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Optimal power flow application issues in the Pool paradigm

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Abstract

This paper focuses on the application of the Optimal Power Flow (OPF) to competitive markets. Since the OPF is a central decision-making tool its application to the more decentralized decision-making in the competitive electricity markets requires considerable care. There are some intrinsic challenges associated with the effective OPF application in the competitive environment due to the inherent characteristics of the OPF formulation. Two such characteristics are the flatness of the optimum surface and the consequent continuum associated with the optimum. In addition to these OPF structural characteristics, the level of authority vested in the central decision-making entity has major ramifications. These factors have wide ranging economic impacts, whose implications are very pronounced due to the fact that, unlike in the old vertically integrated utility environment, various market players are affected differently. The effects include price volatility, financial health of various players and the integrity of the market itself. We apply appropriate metrics to evaluate market efficiency and how the various players fare. We study the impacts of OPF applications in the Pool paradigm, with both supply and demand side explicitly modeled, and provide extensive numerical results on systems based on IEEE 30-bus and 118-bus networks. The results show the variability of nodal prices and the skew possible in different 'optimal' allocations among competing suppliers. Such variability in the results may lead to serious disputes among the players and the central decision-making authority. Directions for future research are discussed.

Keywords: Pool paradigm; Optimal power flow; Nodal prices; Market dispatch

1. Introduction

The optimal power flow (OPF) was developed to bring explicit consideration of some system performance measures into the steady state analysis of the power system. The basic idea is to define an objective function, which is optimized subject to the steady state conditions of the power system and certain physical, operational and policy constraints, which need to be satisfied for the generation and delivery of electricity in a bulk power system. The steady state conditions are represented by the power flow relations—a set of algebraic equations—and the other constraints typically arise from the consideration of the physical capability of each facility/equipment installation, the security criteria used, the operational/policy requirements of the system and specific physical/engineering requirements to ensure secure system operations. The objective is defined to be some quantifiable metric—typically some economic measure of system performance such as production costs or transmission losses or some system characteristic such as available transfer capability. There are various applications of OPF ranging from real-time operations—e.g. economic or security-constrained dispatch—to operational planning—e.g. production cost optimization—to longer term planning, such as reactive support investment decisions. The OPF problem is concerned with the optimization of the static power system for a specified point in time. The decision maker is the power system operator or, in certain cases, the interconnected system operator. The solution of the OPF determines the optimal policy from the central authority viewpoint to meet the specified objective under the given constraints. In addition, there is associated sensitivity information with direct economic interpretations that can be readily derived from the optimum.

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The analytical formulation of the problem can be stated as follows

$$\min f(\underline{\mathbf{u}}, \underline{\mathbf{x}}) \quad s.t.\ \underline{\mathbf{g}}(\underline{\mathbf{u}}, \underline{\mathbf{x}}) = \underline{0} \quad \underline{\mathbf{h}}(\underline{\mathbf{u}}, \underline{\mathbf{x}}) \leq \underline{0} \tag{1}$$

where

$\underline{\boldsymbol{u}}$	the vector of m decision variables
$\underline{\boldsymbol{x}}$	the vector of n state variables
$f : \Re^m \times \Re^n \rightarrow \Re$	is the objective function
$\underline{\mathbf{g}} : \Re^m \times \Re^n \rightarrow \Re^n$	is the equality constraint function
$\underline{\mathbf{h}} : \Re^m \times \Re^n \rightarrow \Re^q$	is the inequality constraint function

The large-scale non-linear programming problem in (1) has been solved by a wide range of numerical techniques [1]. An important byproduct of the optimum of the OPF is the economic information obtained from the dual variables, which has direct applications in electricity markets. For example, the dual variables associated with the power flow equality constraints can be interpreted as the nodal real and reactive power prices at each bus. We denote the optimal real power price at node i by λ_i. Similarly, dual variables associated with inequality constraints are interpreted as the sensitivity of the objective function to a change in the constraint limit.

The various implementations of OPF differ in terms of the model formulation, the level of detail in the representation of the system and the computational/algorithmic aspects of the solution methodology. Different objectives call on different levels of detail in the model and in the representation of various considerations. However, there is considerable arbitrariness involved in all the aspects of the OPF—model formulation, level of detail in the interconnected system representation and solution methodology/implementation scheme.

The OPF tool was developed for use in the centrally controlled environment, which permeated the vertically integrated utility (VIU) structure that was well entrenched in the electricity sector. In the VIU, the central decision maker is, typically, the utility, i.e. the single entity that operates and controls the generation and transmission plants and has the obligation to serve load. The central decision maker charged with the determination of the optimal policy was also the owner of all the generators and the transmission/distribution grids under his direct control. In the VIU, favoring one generating unit over another has no significant financial implications since all units are owned by the same entity.

In the new environment, the decision maker and the players need no longer be the same entity. Generators belong to different companies that are competing on the market. The load is no longer a fixed quantity to be forecasted but becomes a decision variable of the optimization problem. The demand at each bus is characterized in terms of load elasticity which expresses the willingness to pay of the customer and, hence, gives the responsiveness of the demand to price variations. The objective of the OPF problem becomes the maximization of the social surplus S^{S} that is a measure of the market efficiency. S^{S} is the sum of the total producer surplus S^{G}, the total consumer surplus S^{D} and the merchandising surplus S^{M} [2]. The social welfare thus incorporates each generator's producer surplus, defined as the difference between the revenues for the sale and the variable costs of production, each consumer's surplus, given by the difference between the benefits and the payments for its purchase, and a term that arises due to the presence of congestion. The sum of the surpluses of all the producers is the total producer surplus S^{G} and the sum of the surpluses of all the consumers is the consumer surplus S^{D}. When congestion occurs, differences in nodal prices arise and so the price of electricity may be different at each bus. The sum of the differences between the prices paid and those received multiplied by the corresponding quantities of *MWh* are the so-called *congestion rents* or the *merchandising surplus* S^{M}.

The application of OPF poses major challenges. This central decision-making tool is, in certain ways, very much at odds with the thrust of the competitive environment for increased decentralized decision-making. In competitive markets, the optimal policy determined by the central authority can have major ramifications on the financial health of various market participants as well as the well being of the market. In particular, favoring one generator over another becomes a major problem in the new environment since this may impact the bottom line of different entities. The structural characteristics of the OPF and the level of authority vested in the central decision-making entity are major contributors to the difficulties faced in addressing the problem of effectively applying OPF in the new environment. In competitive markets, bias in favor or against a given generator may result in the bankruptcy or windfall profiteering of a generator. In light of the important economic signals emanating from the OPF results, the application of this tool in the new environment is a challenging task [3,4]. Some of the inherent characteristics of the OPF problem are contributing factors to this reality. But, as we will see in the numerical results, the level of discretion of the decision maker is a key issue in the effective application of this tool.

One issue of particular importance is the ramification of the economic information derived from the OPF. In the new context, this impact includes questions of the fairness of electricity markets. The OPF application can lead to anomalous behavior in a price setting context. Different buses of the grid can experience widely different prices at a fixed point in time. Also, the energy price at a given bus can be subject to significant variations over time. Such a situation leads to volatility in market pricing. Moreover, the OPF application results, due to the presence of the physical power network constraints, can lead to the economically unintuitive outcome of sending energy from a bus with a higher nodal price to one with a lower nodal price [5,6]. One of the most important decision variables in the OPF is the real power output of each generator of the network. However, due to the dependence of the OPF solution on various parameters and factors, as discussed below, slight changes in such parameters/factors can result in different allocations of the power dispatched which lead to different profits for the competing companies. These various ramifications are explored at length through a number of illustrative numerical examples.

We discuss the structural characteristics of the OPF, from which arise some challenges in the effective use of the OPF tool, in Section 2. The discretionary powers of the central decision-making authority also contribute to these challenges and are discussed in Section 3. We devote the entire Section 4 to the presentation of numerical results of our studies on applications of OPF in competitive electricity markets. We summarize the basic results and discuss directions for future work in Section 5.

2. Impacts of OPF structural characteristics

There are certain characteristics of the OPF that have significant impacts on the well being of the electricity markets. One overriding concern arises from the wide degree of arbitrariness that the central decision-making has in the determination of the optimum.

In general, the solution of the OPF problem is considerably ‘flat’. This means that there exists a continuum of ‘optimum’ solutions, which results, in effect, in the same objective function value within a specified ε tolerance. While the different solutions can be considered practically the same from the point of view of the market efficiency, they can impact market players very differently. For example, two solutions with almost the same optimum value can allocate generation power levels and surpluses differently among generators belonging to different companies. As such, the choice of an optimum solution has a great degree of arbitrariness.

Moreover, different solution algorithms can lead to different optima and usually the solutions determined by various algorithms are sensitive to the initial guess. This fact together with the flatness characteristic adds additional latitude to the arbitrariness of the solution in the sense that different initial guesses can lead to solutions that are equally ‘good’ (within the specified ε tolerance) but that correspond to very different values of the decision variables.

The system parameters are often known only within an uncertainty range; the different possible values, for a certain parameter, that may be imposed in the OPF problem may greatly affect the decision variable values at the optimum with remarkable impacts on the market participant and nodal pricing.

These characteristics, combined with the general difficulty of solving non-linear optimization problems, are key in appreciating the limitations of the straightforward application of OPF in unbundled markets. In addition, the important market signals obtained from OPF—in particular, the dual variables that provide the real power nodal price—can vary considerably for different optima. For example, even if the optima are ε-close, the dual variables can be so different as to result in significant market shifts. Such behavior arising out of the mathematical structure of the problem can lead to wide fluctuations in markets. A non-intuitive situation, from an economic point of view, in which the power flows are from higher to lower λ_i may, as well, arise. One outcome is to question the reliability of such signals. A major concern is that some players may use their knowledge of the grid and the OPF application to take unfair advantage of the situation and game the system.

3. The central decision-making authority’s discretionary powers

The application of the OPF is performed by some centralized coordination entity in every structure. In the emerging unbundled electricity markets this central authority is the Independent System Operator (ISO), Transmission System Operator (TSO), Regional Transmission Organization (RTO) for which we use the generic term Independent Grid Operator (IGO), whose responsibility includes congestion management in some explicit or implicit manner [7,8]. This central decision-making authority has many degrees of freedom in specifying the OPF model and solving the resulting formulation. These choices have major impacts on the optimum and on the values of the dual variables.

We categorize the discretionary powers of the central decision-making authority in three principal areas: constraint set formulation, contingency set specification and algorithm selection. The constraint set formulation includes the explicit representation of the constraints (voltage profiles, line flows limits, generator real and reactive limits, voltage stability limits, etc.) that are considered in the OPF model. Since typically not all of these limits are considered simultaneously, consequently, the selection of the constraints explicitly taken into account is done by the central decision-making authority on the basis of some technical considerations. In the same way, the selection of the set of ‘plausible’ contingencies to be used for security analysis has a certain amount of latitude. Which contingencies should be included in the set is, to a certain extent, arbitrary. The selection of the algorithm to solve the optimization problem and the specification of various algorithmic parameters are tasks within the domain of responsibility of the central decision-making entity. Such choices clearly influence the OPF results and the dual variable values. In fact, if the solution is feasible for the discretionary choices made, the set of dual variables is computed and transmitted to the market. If, on the other hand, the solution is not feasible the central entity exercises further discretion in specifying

Table 1
Load elasticity for the 30-bus system

Elasticity	Buses
0.001	10, 15, 21, 24, 26
0.01	3, 4, 7, 8, 12, 14, 23
0.1	2, 17, 18, 19, 20
1	30
10	16, 29

the manner in which certain constraints are relaxed. Such actions may add a further level of arbitrariness.

4. Numerical studies

We next provide illustrative examples of the issues discussed above using systems based on the IEEE 30-bus and on the IEEE 118-bus networks [9]. We consider a Pool paradigm structure in which the objective of the OPF is the maximization of S^S [2]. Each generator's cost function, for the systems considered, has been scaled so that the system marginal cost (system lambda) is one for the lossless unconstrained case. Each generator submits its offer equal to its marginal cost. In addition, each load is elastic and submits a bid [10]. For the 30-bus system the load elasticity is given in Table 1. For the 118-bus-based system a uniform unit elasticity is used. We use a full AC OPF in which the reactive load is modeled as an affine function of real power [11]. The electricity price at each bus is given by the Lagrangian multiplier associated with the real power flow equation. Clearly, the price may be interpreted as the sensitivity of the social surplus to a change in the real power demand at that bus. In the following, examples A, B and C show the impacts of the structural characteristics of the OPF on the market while examples D and E illustrate the arbitrariness of the central decision-making authority.

4.1. The lack of sensitivity of the OPF solution to changes in parameter values

The solution of the OPF remains considerably unchanged as the parameter values are varied around their rated values. For example, we use the IEEE 30-bus-based system and we consider the situation as the reactance parameters of the lines 2–6 and that of the lines 27–28 are varied. We evaluate the optimum corresponding to each parameter value of each line reactance in the range of $[-20\%, +20\%]$ around the rated values using 5% step increments. We consider the changes with respect to the base case, i.e. the value corresponding to the 0% change in the line parameters, and construct the contour plot in Fig. 1. Each contour represents a constant objective percentage change with respect to the base case. This example serves to show that a considerable change in the values of parameters does not affect the optimal solution of the optimization problem. This characteristic of the OPF application gives rise to concerns when we also bring in the level of discretion of the IGO in the selection of data values since the OPF solutions may discriminate against certain players.

4.2. Uncertainty in system parameter values

The level of uncertainty with which the system parameters are known may have significant impacts on the outcomes of the generators. We consider the IEEE 30-bus-based system with a reference line flow limit, $F_{\mathrm{ref}}=0.35$

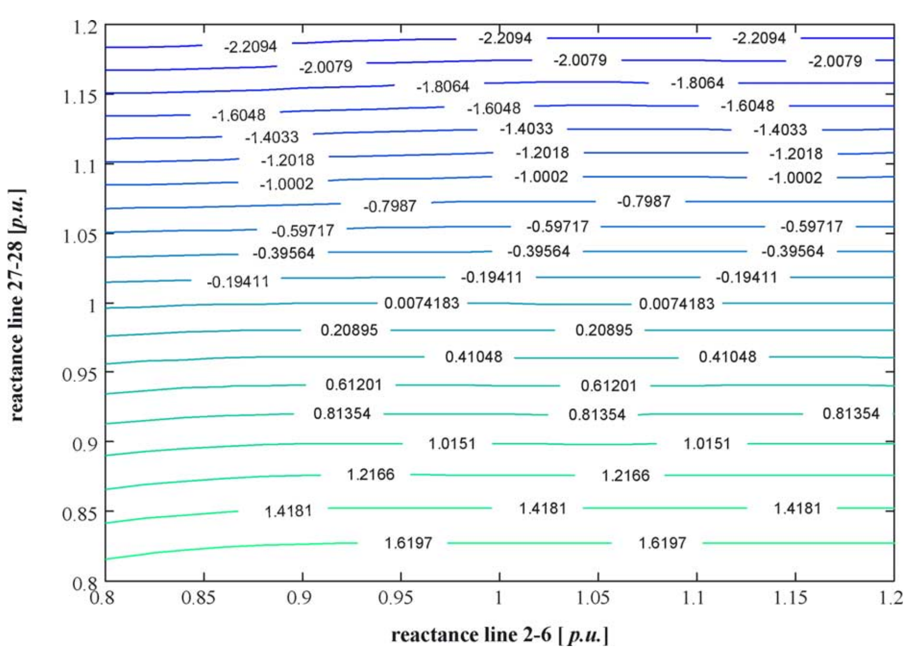

Fig. 1. Contours of constant objective percentage changes at the optimum, with respect to the base case, for various values of the reactances of lines 2–6 and 27–28 (10^{-3}%).

Table 2
Cases for different values of the line flow limit F_{max} of lines 6–8

Case	F_{max} lines 6–8 (p.u.)	F_{max}/F_{ref} lines 6–8
B.0	0.350	1
B.1	0.280	0.80
B.2	0.297	0.85
B.3	0.315	0.90
B.4	0.332	0.95
B.5	0.367	1.05
B.6	0.385	1.10

p.u., on lines 6–8 and we assume that that limit is not known with certainty but may vary in the range [−20%, +10%] of its reference value. The cases with the different line flow limits F_{max} considered are listed in Table 2. The base case *B*.0 has a non-binding line flow limit and so any increase in the line flow limit has no impact. On the other hand, a reduction in the line flow limit, as in cases *B*.1–*B*.4, causes the line flow limit to become binding. The impacts are more marked as the flow limit decreases. Fig. 2 shows that even for the relatively narrow interval of uncertainty considered, the volatility in the λ_i may be marked. In cases *B*.0–*B*.4, λ_8 experiences a variation in the range [1.021, 5.321] while the λ_{26} variation lies in the range [1.034, 1.902]. The changes in the generators' power outputs are major as shown in Fig. 3 and lead to considerable swings in S^G. The widest variations, as seen in Fig. 4, affect the generator at bus 27 which has an increase of S^G of about 260% in case *B*.1 and of about 50% in case *B*.4. The generator at bus 23 has a doubling of S^G in case *B*.1 and a decrease of about 20% in *B*.3. The market efficiency changes measured by the changes in S^S with respect to the case *B*.0, are imperceptibly small with a magnitude less than 0.1%. This example is

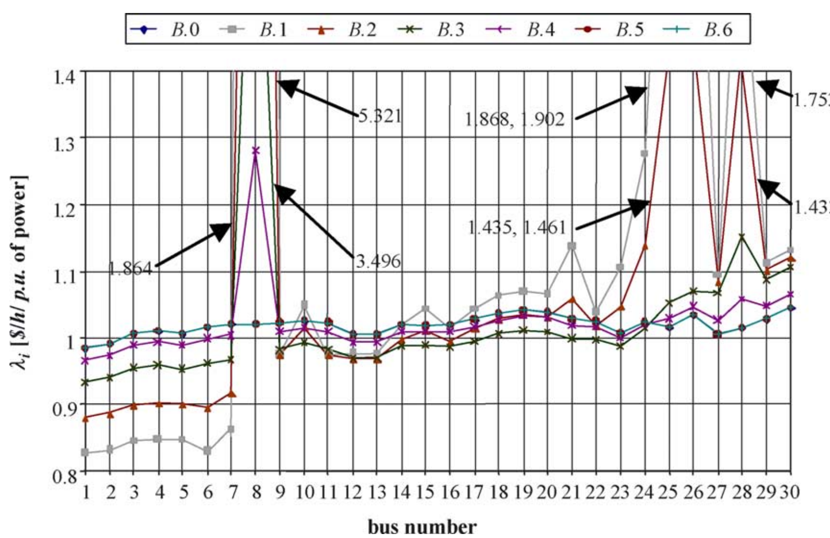

Fig. 2. The change in λ_i for cases of uncertain in system parameter values.

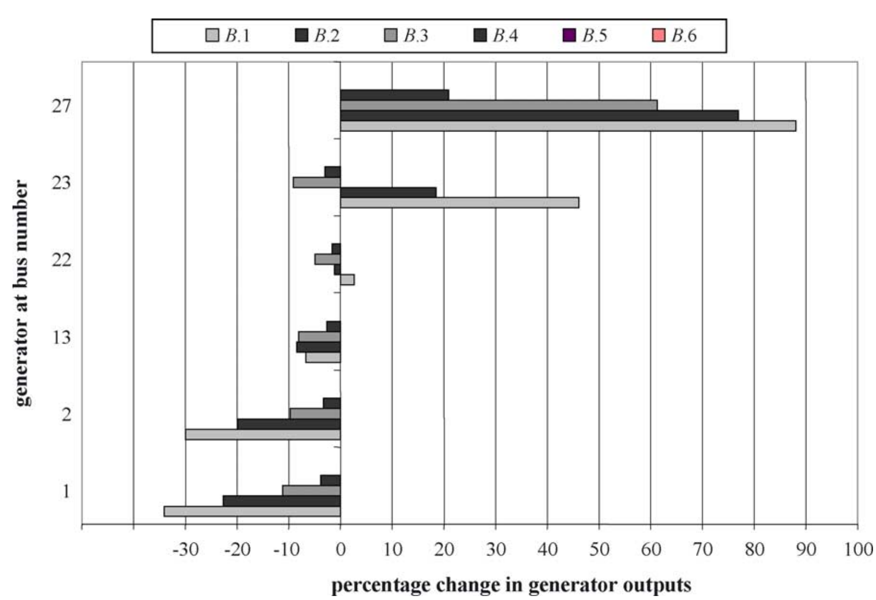

Fig. 3. The change in the generator outputs, for cases of uncertain in system parameter values, with respect to the base case.

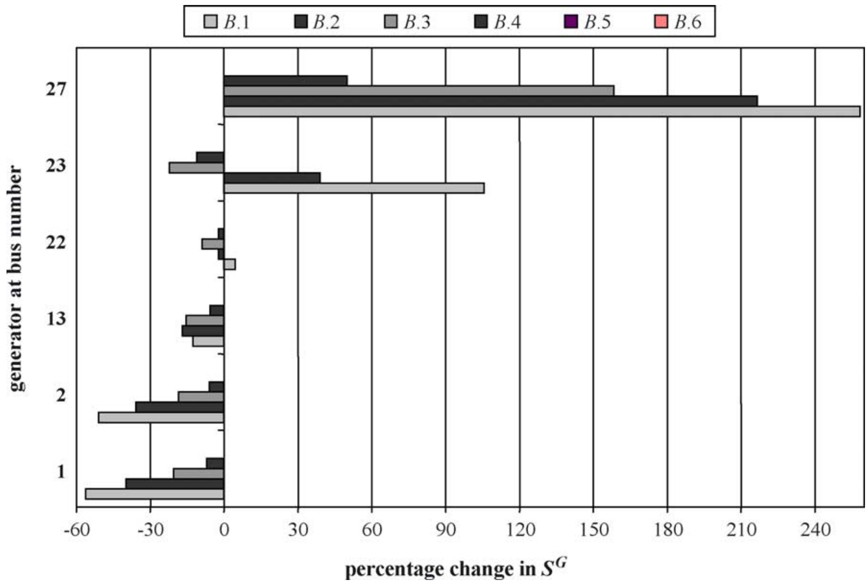

Fig. 4. The changes in the generators' surplus, for cases of uncertain in system parameter values, with respect to the base case.

representative of the impacts of uncertainty in system parameter values both on the market prices and the surpluses of each generator without much change in the market efficiency. Since the generators may be independent entities, disputes between generators and the central decision-making authority may arise.

4.3. Non-economic flows

We illustrate the notion of 'non-economic flows' on the system based on the 30-bus network, with the given line flow limits and an additional 8 MVA limit imposed on lines 15–23. The flows on a subset of lines of the system and the λ_i at their terminal buses are given in Table 3. The results indicate the unintuitive behavior, at least from an economic point of view, that arises at the optimum in terms of flow of power from a higher to a lower priced node. Consider, for example, the flow from buses 28–6 with $\lambda_6=0.975$ and $\lambda_{28}=1.120$. Similar flows are observed between the bus pairs {25, 24}, {24, 22}, {27, 25}, {21, 10}, {22, 10} and {15, 14}. Such flows are rather unintuitive since in the transport of other commodities the direction is, typically, from a lower to a higher priced node. This behavior is due to the network constraints.

Table 3
Non-economic flows

Line		Real power flow from i to j (p.u.)	Units are \$/h per p.u. of power		
Bus i	Bus j		λ_i	λ_j	$\Delta\lambda$
28	6	0.040	0.975	1.120	0.145
25	24	0.103	1.015	1.044	0.029
24	22	0.054	1.004	1.015	0.011
27	25	0.141	1.044	1.055	0.011
21	10	0.007	1.001	1.006	0.005
22	10	0.029	1.001	1.004	0.003
15	14	0.005	0.999	1.001	0.002

4.4. Impacts of voltage limit specification

We studied the impacts of voltage limit specification on the OPF result. For a representative illustration, we modified the IEEE 30-bus network by relaxing the line power flow limits. We evaluated the impacts of five different voltage profile specifications on nodal prices and allocations of supply among the generators. The profile specifications are given in Table 4 and plotted in Fig. 5. The most significant impacts are in the nodal price volatility, as shown in Fig. 6; the nearly flat nodal prices of the base case $D.0$ undergo wide variations in two of the four change cases. Case $D.4$ results in a wide range [0.942, 2.755] and case $D.3$ results in the widest range [0.501, 2.619]. Cases $D.1$ and $D.2$ maintain the flatness of nodal prices with variations in the ranges [0.977, 1.048] and [0.98, 1.049], respectively.

Even more pronounced changes arise in the allocation of supply among the generators and their associated surpluses. The wide changes in the S^G with respect to the base case are shown in Fig. 7. For example, the generator at bus 23 can experience swings in the range [−45%, +45%] for the specified voltage profiles. In OPF applications, different specifications of the voltage profile may appear as discriminatory in favoring certain market players over others and may give rise to disputes. We note that these changes are not accompanied by any noticeable changes in

Table 4
Voltage profile specifications

Case	Voltage profile (p.u.)
$D.0$	$0.95\le V_i\le 1.05\,\forall i$
$D.1$	$V_i=1.0$ $i=3, 4, 10$ and $0.95\le V_i\le 1.05$ $i\ne 3, 4, 10$
$D.2$	$0.98\le V_i\le 1.02$ $\forall i$
$D.3$	$0.98\le V_i\le 0.99$ $i=10, 11, 14, 20, 26$ and $0.95\le V_i\le 1.05$ otherwise
$D.4$	$V_i=0.98$ $i=9, 19, 21$ and $0.95\le V_i\le 1.05$ $i\ne 9, 19, 21$

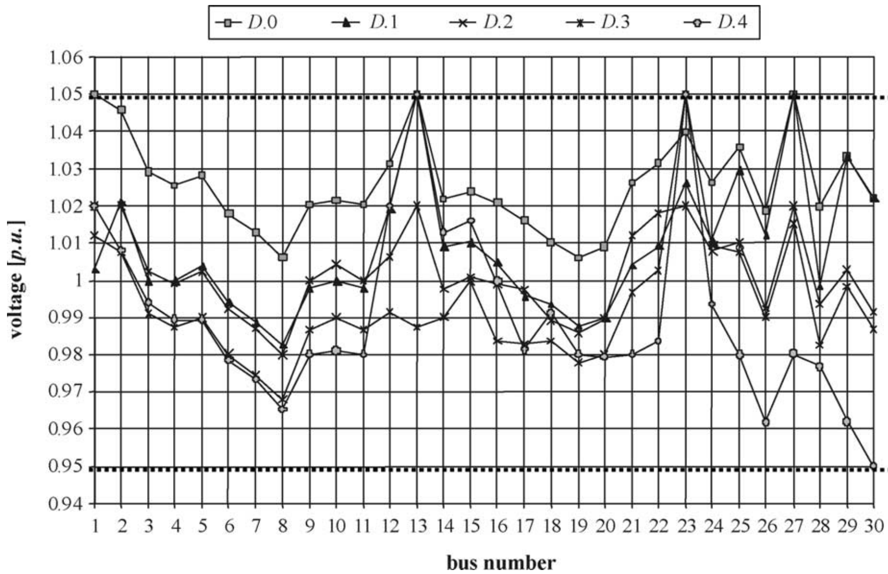

Fig. 5. Voltage profiles for different voltage limit specifications.

the social surplus and consequently the market efficiency is not impacted.

4.5. Impacts of line flow limit specification

Line flow limits may have seasonal variations and are strongly dependent on ambient conditions. The limit values are typically not known with certainty and so their specification is done by the IGO and involves a certain amount of arbitrariness. We reproduce a representative illustration of the impacts of the specification using the system based on the IEEE 118-bus network. The bus voltages are permitted to vary in the range [0.9, 1.1] p.u. We define as the base case $E.0$ without any line flow limits and we consider five additional cases of line flow limits, as shown in Table 5.

The different line flow limit specifications lead to price volatility as depicted in Fig. 8. For example, λ_{73} and λ_{26} experience similar wide ranges of variation of [0.833, 0.956] and [0.854, 0.973], respectively. Figs. 9 and 10 represent the corresponding changes in the generation levels and the generator surpluses, for a subset of the generators of the system. The different line flow limits impact the generator outputs and the associated S^{G}. For example, in case $E.1$, generator at bus 25 reduces its output by 25% with respect to case $E.0$, and by about 5% in case $E.5$. The impacts on S^{G} are even more marked at 45% and nearby 10%, respectively. While the efficiency of the market is not affected considerably,

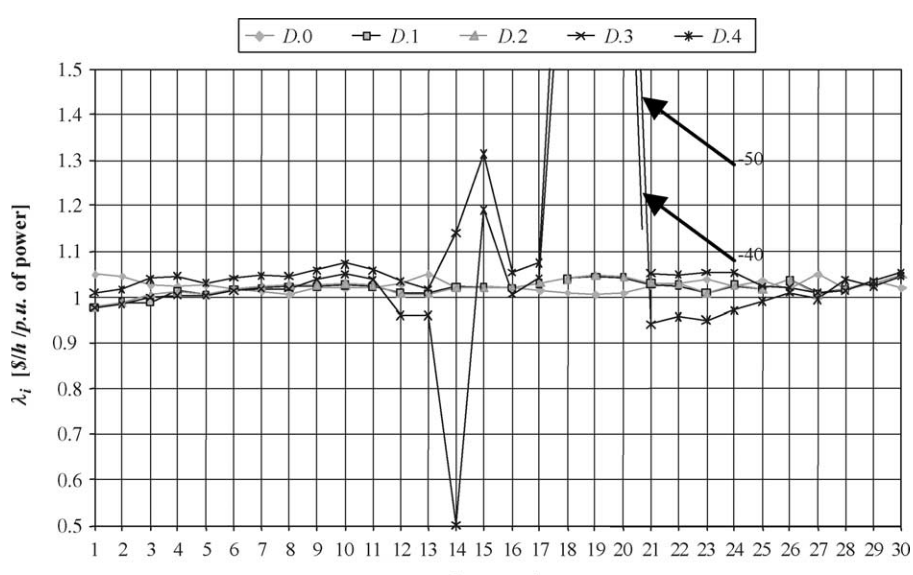

Fig. 6. The change in λ_i for different voltage limit specifications.

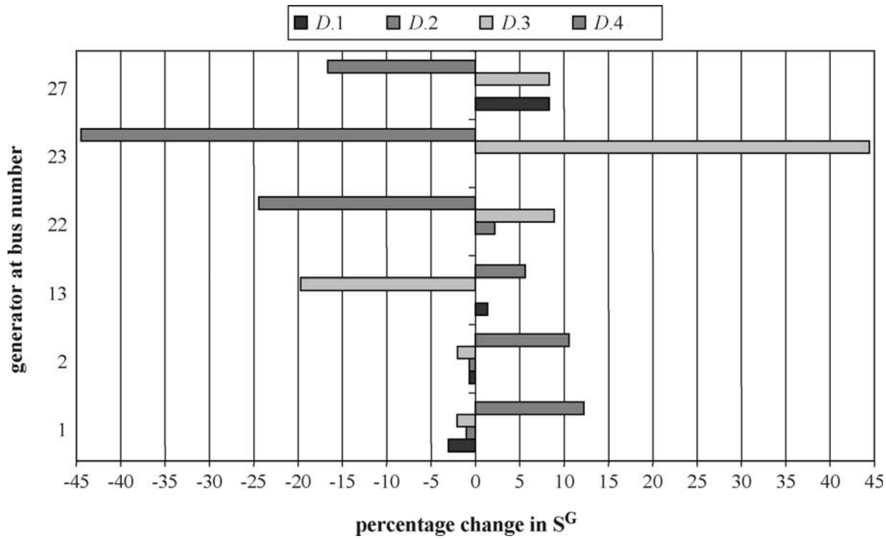

Fig. 7. The changes in the generators' surplus, for different voltage limit specifications, with respect to the base case.

Table 5
Line flow limits specifications

Case	Line flow limits (p.u.)
E.0	None
E.1	Line 26–30 = 0.75
E.2	Line 30–38 = 1.35
E.3	Line 63–65 = 0.8
E.4	Line 26–30 = 0.83, line 38–65 = 0.68, line 81–80 = 0.9, line 100–103 = 1.13
E.5	Line 70–71 = 0.9, line 68–69 = 1.2, line 17–113 = 0.9, line 37–38 = 1

S^M undergoes relatively high variations for the different line flows specifications, as shown in Fig. 11. As such the IGO may affect its own surplus under the different specifications. This example shows how different choices of a set of operational constraints may affect the market prices differently and allocate the producer surplus in favor of certain market players. Such situations are likely to cause disputes between the market players and the central decision-making authority.

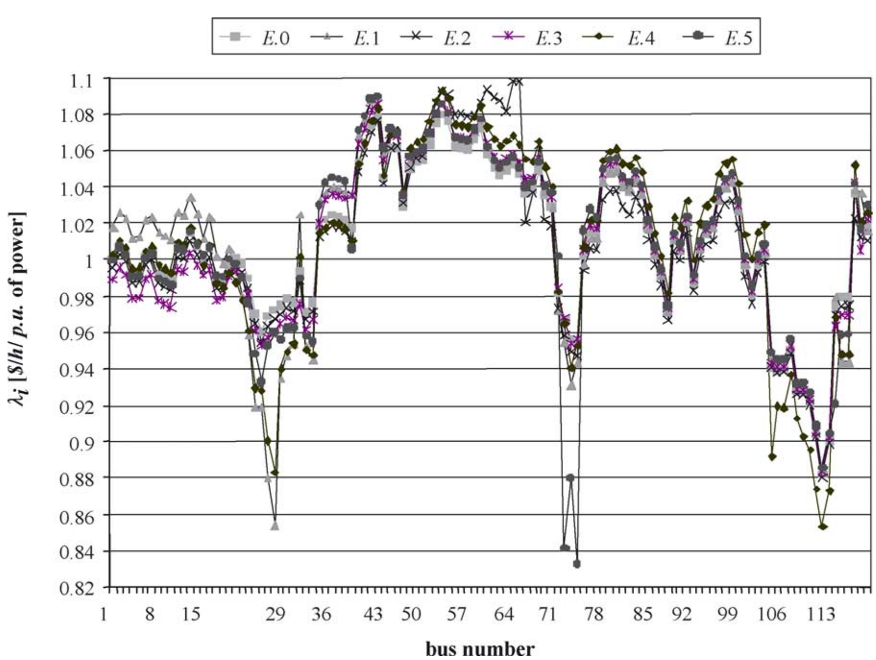

Fig. 8. The change in λ_i for different line flow limit specifications.

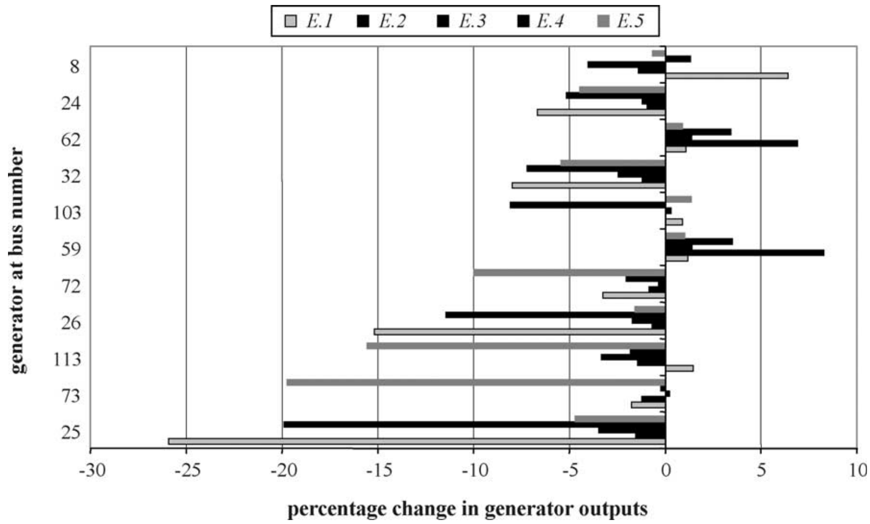

Fig. 9. The change in the generator outputs, for different line flow limit specifications, with respect to the base case.

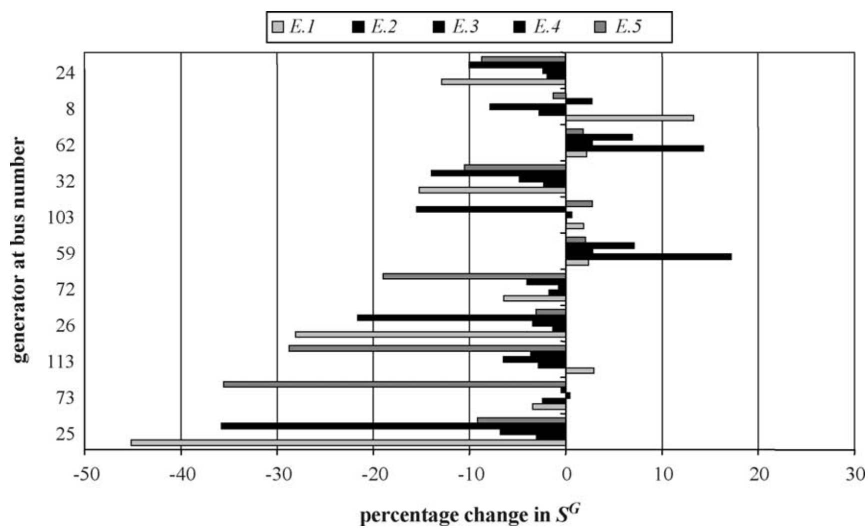

Fig. 10. The changes in the total generators surplus, for different line flow limit specifications, with respect to the base case.

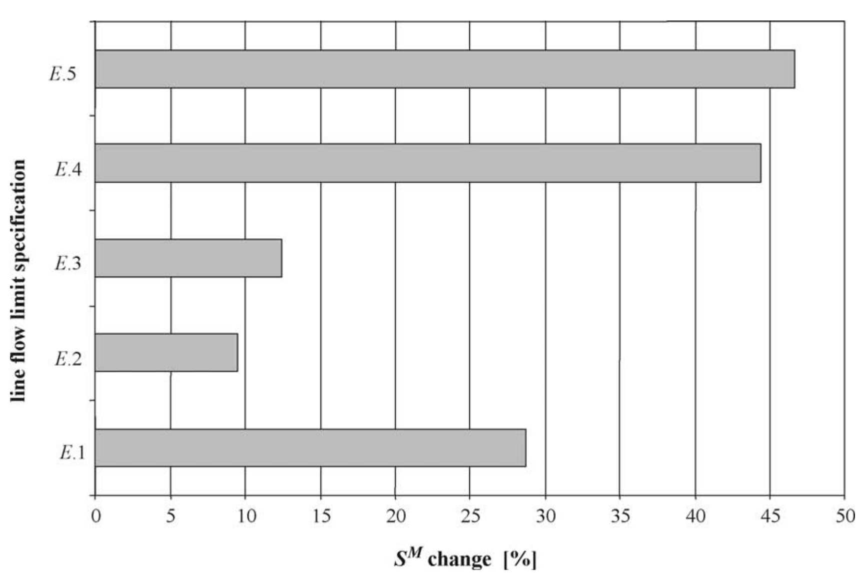

Fig. 11. S^M changes, for different line flow limit specifications, with respect to the base case.

5. Concluding remarks

The OPF, a central decision-making tool, requires particular care in the application to the more decentralized decision-making environment of competitive electricity markets. The inherent characteristics of the OPF, such as the flatness of the objective function around the optimum and the level of discretionary power vested in the IGO — the central decision-making authority-impact the market outcomes and affect the market players differently. In particular, major shifts in the power generated among competing producers and corresponding surpluses and nodal prices are possible, even though market efficiency remains virtually unchanged. Moreover, in terms of nodal prices, the variability impacts the market prices widely both spatially and over time giving rise to a marked volatility. A broad range of examples illustrating the possible outcomes in the electricity markets was discussed in detail in the paper. The variability effects in the OPF results may lead to disputes among the players. Consequently, mechanisms need to be developed for effective dispute resolution.

In addition, OPF application to competitive electricity markets would benefit from research in a number of areas. One is the formulation of the OPF to incorporate a more detailed representation of the role of loads as active players in electricity markets. The explicit modeling of load responsiveness may have major impacts on the OPF applications. The contingency selection requires considerable attention due to the major impacts shown to have. Key questions focus on the contingency selection and ranking criteria so as not to result in perceived discriminatory behavior against any market players. A second area is the development of some post-OPF solution processing mechanism aimed at smoothing out the variability of the OPF results. Research results on these topics will be reported in future papers.

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