

## Discussion of “A Physical-Flow-Based Approach to Allocating Losses in a Transaction Framework<sup>1</sup>”

J. W. Bialek

I would like to congratulate the authors on this very timely and interesting contribution to the current restructuring debate.

For the loss allocation purposes, it would be convenient to represent the total loss as the sum of components due to individual transactions. This however is difficult to achieve as the losses are a nonseparable function of all the transactions. To circumvent this problem, the authors' approach seems to be to stop half-way in their derivations by substituting only one  $(\theta_i - \theta_j)$  term in the loss equation (12) by (17), i.e., by the linear combination of transactions. Consequently the loss expressed by (21) is shown to be the sum of components due to individual transactions with the remaining  $(\theta_i - \theta_j)$  terms in (20) treated as coefficients which are presumably calculated from the load flow. Obviously, as the authors show in (22), if their derivations are logically carried all the way through and the remaining  $(\theta_i - \theta_j)$  terms in (20) are also substituted by (17), the loss formula would contain both the main quadratic terms due to individual transactions and cross-terms due to interactions between the transactions. Thus, it seems that the main approximation of the methodology (apart from using the dc load flow model) lies in treating the  $(\theta_i - \theta_j)$  terms in (20) as constant coefficients while in fact they are also functions of the transactions. I wonder if the authors would agree with this interpretation.

As the authors' methodology is based on the standard dc load flow model, it would be interesting to check if their methodology is consistent with the traditional nodal marginal loss charging [9]. This could be easily checked for bilateral transactions, when a power transfer from node  $i$  to node  $j$  would be allocated a loss equal to the amount of transacted power multiplied by the difference between the marginal loss factors in nodes  $j$  and  $i$  (and divided by 2 to circumvent the problem of loss over-recovery). To be consistent with the authors' assumptions, the marginal loss factors would have to be derived from the dc load flow model. I wonder if the authors undertook that sort of tests and, if not, whether they think the results would be similar.

The next question is whether the results of the proposed loss allocation methodology depend on the choice of the marginal (slack) node. As the authors correctly state in the introduction “transacting entities need *a priori* information [on the loss allocation] to evaluate impacts of various transactions under consideration.” However the location of the marginal node is known only *a posteriori*. If the loss allocation due to the proposed methodology does depend on the choice of the marginal node then this nonuniqueness could open possibility for a legal challenge from the users disadvantaged by a particular choice of the marginal node. By contrast, the scheme proposed in [4] does not depend on the assumed location of the marginal node.

And finally, the paper seems to include a small but frustrating mistake. In the last row of (19), the second symbol  $\sum_{m=1}^M$  seems to be superfluous. This can be checked by comparing with (20).

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<sup>1</sup>G. Gross and S. Fao, *IEEE Trans. Power Systems*, vol. 15, no. 2, pp. 631–637, May 2000.

## Closure to Discussion of “A Physical-Flow-Based Approach to Allocating Transmission Losses in a Transaction Framework”

George Gross and Shu Tao

The authors are grateful to Dr. Bialek for reading our paper<sup>1</sup> with care and for his thoughtful comments.

The comments focus on the expression for the losses and for the approximation used. Dr. Bialek pointed out a typing error in the paper. The second summation sign in (19) is superfluous and should be omitted. Unfortunately, we failed to notice this mistake until Dr. Bialek brought it to our attention.

Concerning Dr. Bialek's interpretation of the expression in (20) for the loss approximation term, his observations are right on target. As he noted, we rewrote the second  $(\theta_i - \theta_j)$  term as a linear function of all the network transactions using the D.C. power flow results. However, we left the first  $(\theta_i - \theta_j)$  term in (19) as a “parameter” whose value is obtained from the A.C. power flow results. Consequently, we represented the total system losses explicitly in terms of all the transactions in such a way as to lead to a physically meaningful allocation mechanism. We used the expression for  $\lambda^{(m)}$  defined by (22)–(23) to reflect the quadratic nature of the total  $|I|^2 R$  loss term expressed in (21).

The second question raised by Dr. Bialek concerned the issue of marginal loss factors. We did not undertake the tests referred to by Dr. Bialek. But, the discussion in the Appendix following the statement in (27) indicates that  $\lambda^{(m)}$  in (20) approximates the loss sensitivity with respect to the amount of transaction  $m$ ,  $\partial l / \partial t^{(m)}$ . Retracing the steps and definitions used in the derivation of  $\lambda^{(m)}$ , it seems reasonable that the statement of Dr. Bialek is correct.

Let us next discuss the question on the choice of the slack bus. The allocation scheme is developed using an arbitrary slack bus but, in fact, is independent of the choice of the system slack bus. Consider the expression in (19). Since the A.C. and D.C. power flow results  $(\theta_i - \theta_j)$  and  $(\hat{\theta}_i - \hat{\theta}_j)$ , respectively, are independent of the choice of the slack bus,  $\lambda^{(m)}$  in (20) is also independent of the choice of the slack bus. While a different choice of the slack bus may lead to a change in the value of  $l$ , such a difference is very small. In addition, the extension of the methodology presented in [1] shows the evaluation of the losses for multiple slack buses.

We thank Dr. Bialek for his comments and for giving us the opportunity to clarify specific points in the paper.

### REFERENCES

- [1] S. Tao and G. Gross, “Transmission loss compensation in multiple transaction networks,” *IEEE Trans. Power Systems*, vol. 15, no. 3, pp. 909–915, Aug. 2000.

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<sup>1</sup>G. Gross and S. Tao, *IEEE Trans. Power Systems*, vol. 15, no. 2, pp. 631–637, May 2000.