales amount as a function of the bilateral transaction volume

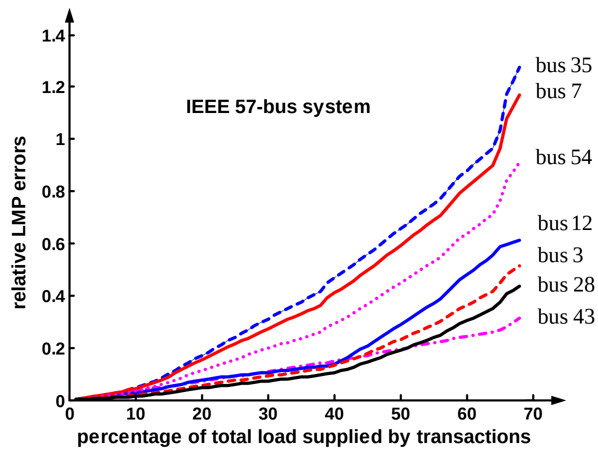

Fig. 6. The relative *LMP* error as a function of bilateral transaction volume

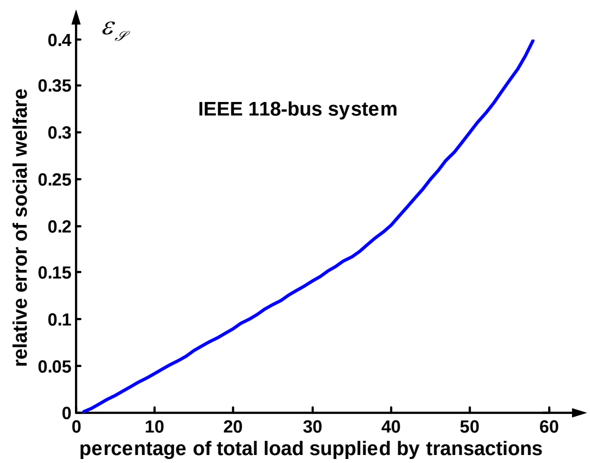

Fig. 7. The relative error on $\mathscr{S}$ as a function of bilateral transaction volume

Fig. 6 indicates the monotonic increase of the absolute relative *LMP* errors ε_{μ_n} as the scaling factor increases for the IEEE 57-bus network. Clearly, for a small value of the scaling factor, the *GTSP* and *TSP* solutions are identical. As the volume of the bilateral transactions reaches 68% of the total load, the *TSP* problem becomes infeasible while the *GTSP* solution continues to exist. Thus, the *GTSP* provides versatility of determining schedules even in cases where the *TSP* is not solvable. The plot of the relative errors of the social welfare in Fig. 7 for the 118-bus network indicates their monotonic behavior. In addition, note that the slope of the curve, i.e., the rate of change or the sensitivity of the relative errors increases with the scaling factor.

These observations substantiate the effectiveness of the proposed *GTSP* formulation in cases when the load supplied by the bilateral transactions constitutes a significant portion of the total load in the system.

VI. Conclusions

In this paper, we proposed a generalized transmission scheduling problem (*GTSP*) formulation for the *IGO* to determine the provision of nondiscriminatory transmission services to both the pool players and the bilateral transactions. By exploring the relationships between the bilateral customers' willingness to pay and their contributions to the social welfare, the proposed *GTSP* formulation captures more appropriately the social welfare and market efficiency loss compared to the conventional *TSP* formulations. In addition, the solution of the *GTSP* formulation provides more efficient market outcomes that may lead to higher social welfare and less market efficiency loss in the presence of congestion. Furthermore, the *GTSP* solutions provide more appropriate *LMPs* and, consequently, *LMP* differences for the determination of the congestion charges and the *FTR* payoffs. Our numerical results on several test systems provide persuasive support for the effectiveness of the proposed formulation and its superiority over the conventional *TSP* solutions.

While the proposed *GTSP* formulation has demonstrated its advantages compared to the conventional *TSP*, a number of extensions are desirable. In this paper, we based our discussions on the lossless DC power flow model. As we pointed out in the Introduction, we can conceptually extend the formulation to include losses and nonlinear effects by using the full AC power flow. The challenges are the huge computational problems – the complexity and speed – of the AC power flow model which need to be addressed to achieve computational tractability of the approach for market applications. Another important extension is to provide a uniform basis for considering ancillary services for the pool customers and bilateral transactions. A topic of intense interest is transmission investment in competitive electricity markets. The proposed *GTSP* formulation provides a basis for the study of investment issues such as incentives and transmission adequacy. In addition, the notion of substitutability of transmission by generation for congestion relief is a closely linked topic and can also be studied using the *GTSP* formulation. Results of our work will be presented in future publications.

Appendix: Mathematical proofs

A. Proof of the Theorem

For simplicity of the notation, we define

$$\mathscr{R}(t^w) \triangleq \mathscr{B}(t^w) - \mathscr{C}(t^w) . \qquad \text{(A.1)}$$

The net profits of the transaction are

$$\pi^w = \mathscr{R}(t^w) - (\mu^*_{n^w} - \mu^*_{m^w})\, t^{w*} \qquad \text{(A.2)}$$

where the *LMPs* at node n^w and m^w are given by $\mu^*_{n^w}$ and $\mu^*_{m^w}$ and $t^{w*} \in [\,0, \bar{t}^{\,w}]$ is the transaction amount for which transmission services are provided. At the optimal of the *GTSP*, the optimal variables $\mu^*_{n^w}$, $\mu^*_{m^w}$ and t^{w*} depend on $\alpha^w(\bullet)$. We indicate this dependence by writing $\mu^*_{n^w}(\alpha^w)$, $\mu^*_{m^w}(\alpha^w)$ and $t^{w*}(\alpha^w)$ and drop the brackets in $\alpha^w(\bullet)$ to simplify the notation. Given the assumption that $\mu^*_{n^w} - \mu^*_{m^w}$ is independent of $\alpha^w(\bullet)$, the net profits of transaction *w* may be expressed as

$$\pi^w = \mathscr{R}\left[t^{w*}(\alpha^w) \right] - (\mu^*_{n^w} - \mu^*_{m^w})\; t^{w*}(\alpha^w) . \qquad \text{(A.3)}$$

To maximize profits, the bilateral transaction selects the willingness-to-pay function $\alpha^w(\bullet)$ accordingly.

To explore the property of the function $t^{w*}(\alpha^w)$, we apply the Lagrangian relaxation approach [22] to the *GTSP* formulation in (6)-(8). Since the objective function is concave and the set of feasible solutions determined by the constraints is compact, the optimal solution of (6)-(8) is identical to the optimal solution of the following optimization problem

$$\min_{\mu_0, \underline{\boldsymbol{\mu}}, \underline{\boldsymbol{\lambda}} \ge \underline{\mathbf{0}}} \; \max_{(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}, \underline{\boldsymbol{\theta}}) \in \boldsymbol{\Omega}} \Big\{ \sum_{n=0}^{N} \big(\beta_n^b(p_n^b) - \beta_n^s(p_n^s) \big) + \sum_{w=1}^{W} \alpha^w(t^w) - \underline{\boldsymbol{\lambda}}^T (\underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}}\, \underline{\boldsymbol{\theta}} - \underline{\boldsymbol{f}}^{max}) - \underline{\boldsymbol{\mu}}^T (\underline{\boldsymbol{B}}\underline{\boldsymbol{\theta}} - \underline{\boldsymbol{p}}^s + \underline{\boldsymbol{p}}^b - \underline{\boldsymbol{p}}^\tau) - \mu_0 (\underline{\boldsymbol{b}}_0^T \underline{\boldsymbol{\theta}} - p_0^s + p_0^b - p_0^\tau) \Big\} \qquad \text{(A.4)}$$

where

$$\boldsymbol{\Omega} \triangleq \left\{ \left(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}, \underline{\boldsymbol{\theta}} \right) : \underline{\boldsymbol{t}} \in \left[\underline{\mathbf{0}}, \underline{\bar{\boldsymbol{t}}} \right], \underline{\boldsymbol{\theta}} \in \mathbb{R}^N, p_n^s \in \left[0, \bar{p}_n^s \right], p_n^b \in \left[0, \bar{p}_n^b \right], n = 0, 1, \cdots N \right\}$$

It is proved in [18] that the optimal solution of (6)-(8) satisfies

$$\underline{\boldsymbol{\mu}}^{*T} \underline{\boldsymbol{B}}\, \underline{\boldsymbol{\theta}}^* - \mu_0^* \underline{\boldsymbol{b}}_0^T \underline{\boldsymbol{\theta}}^* - \underline{\boldsymbol{\lambda}}^{*T} \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}}\, \underline{\boldsymbol{\theta}}^* = 0 . \qquad \text{(A.5)}$$

Consequently, we may drop the terms containing $\underline{\boldsymbol{\theta}}$ in (A.4). Then, for the given optimal value $\left(\mu_0^*, \underline{\boldsymbol{\mu}}^*, \underline{\boldsymbol{\lambda}}^* \right)$ of the dual variables, (A.4) is equivalent to

$$\max_{(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}) \in \boldsymbol{\Omega}} \Big\{ \sum_{n=0}^{N} \big(\beta_n^b(p_n^b) - \beta_n^s(p_n^s) \big) + \sum_{w=1}^{W} \alpha^w(t^w) - \underline{\boldsymbol{\mu}}^{*T} \left(\underline{\boldsymbol{p}}^b - \underline{\boldsymbol{p}}^s - \underline{\boldsymbol{p}}^\tau \right) - \mu_0^* \left(p_0^b - p_0^s - p_0^\tau \right) \Big\} \qquad \text{(A.6)}$$

Note that the variables are decoupled. Therefore, we may decompose (A.6) and determine the optimal value of each variable separately. The sub-problem that determines t^{w*} is

$$\max_{t^w \in [\,0, \bar{t}^{\,w}]} \;\; \alpha^w(t^w) - (\mu^*_{n^w} - \mu^*_{m^w})\;\, t^w . \qquad \text{(A.7)}$$

That is,

$$t^{w*}(\alpha^w) = \underset{t^w \in [\,0,\,\bar{t}^{\,w}\,]}{argmax} \quad \alpha^w(t^w) - (\mu^*_{n^w} - \mu^*_{m^w})\ t^w . \qquad \text{(A.8)}$$

We use (A.8) to rewrite the profit maximization problem of transaction w as

$$\underset{\gamma \in \mathcal{A}}{max} \qquad \pi^w = \mathscr{R}\left[t^{w*}(\gamma)\right] - \left(\mu^*_{n^w} - \mu^*_{m^w}\right)\ t^{w*}(\gamma) \qquad \text{(A.9)}$$

where

$\mathcal{A} \triangleq \left\{ \gamma(\bullet) : [0, \bar{t}^{\,w}] \rightarrow \mathbb{R}, \text{concave,differentiable and } \gamma(\cdot) \geq 0 \right\}$

is the set of all the feasible willingness-to-pay functions. Clearly, $\mathscr{R}(\bullet) \in \mathcal{A}$. We define

$$\hat{\gamma}(\bullet) = \mathscr{R}(\bullet) \qquad \text{(A.10)}$$

and prove, by contradiction, that $\hat{\gamma}(\bullet)$ is the optimal solution of (A.9). We assume $\exists\ \ \gamma'(\bullet) \in \mathcal{A},\ \gamma'(\bullet) \neq \hat{\gamma}(\bullet)$ such that

$$\mathscr{R}\left[t^{w*}(\gamma')\right] - \left(\mu^*_{n^w} - \mu^*_{m^w}\right)\ t^{w*}(\gamma') \ > \ \mathscr{R}\left[t^{w*}(\hat{\gamma})\right] - \left(\mu^*_{n^w} - \mu^*_{m^w}\right)\ t^{w*}(\hat{\gamma}) .$$

Let $t' \equiv t^{w*}(\gamma')$, then $t' \in [0, \bar{t}^{\,w}]$, $t' \neq t^{w*}(\hat{\gamma})$ and

$$\mathscr{R}(t') - \left(\mu^*_{n^w} - \mu^*_{m^w}\right) t' > \mathscr{R}\left[t^{w*}(\hat{\gamma})\right] - \left(\mu^*_{n^w} - \mu^*_{m^w}\right) t^{w*}(\hat{\gamma}) . \qquad \text{(A.11)}$$

However, by definition,

$$\begin{aligned} t^{w*}(\hat{\gamma}) &= \underset{t^w \in [0,\,\bar{t}^{\,w}]}{argmax} \quad \hat{\gamma}\,(t^w) - (\mu^*_{n^w} - \mu^*_{m^w})\, t^w \\ &= \underset{t^w \in [0,\,\bar{t}^{\,w}]}{argmax} \quad \mathscr{R}(t^w) - (\mu^*_{n^w} - \mu^*_{m^w})\, t^w . \qquad \text{(A.12)} \end{aligned}$$

This contradicts the inequality in (A.11) and implies that

$$\hat{\gamma}^w(\bullet) = \mathscr{R}(\bullet) = \mathscr{B}(\bullet) - \mathscr{C}(\bullet)$$

is the optimal solution of (A.9). In other words, the transaction maximizes its net profits by setting the willingness-to-pay function equal to its contribution to the social welfare. □

B. *Proof of inequality* $\left(\mu^*_{n^w} - \mu^*_{m^w}\right) t^{w*} \leq \alpha^w(t^{w*})$

The Lagrangian of the *GTSP* given by (6)-(8) is

$$\begin{aligned} \mathscr{L} = &\sum_{n=0}^{N}\left(\beta^b_n(p^b_n) - \beta^s_n(p^s_n)\right) + \sum_{w=1}^{W}\alpha^w\left(t^w\right) - \\ &\underline{\mu}^T(\underline{B\theta} - \underline{p}^s + \underline{p}^b - \underline{p}^\tau) - \mu_0\,(\underline{b}^T_0\underline{\theta} - p^s_0 + p^b_0 - p^\tau_0) - \\ &\underline{\lambda}^T(\underline{B}_d\,\underline{A\theta} - \underline{f}^{\,max}) - \underline{\eta}^T_+\left(\underline{t} - \underline{\bar{t}}\right) + \underline{\eta}^T_-\ \underline{t} - \qquad \text{(A.13)} \\ &\sum_{n=0}^{N}\left[\varsigma_{n+}\left(p^s_n - \bar{p}^s_n\right) + \varsigma_{n-}\ p^s_n\right] - \sum_{n=0}^{N}\left[\zeta_{n+}\left(p^b_n - \bar{p}^b_n\right)\zeta_{n-}\ p^b_n\right] \end{aligned}$$

The necessary conditions of optimality [22] include

$$\frac{\partial\mathscr{L}}{\partial t^w} = \frac{\partial\alpha^w}{\partial t^w}\left(t^{w*}\right) + \mu^*_{m^w} - \mu^*_{n^w} - \eta^{w*}_+ + \eta^{w*}_- = 0 \qquad \text{(A.14)}$$

for all $t^w \in [0, \bar{t}^{\,w}]$, $w = 1, 2, \cdots, W$ and

$$\begin{cases} \eta^{w*}_+\left(t^{w*} - \bar{t}^{\,w}\right) = 0\,; & \eta^{w*}_+ \geq 0\,; \\ \eta^{w*}_-\ t^{w*} = 0\,; & \eta^{w*}_- \geq 0\,; \end{cases} \quad \text{for } w = 1, 2, \cdots, W \qquad \text{(A.15)}$$

From (A.14), we obtain

$$\mu^*_{n^w} - \mu^*_{m^w} = \frac{\partial\alpha^w}{\partial t^w}\left(t^{w*}\right) - \eta^{w*}_+ + \eta^{w*}_- \qquad \text{(A.16)}$$

which, together with (A.15), implies the following relationship between t^{w*} and the *LMP* differences:

$$\mu^*_{n^w} - \mu^*_{m^w} \begin{cases} \geq \dfrac{\partial\alpha^w}{\partial t^w}\left(t^{w*}\right) & if \qquad t^{w*} = 0 \\ \leq \dfrac{\partial\alpha^w}{\partial t^w}\left(t^{w*}\right) & if \qquad t^{w*} = \bar{t}^{\,w}\,; \\ = \dfrac{\partial\alpha^w}{\partial t^w}\left(t^{w*}\right) & otherwise \end{cases}$$

Since $\alpha^w(\bullet)$ is concave, we conclude that

$$\left(\mu^*_{n^w} - \mu^*_{m^w}\right) t^{w*} \leq \frac{\partial\alpha^w}{\partial t^w}\left(t^{w*}\right) t^{w*} \leq \alpha^w\left(t^{w*}\right),\ \forall\, t^{w*} \in [0, \bar{t}^{\,w}] .$$

□

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