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Generation Supply Bidding in Perfectly Competitive Electricity Markets*

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Abstract

This paper reports on the development of a comprehensive framework for the analysis and formulation of bids in competitive electricity markets. Competing entities submit offers of power and energy to meet the next day's load. We use the England and Wales Power Pool as the basis for the development of a very general competitive power pool (CPP) framework. The framework provides the basis for solving the CPP dispatcher problem and for specifying the optimal bidding strategies. The CPP dispatcher selects the winning bids for the right to serve load each period of the scheduling horizon. The dispatcher must commit sufficient generation to meet the forecasted load and reserve requirements throughout the scheduling horizon. All the unique constraints under which electrical generators operate including start-up and shut-down time restrictions, reserve requirements and unit output limits must be taken into account. We develop an analytical formulation of the problem faced by a bidder in the CPP by specifying a strategy that maximizes his profits. The optimal bidding strategy is solved analytically for the case of perfect competition. The study in this work takes into account the principal sources of uncertainty—the load forecast and the actions of the other competitors. The formulation and solution methodology effectively exploit a Lagrangian relaxation based approach. We have conducted a wide range of numerical studies; a sample of numerical results are presented to illustrate the robustness and superiority of the analytically developed bidding strategies.

Keywords: uniform price auction, competitive power pool, Lagrangian relaxation, unit commitment, generator optimal bidding policy

1. Introduction

The privatization of the electric supply industry in England and Wales marks a significant development in the restructuring of the electricity business around the world. The most striking feature of the new system is the England and Wales Power Pool (EWPP) which plays a key role in enabling competition in electricity markets. The Pool is a centralized entity that controls the scheduling and dispatch of generation to meet load around the clock and operates the electricity spot market. Virtually all power is transacted through the Pool and the multiple buyers and sellers have set up what has become the largest competitive electricity market in the world. It is this pool-based competitive market for power that provides the basis for the work in this paper. Our focus is on the position of a seller in

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such a structure. In particular, we consider the task such a seller faces in constructing the offer to sell power, or bid, to take best advantage of the sealed bid auction given the generating resources, costs and constraints. We refer to this as the *optimal bidding strategy problem*. To formulate and attack this problem, we develop a mathematical framework of the operation of a competitive power pool. Our work explicitly considers the competitive bidding mechanism in electricity and takes into account the unique features and problems associated with the generation of electrical power.

We provide a brief review of the EWPP operation (White et al., 1990). The Pool dispatcher is charged with determining on a daily basis the schedule for the so-called availability declaration period (ADP), a 39-hour period running from 9:00 p.m. on *day* 0, the bid submittal day, to 12:00 noon on *day* 2. The generation schedule for the period known as the *schedule day*, running from 5:00 a.m. on *day* 1 to 5:00 a.m. on *day* 2, is then accepted as the actual schedule for the next day. By 10:00 a.m. each day, the dispatcher produces a forecast of national demand for every half hour of the ADP. Also by 10:00 a.m., each bidder must submit an *offer file* for each of his *gensets*. A genset is a unit or a group of units which are considered together for the purposes of the dispatch. The offer file contains information on the *availability* [maximum capacity] of the genset for each of the 78 ADP half hours; the *offer price* of the genset; the genset *start-up prices*; and, the genset operational characteristics. The prices charged from the Pool for operation and start up need not have any relation to actual costs. There is no obligation on a bidder to reveal its genset's true costs. The genset offer price is specified as a piece-wise linear function known as the Willans line (Littlechild, 1991). A maximum of three segments can be submitted per genset. Figure 1 shows an example.

The Willans line is completely specified by at most 8 parameters: the no-load price b_i^o, three *incremental prices* $\eta_i^1, \eta_i^2, \eta_i^3$, two *elbow points* $\epsilon_i^1, \epsilon_i^2$, and the minimum and maximum power output of the genset $p_i^{\min}$ and $p_i^{\max}$.

Using the information submitted by the generators, the dispatcher determines the schedule of generation to meet the forecasted demand at minimum cost to the Pool. This problem is essentially the unit commitment. The Pool Rules specify the use of a scheduling algorithm known as *Settlement Goal*, which uses heuristics to perform this task (Executive Committee

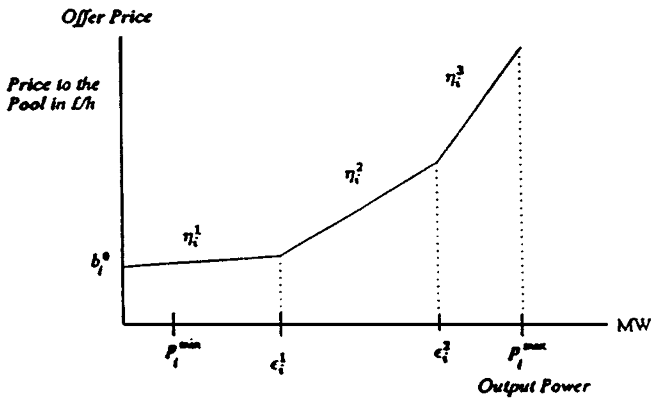

Figure 1. Example of a Willans line.

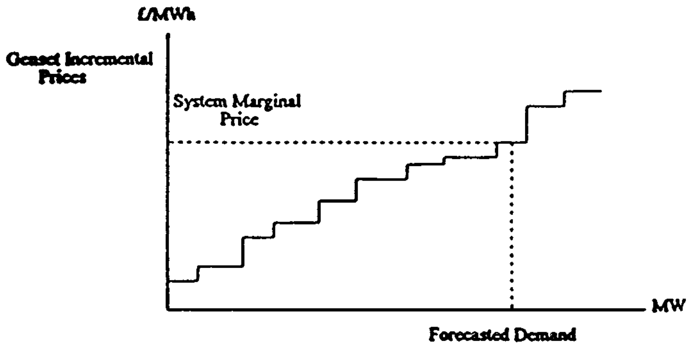

Figure 2. Determination of the system marginal price.

for the Pooling System in England and Wales, 1993). Basically, the dispatcher arranges the bidding genset blocks in order of increasing price to form a *merit order list* for each ADP half hour.

The price of the most expensive genset dispatched in any half hour t is designated as the System Marginal Price (SMP_t) (Executive Committee for the Pooling System in England and Wales, 1993), for that half hour, as is shown in figure 2. Each genset that operates during half hour t receives a payment that includes SMP_t for each MWh of energy generated during that time. Hence every genset is paid more than or equal to the price specified in the offer file. Generators also receive the submitted start-up price each time the unit is started up.

The generation schedule that results is known as the unconstrained schedule since transmission systems constraints are ignored. Once the unconstrained schedule is determined, the operating schedule for each genset is provided by the Pool dispatcher.

The bidder, thus, submits a bid for the *right to serve load*. Under competitive conditions, the bidder prices must be sufficiently low for the Pool dispatcher to select the unit to be included in the commitment list. Since the bidder receives a payment which is greater or equal to its bid price the challenge is to formulate a bid that permits the bidder to maximize profits. Given the large-scale and nonlinear nature of the problem, the auction theory literature (Vickrey, 1961; Milgrom, 1981; Wilson, 1977) has limited application. The approach developed here is new and exploits well the structural characteristics of the analytical CPP framework. The optimal strategy under conditions of perfect competition constructed with the analytical approach results in bidding at cost and offering capacity for dispatch to the maximum extent. This result is well known in microeconomics for a general commodity without the complex constraints such as are present in power system operations. The remarkable contribution of this work lies in the explicit representation of the various constraints and considerations under which power systems operate. The analytical development not only allows the optimal bidding strategy formulation but also is useful in providing estimates of bidder profit volatility and analytical expressions for the evaluation of the returns on investments aimed at improving the performance of generating units. We have conducted a wide range of numerical studies; a sample of numerical results are given to illustrate the robustness and superiority of the analytically developed optimal bidding strategies.

The paper has four additional sections. In Section 2 we develop the general framework of a Competitive Power Pool (CPP) in which generators compete to serve load. This very general model includes the EWPP as a special case. In Section 3, we use the mathematical CPP framework to formulate the general optimal bidding strategy problem. We solve this for the case of perfect competition. Section 4 presents the numerical results, which illustrate and elucidate the analytical results. The final section summarizes the key results and presents some directions of future research.

2. The Competitive Power Pool Framework

We develop a general *competitive power pool* (CPP) framework which we use to formulate and analyze optimal bids. The commitment and dispatch of units in the CPP are based on a competitive auction procedure. The market sellers, typically generators, submit a sealed bid stating the price at which they are willing to sell power. The CPP dispatcher, the entity responsible for coordinating all energy transactions with the CPP, selects the set of least expensive units to meet the forecasted demand. The CPP structure incorporates the salient features of the EWPP.

We formulate the CPP dispatcher problems by considering the bids received from the set of M bidders. Each bid β_i has three components:

- *The bid variable price* $b_i^f(\cdot)$: describes the per hour cost to the CPP as a function of MW provided. We assume that the bid variable price is a piecewise linear function mapping $\mathcal{L}[p_i^{\min}, p_i^{\max}]$ into $\mathcal{R}$, the set of real numbers where $p_i^{\min}(p_i^{\max})$ is the minimum (maximum) output of bidder i's unit.
- *The bid start-up price* $b_i^s(\cdot)$: describes the cost incurred by the CPP whenever unit i is started up. We assume that the start-up price is a function of the down time of the unit with $b_i^s : [0, \infty) \to \mathcal{R}$.
- *The bid offered capacity* $\underline{a}_i = [a_{i,1}, a_{i,2}, \ldots, a_{i,T}]^T$: a vector whose tth component $a_{i,t}$ is the maximum capacity offered by bidder i to the CPP dispatcher for use in time period t.

We assume that the bidder submits the correct operational data for the unit. These consist of $p_i^{\min}$, $p_i^{\max}$, $r_i^{\max}$, the maximum spinning reserve capability of unit i and τ_i^u and τ_i^d the unit i minimum up and down times, respectively. The generator is *not* obliged to reveal any information concerning true costs. However, since a bidder must fulfill any schedule requested by the CPP dispatcher, the operational data must appropriately reflect actual operational information lest a schedule be imposed which is physically infeasible. The unit operational data is *not* a decision variable for bidder i. On the other hand, the bid variable price, bid start-up price and bid offered capacity are *strategic* decision variables that the bidder selects to maximize profits.

We define a bid of bidder i to be the triple $\beta_i = \{b_i^f(\cdot), b_i^s(\cdot), \underline{a}_i\}$. A bid β_i is admissible if $b_i^f(\cdot) \in \mathcal{L}[p_i^{\min}, p_i^{\max}]$, $b_i^s(\cdot) \in \mathcal{C}[0, \infty)$ and $\underline{a}_i \geq 0 \in \mathcal{R}^T$, where $\mathcal{C}[a, b](\mathcal{L}[a, b])$ denotes the set of continuous (piece-wise linear) functions on the interval [a, b].

The CPP dispatcher must commit sufficient capacity to supply the forecasted load plus the reserve requirements. Security and reliability considerations impose reserve requirements to allow the system to respond to any contingencies that may occur due to uncertainty in forecasted operating conditions. Examples are load forecast errors, sudden surges of demand and forced outages of equipment. The reserve requirements are typically specified as a deterministic quantity and are a function of the system load and the capacity of the largest unit committed. Contributors to reserve are synchronized units not operating at full capacity, fast start units, such as gas turbines, and interruptible loads. The *t*-minute generating unit reserve is the amount of additional load with respect to its current operating point, which the unit is capable of picking up in *t*-minutes. The *t*-minute system reserve is the sum of all *t*-minute generating unit reserves of the committed units. The reserve requirements may be specified as spinning reserves, usually defined as the 5-minute system reserve, or as operating reserves, usually defined as the 10-minute or the 30-minute system reserve. Each unit may offer a reserve capacity that does not exceed its physical capability given its operating level.

A key consideration in the commitment of units is the limitations due to the thermal characteristics of generating plants. The minimum up time (down time) is the limit constraining a unit to require it to operate (remain shut) once it is committed (shut down). These limits are imposed to provide time for temperature equalization within the turbine so as to maintain thermal stresses due to temperature differentials within limits of safety. The limits are a function of unit size and type.

We state the CPP dispatcher problem using the notation of Table 1 and the definition of the T-dimensional vectors $\underline{D} = [D_1, D_2, \ldots, D_T]^T$, $\underline{R} = [R_1, R_2, \ldots, R_T]^T$, $\underline{u}_i = [u_{i,1}, u_{i,2}, \ldots, u_{i,T}]^T$, $\underline{p}_i = [p_{i,1}, p_{i,2}, \ldots, p_{i,T}]^T$ and $\underline{r}_i = [r_{i,1}, r_{i,2}, \ldots, r_{i,T}]^T$ and the MT-dimensional vectors $\underline{u} = [\underline{u}_1^T, \underline{u}_2^T, \ldots, \underline{u}_M^T]^T$, $\underline{p} = [\underline{p}_1^T, \underline{p}_2^T, \ldots, \underline{p}_M^T]^T$, and $\underline{r} = [\underline{r}_1^T, \underline{r}_2^T, \ldots, \underline{r}_M^T]^T$. The CPP dispatcher problem determines the most economic dispatch that satisfies the forecasted demands and required reserves without violating physical and operating constraints. This is denoted by

$$P(\underline{D}, \underline{R}) = \min_{\underline{u}, \underline{p}, \underline{r}} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) u_{i,t} + b_i^s(\tau_{i,t-1})(1 - u_{i,t-1}) u_{i,t} \right] \right\} \tag{1}$$

$$\text{subject to} \quad \left. \begin{array}{l} D_t - \sum_{i=1}^{M} p_{i,t} u_{i,t} = 0 \\ R_t - \sum_{i=1}^{M} r_{i,t} u_{i,t} \le 0 \end{array} \right\} \quad \forall t = 1, 2, \ldots, T \tag{2}$$

$$\left. \begin{array}{l} p_i^{\min} \le p_{i,t} \le p_i^{\max} \\ 0 \le p_{i,t} \le a_{i,t} \\ 0 \le r_{i,t} \le \min\left\{ r_i^{\max}, p_i^{\max} - p_{i,t} \right\} \\ u_{i,t} \in \{0, 1\} \\ \tau_{i,t} \text{ satisfies the } \tau_i^d \text{ and } \tau_i^u \text{ constraints} \\ \tau_{i,0} \text{ is given} \end{array} \right\} \quad \begin{array}{l} \forall i = 1, \\ 2, \ldots, M \\ t = 1, \\ 2, \ldots, T \end{array} \tag{3}$$

We refer to Eqs. (1)–(3) as the *primal form* of the CPP dispatcher problem. The triple $\sum_i = \{\underline{u}_i, \underline{p}_i, \underline{r}_i\}$, is called an *operating schedule for unit i* and $\sum = \{\underline{u}, \underline{p}, \underline{r}\}$ is a *system schedule*. The CPP dispatcher problem is the determination of the optimum system schedule

Table 1. Notation.

Time parameters	T is the number of time periods in the scheduling horizon
	$t = 1, 2, \ldots, T$, is the time period index
System parameters	D_t is the system demand in time period t
	R_t is the system reserve requirement in time period t
Bidder data	M is the number of bidders participating in the CPP
	$i = 1, 2, \ldots, M$ is the bidder index
Unit variables	$u_{i,t} = \begin{cases} 1 & \text{if the unit is in operation} \\ 0 & \text{if the unit is shut down} \end{cases}$ is the *status* of unit i in time period t
	$p_{i,t}$ is the real power output of unit i in time period t
	$r_{i,t}$ is the reserve provided by unit i in time period t
	$\tau_{i,t}$ is the downtime of unit i at the end of time period t

$\sum^{opt} = \{\underline{u}^{opt}, \underline{p}^{opt}, \underline{r}^{opt}\}$ that minimizes total cost to the CPP. We denote by

$$\Omega_i(\underline{a}_i) \triangleq \left\{ \sum_i : \sum_i = \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \text{ satisfies Eq. (3)} \right\} \tag{4}$$

the set of feasible operating schedules for unit i.

The objective function in Eq. (1) is the sum of the variable and the start-up prices. For each unit i, the cost incurred by the CPP when unit i serves demand $p_{i,t}$ in period t is given by $b_i^f(p_{i,t})u_{i,t}$. Start-up costs are incurred by the CPP if unit i is shut down in time period $t-1$ and is operating in period t, i.e., if $u_{i,t-1} = 0$ and $u_{i,t} = 1$. The downtime when started up is the downtime of unit i at the end of period $t-1$, $\tau_{i,t-1}$. Note that with Δ_T as the length of the time period we express $\tau_{i,t}$ recursively in terms of $u_{i,t}$, $t = 1, \ldots, T$ and $\tau_{i,0}$:

$$\begin{aligned} \tau_{i,t} &= (\tau_{i,t-1} + \Delta_T)(1 - u_{i,t}) \quad \forall t = 1, 2, \ldots, T \\ &\tau_{i,o} \text{ is given.} \end{aligned} \tag{5}$$

The objective function is nonconvex. The state space admits complex minimum up and down time constraints and is discrete in $\underline{u}_i$, which introduces nonconvexity into the set of feasible schedules. Considering that the time frame in the EWPP is 78 half hour periods and the number of units can exceed 200, this is a large scale and complex nonlinear optimization problem.

The use of Lagrangian relaxation (Shaw et al., 1985; Luenberger, 1969) in the solution of the CPP dispatcher problem may be effectively exploited. This approach leads to the decomposition of the problem in terms of each bidder and results in the economic interpretation of the Lagrange multipliers as prices. The Lagrangian relaxation technique involves the construction and solution of a modified problem in which the system-wide constraints on demand and reserve constraints, which couple all bidders, are used to augment the primal objective function with their associated Lagrange multipliers. The new problem does not enforce the demand and reserve constraints and is therefore "relaxed". All bidder constraints, however, are enforced.

We define the T-dimensional vectors $\underline{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_T]^T$ and $\underline{\mu} = [\mu_1, \mu_2, \ldots, \mu_T]^T$. Here λ_t and $\mu_t \geq 0$ are the Lagrange multipliers, which are non-negative for inequality constraints (Luenberger, 1969), for the demand and reserve constraints in time period t, respectively. The *Lagrangian relaxation* of the CPP dispatcher problem is

$$\begin{aligned}
\min_{\underline{u},\underline{p},\underline{r}} \quad & \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t-1})(1-u_{i,t-1}) \right] u_{i,t} \right. \\
& \left. + \sum_{t=1}^{T} \lambda_t \left(D_t - \sum_{i=1}^{M} p_{i,t} u_{i,t} \right) + \sum_{t=1}^{T} \mu_t \left(R_t - \sum_{i=1}^{M} r_{i,t} u_{i,t} \right) \right\} \qquad (6) \\
\text{subject to} \quad & \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \in \Omega_i(\underline{a}_i) \quad \forall i = 1, 2, \ldots, M
\end{aligned}$$

We can rewrite the Lagrangian relaxation as

$$\begin{aligned}
\min_{\underline{u},\underline{p},\underline{r}} \quad & \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t-1})(1-u_{i,t-1}) - \lambda_t p_{i,t} - \mu_t r_{i,t} \right] u_{i,t} \right\} \\
& + \underline{\lambda}^T \underline{D} + \underline{\mu}^T \underline{R} \qquad (7) \\
\text{subject to} \quad & \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \in \Omega_i(\underline{a}_i) \quad \forall i = 1, 2, \ldots, M.
\end{aligned}$$

we removed the constant terms $\underline{\lambda}^T \underline{D}$ and $\underline{\mu}^T R$ from the minimand. The Lagrangian function

$$\begin{aligned}
& \phi(\underline{\lambda}, \underline{\mu}; \underline{D}, \underline{R}) \\
& \triangleq \min_{\underline{u},\underline{p},\underline{r}} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t-1})(1-u_{i,t-1}) - \lambda_t p_{i,t} - \mu_t r_{i,t} \right] u_{i,t} \right. \\
& \qquad \left. + \underline{\lambda}^T \underline{D} + \underline{\mu}^T \underline{R} : \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \in \Omega_i(\underline{a}_i), i = 1, 2, \ldots, M \right\} \qquad (8)
\end{aligned}$$

is separable in terms of bidders as there is no inter-unit coupling in the constraints. This allows us to decompose the problem into M subproblems. The subproblem for bidder $i = 1, 2, \ldots, M$ is

$$\begin{aligned}
\phi_i(\underline{\lambda}, \underline{\mu}) = \min_{\underline{u}_i,\underline{p}_i,\underline{r}_i} & \left\{ \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t-1})(1-u_{i,t-1}) - \lambda_t p_{i,t} - \mu_t r_{i,t} \right] u_{i,t} \right. \\
& \left. : \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \in \Omega_i(\underline{a}_i) \right\} \qquad (9)
\end{aligned}$$

For given $\underline{\lambda}$ and $\underline{\mu}$, the M subproblems can be independently solved in an efficient manner. Hence the Lagrangian relaxation of the CPP dispatcher problem can be solved efficiently for particular values of $\underline{\lambda}$ and $\underline{\mu}$, giving the value $\phi(\underline{\lambda}, \underline{\mu}; \underline{D}, \underline{R})$. It can be shown that the Lagrangian function $\phi(\underline{\lambda}, \underline{\mu}; \underline{D}, \underline{R})$ provides a lower bound for $P(\underline{D}, \underline{R})$ (Finlay, 1995) i.e.,

$$P(\underline{D}, \underline{R}) \geq \phi(\underline{\lambda}, \underline{\mu}; \underline{D}, \underline{R}) \tag{10}$$

In particular, if $(\underline{\lambda}^*, \underline{\mu}^*)$ is the optimal Lagrange multipliers that maximize ϕ, i.e.,

$$\phi(\underline{\lambda}^*, \underline{\mu}^*; \underline{D}, \underline{R}) = \max\{\phi(\underline{\lambda}, \underline{\mu}; \underline{D}, \underline{R}) : \underline{\lambda}, \underline{\mu} \geq \underline{0}\} \tag{11}$$

then,

$$L(\underline{D}, \underline{R}) \stackrel{\Delta}{=} \phi(\underline{\lambda}^*, \underline{\mu}^*; \underline{D}, \underline{R}) \tag{12}$$

provides a tighter lower bound on the optimal cost $P(\underline{D}, \underline{R})$ of the primal problem

$$P(\underline{D}, \underline{R}) \geq L(\underline{D}, \underline{R}) \tag{13}$$

As a by-product of the process of maximizing $\phi(\underline{\lambda}, \underline{\mu}; \underline{D}, \underline{R})$, we obtain the optimal Lagrange multipliers $\underline{\lambda}^*$ and $\underline{\mu}^*$ and a system schedule $\sum^* = \{\underline{u}^*, \underline{p}^*, \underline{r}^*\}$ resulting from the solution to the Lagrangian relaxation for $\underline{\lambda} = \underline{\lambda}^*$ and $\underline{\mu} = \underline{\mu}^*$. The schedule $\sum^* = \{\underline{u}^*, \underline{p}^*, \underline{r}^*\}$ must satisfy the bidder constraints given in Eq. (3), i.e., $\sum_i^* = \{\underline{u}_i^*, \underline{p}_i^*, \underline{r}_i^*\} \in \Omega_i(\underline{a}_i)$ for all $i = 1, \ldots, M$. In certain cases, $\sum^*$ does satisfy the demand and reserve constraints making it feasible for the primal problem. If, in addition, $\sum^*$ satisfies the complementary slackness condition, $\sum^*$ is, in fact, the optimal schedule to the primal problem, i.e., $\sum^* = \sum^{opt}$ (Luenberger, 1969). Practical approaches for computing a *near-optimal* schedule have been developed (Merlin and Sandrin, 1983; Shaw et al., 1985). For all practical purposes the difference between $\sum^{opt}$ and the near optimal schedule $\sum^*$ is assumed to be negligible. Moreover, we also assume that the optimal Lagrange multiplier $\underline{\lambda}^*(\underline{\mu}^*)$ associated with demand (reserve) in time period t, differs negligibly from the marginal price (reserve price) in the same period.

3. Bidding Strategy Formulation

We use the CPP framework constructed in Section 2 to solve the bidder's problem: formulation of a bidding strategy to earn maximum return. We consider the problem of bidder i who submits bid β_i. For bidder i, the bids β_j, $j = 1, 2 \ldots, i-1, i+1, \ldots, M$ of the other bidders are *fixed but unknown*. The CPP dispatcher determines the optimal system price pair $(\underline{\lambda}^*, \underline{\mu}^*)$, the optimal system schedule $\sum^* = \{\underline{u}^*, \underline{p}^*, \underline{r}^*\}$ and from this the operating schedule $\sum_i^* = \{\underline{u}_i^*, \underline{p}_i^*, \underline{r}_i^*\}$ for unit i. The prices depend on the bids of all generators in the CPP, β_j, $j = 1, \ldots, M$. However, bidder i exerts control only over β_i; hence, we can write $\underline{\lambda}^* = \underline{\lambda}^*(\beta_i)$ and $\underline{\mu}^* = \underline{\mu}^*(\beta_i)$. The dependence of $\sum_i^*$ on the bid β_i is suppressed for notational simplicity; $\sum_i^* = \{\underline{u}_i^*, \underline{p}_i^*, \underline{r}_i^*\}$ satisfies the subproblem associated with unit i

from the Lagrangian relaxation of the CPP dispatcher's problem

$$\min_{\underline{u}_i,\underline{p}_i,\underline{r}_i} \left\{ \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t})(1-u_{i,t-1}) - \lambda_t^*(\beta_i)p_{i,t} - \mu_t^*(\beta_i)r_{i,t}\right]u_{i,t} \right.$$
$$\left. : \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \in \Omega_i(\underline{a}_i) \right\} \tag{14}$$

where, the set $\Omega_i(\underline{a}_i)$ is as defined in Eq. (4). The bidder i generation costs incurred in each time period t is the sum of the variable costs and start-up costs $c_i^f(p_{i,t}^*)u_{i,t}^* + c_i^s(\tau_{i,t-1}^*)(1-u_{i,t-1}^*)u_{i,t}^*$. We use $c_i^f(\cdot)[c_i^s(\cdot)]$ to denote the fuel and variable operations and maintenance costs [start-up costs] of unit i. We assume both functions to be continuous. The total costs of unit i are $\sum_{t=1}^{T}[c_i^f(p_{i,t}^*) + c_i^s(\tau_{i,t-1}^*)(1-u_{i,t-1}^*)]u_{i,t}^*$ and the amount paid to generator i in each time period is $\lambda_t^*(\beta_i)$ per MWh of energy and $\mu_t^*(\beta_i)$ per MW of reserve served. It follows that the profits $\Pi_i(\beta_i, \underline{\lambda}^*(\beta_i), \underline{\mu}^*(\beta_i))$ of bidder i are equal to the revenues less the costs incurred. Thus,

$$\Pi_i(\beta_i; \underline{\lambda}^*(\beta_i), \underline{\mu}^*(\beta_i)) = \sum_{t=1}^{T} \left[\lambda_t^*(\beta_i)p_{i,t}^* + \mu_t^*(\beta_i)r_{i,t}^* \right.$$
$$\left. - c_i^f(p_{i,t}^*) - c_i^s(\tau_{i,t}^*)(1-u_{i,t-1}^*)\right]u_{i,t}^* \tag{15}$$

The optimal bidding strategy calls for the maximization of $\Pi_i(\beta_i, \underline{\lambda}^*(\beta_i), \underline{\mu}^*(\beta_i))$ over the set of admissible bids, i.e.,

$$\max_{b_i^f(\cdot),b_i^s(\cdot),\underline{a}_i} \left\{ -\left[\sum_{t=1}^{T} \left[c_i^f(p_{i,t}^*) + c_i^s(\tau_{i,t-1}^*)(1-u_{i,t-1}^*) - \lambda_t^*(\beta_i)p_{i,t}^* - \mu_t^*(\beta_i)r_{i,t}^*\right]u_{i,t}^*\right] \right.$$
$$\left. : b_i^f \in \mathcal{L}\left[p_i^{\min}, p_i^{\max}\right], b_i^s \in \mathcal{C}[0,\infty), \underline{a}_i \geq \underline{0} \right\} \tag{16}$$

$\{\underline{u}_i^*, \underline{p}_i^*, \underline{r}_i^*\}$ minimizes the problem

$$\min_{\underline{u}_i,\underline{p}_i,\underline{r}_i} \left\{ \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t-1})(1-u_{i,t-1}) - \lambda_t^*(\beta_i)p_{i,t} - \mu_t^*(\beta_i)r_{i,t}\right]u_{i,t} \right.$$
$$\left. : \{\underline{u}_i, \underline{p}_i, \underline{r}_i\} \in \Omega_i(\underline{a}_i) \right\} \tag{17}$$

with $\Omega_i(\underline{a}_i)$ as defined in Eq. (4).

We next introduce the assumption of *perfect competition* in the CPP. Under such a condition, no single bidder may affect prices and is consequently a price taker. In other words, any change in the bid submitted by bidder i will have a small effect on the prices determined by the CPP dispatcher. Formally, we state the

Perfect Competition Assumption. The bid of any bidder has a negligible effect on the system marginal and reserve prices.

This assumption holds when no single bidder controls a significant portion of the total CPP generation and capacity.[1] The market price is determined by the bids of the set of competing bidders. From the viewpoint of bidder i, the market clearing prices are independent of β_i so that

$$\underline{\lambda}^*(\beta_i) = \underline{\lambda}^o \quad \text{and} \quad \underline{\mu}^*(\beta_i) = \underline{\mu}^o \tag{18}$$

It is convenient to define the loss function $\Lambda_i \triangleq -\Pi_i$ and replace the maximization in Eq. (16) by the minimization of Λ_i. We restate the problem as

$$\min_{b_i^f(\cdot), b_i^s(\cdot), \underline{a}_i} \left\{ \sum_{t=1}^{T} \left[c_i^f(p_{i,t}^*) + c_i^s(\tau_{i,t}^*)(1 - u_{i,t-1}^*) - \lambda_t^o p_{i,t}^* - \mu_t^o r_{i,t}^*\right] u_{i,t}^* \right. \\ \left. : b_i^f \in \mathcal{L}\left[p_i^{\min}, p_i^{\max}\right], b_i^s \in \mathcal{C}[0, \infty), \underline{a}_i \geq \underline{0} \right\} \tag{19}$$

where, $\{\underline{u}_i^*, \underline{p}_i^*, \underline{r}_i^*\}$ minimizes the problem

$$\min_{\underline{u}_i, \underline{p}_i, \underline{r}_i} \left\{ \sum_{t=1}^{T} \left[b_i^f(p_{i,t}) + b_i^s(\tau_{i,t-1})(1 - u_{i,t-1}) - \lambda_t^o p_{i,t} - \mu_t^o r_{i,t}\right] u_{i,t} \right. \\ \left. : (\underline{u}_i, \underline{p}_i, \underline{r}_i) \in \Omega_i, (\underline{a}_i) \right\} \tag{20}$$

Given the structural similarity between the minimizations in Eqs. (19) and (20) we have the following

Fundamental Theorem. *A global optimal solution to the problem in* (19) *and* (20) *is the bid* $\beta_i^{opt} = \{b_i^f, b_i^s, \underline{a}_i\}$, *where*

$$\begin{aligned} b_i^f(p) &= c_i^f(p) \quad \forall p \in \left[p_i^{\min}, p_i^{\max}\right] \\ b_i^s(\tau) &= c_i^s(\tau) \quad \forall \tau \geq 0 \\ a_{i,t} &= p_i^{\max} \quad \forall t = 1, 2, \ldots, T \end{aligned} \tag{21}$$

The proof of this theorem is an application of the lemma in the Appendix (Finlay, 1995). $\beta_i^{opt} = \{c_i^f, c_i^s, p_i^{\max}\}$ is a globally optimal bidding strategy. No other bidding strategy can result in a greater profit to the bidder. This does not preclude some other bid from also achieving the same profit. We also note that the global optimality is independent of the system price pair $(\underline{\lambda}^o, \underline{\mu}^o)$. Regardless of the prices that may be realized during the schedule horizon, the bid is optimal if it equals β_i^{opt}.

A bidder whose unit has a long minimum up time may feel it is to his advantage to underbid his cost to compensate for the fact that the constraints on the unit will hinder its ability to get scheduled. Such a strategy abandons a known global optimum for one which may not necessarily be so. In a complementary situation, a generator with a short minimum up time may be inclined to overbid, believing that he can cash in on the unit's higher level of dispatchability. Again, the Fundamental Theorem shows this to be an ineffective strategy. Numerical results presented in the next section demonstrate these analytic results.

A salient feature of this optimal bidding strategy is that it reveals the true cost of operation of the unit to the CPP dispatcher. This highly desirable outcome is due to the construction of this auction for the right to serve load in the CPP. We can refer to the optimal bid as *bidding at cost* or the *truth-revealing bid*. Note that while the Fundamental Theorem was derived for the case of each bidder having a single unit in his possession, the result extends to cases in which a generator owns multiple units. The reason for this is simple. The bid of any one unit does not affect the system marginal and reserve prices by the Perfect Competition Assumption. The problem stated in (19)–(20) is simply applied independently to each of the bidder's units. Hence, the global strategy for each unit in such cases is to bid at cost.

The expression for the profit realized by bidder i under the optimal bidding at cost strategy is thus

$$\begin{aligned}
\Pi_i^*\big(\underline{\lambda}, \underline{\mu} &\triangleq \Pi_i\big(\beta_i^{opt}, \underline{\lambda}, \underline{\mu}\big)\big) \\
&= \Pi_i\big(\big\{c_i^f(\cdot), c_i^s(\cdot)\underline{p}_i^{\max}\big\}; \underline{\lambda}, \underline{\mu}\big) \\
&= \max_{\underline{\mu}_i, \underline{p}_i, \underline{r}_i} \left\{ \sum_{t=1}^{T} \big[\lambda_t p_{i,t}, + \mu_t r_{i,t} - c_i^f(p_{i,t}) - c_i^s(\tau_{i,t-1})(1 - u_{i,t-1})\big] u_{i,t} \right. \\
&\qquad \left. : \{u_i, p_i, r_i\} \in \Omega_i(\underline{a}_i) \right\} \qquad (22)
\end{aligned}$$

Here, $\underline{p}_i^{\max}$ is the T vector with each component equal to $p_i^{\max}$. This expression for the optimal profit $\Pi_i^*(\underline{\lambda}, \underline{\mu})$ allows the identification of several key properties including convexity and nonnegativity (Finlay, 1995). These are useful in developing sensitivity information and in quantifying the effects of volatility in the prices $(\underline{\lambda}, \underline{\mu})$ on the optimal profit of bidder i. Moreover, these properties can be used in evaluating the impacts on profits of a change in the costs of the bidder to assess the return on possible investments aimed at improving the performance of the unit (Finlay, 1995).

4. Numerical Results

We illustrate the formulation of optimal bidding strategies in the CPP under the assumption that the system prices are independent of the bid of any one generator. Given the system price $(\underline{\lambda}, \underline{\mu})$ we simulate the profit $\Pi_i(\beta_i; \underline{\lambda}, \underline{\mu})$. For the numerical studies reported here, corresponding to the bid β_i the bid variable price b_i^f is submitted as a piecewise linear function of unit power (Willans line) as is the case in the EWPP. We examine the variation of the profit of bids with respect to various parameter values and compare that to the optimal strategy bid. We ignore the reserve price in these numerical studies so as not to detract

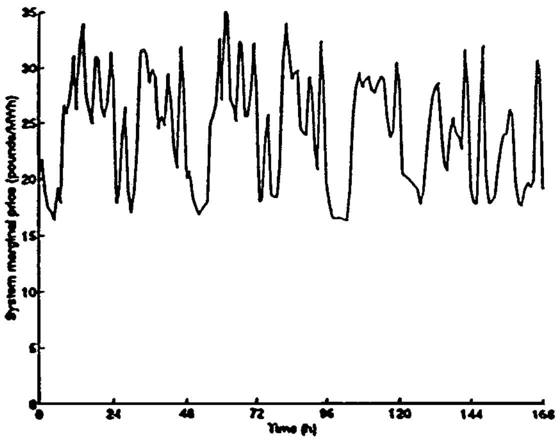

Figure 3. System marginal price data.

from the focus on the system marginal price. There is no conceptual difficulty in including reserve price in the numerical simulations.

We use system marginal price data derived from the EWPP to be the *driving function* for the commitment and dispatch of a unit. A set of half hourly values of λ_t^* for a week was constructed using data in Littlechild (1993). The time plot of the week in units of £/MWh is shown in figure 3. Hour 0 corresponds to Sunday midnight.

To evaluate a bid, we calculate the profit corresponding to the bid for the week of system marginal price data. Given the generator's bid and the assumed system marginal price data, we can determine the unit's schedule for the week and consequently its profits. The variable costs of the bidding unit are represented by a piecewise linear function of the unit output power, with three segments. The parameters that describe this function are no-load cost, c^o, the two *elbow* or *break points*, e^1, and e^2 and the slopes of the linear segments, m^1, m^2, m^3. A plot of the variable cost function for the unit is shown in figure 4.

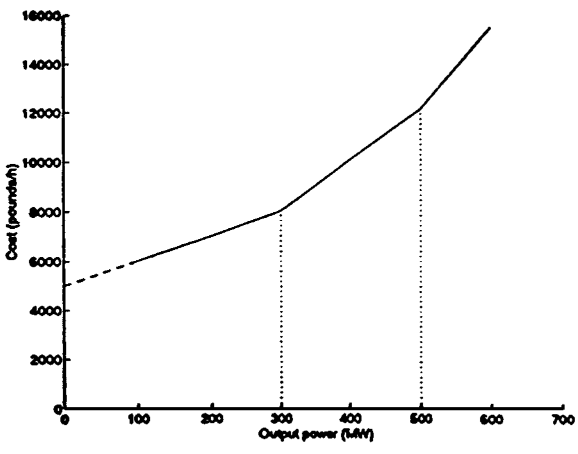

Figure 4. Variable cost in £/h for the bidding unit.

Table 2. Cost and operational parameters for the bidding unit.

Parameter	$p_i^{\min}$	$p_i^{\max}$	τ^d	τ^u	c^0	e^1	e^2	m^1	m^2	m^3	$c^s(0)$	$c^s(\infty)$	τ^c
Value	100	600	4	3	5,000	300	500	10.04	20.68	33.52	2,000	3,000	2
Units	MW	MW	h	h	£/h	MW	MW	£/MWh	£/MWh	£/MWh	£	£	h

The start-up costs of the unit are assumed to be an exponential function of cooling time

$$c^s(\tau) = c^s(0) + (c^s(\infty) - c^s(0))[1 - \exp(-\tau/\tau^c)]$$

τ^c is the *cooling time constant* for the unit. The selection of $c^s(0)$, $c^s(\infty)$ and τ^c completely specifies this function. We refer to the set of values of $\{c^0, e^1, e^2, m^1, m^2, m^3, c^s(0), c^s(\infty), \tau^c, c_i^s(0), c_i^s(\infty), \tau_i^c\}$, which specify the variable and start-up cost functions as the *cost parameters* of the unit. The cost parameters, the minimum up and down times and the minimum and maximum outputs of the bidding unit are presented in Table 2.

We consider for the bidding unit the effect on profits of the submission of bids different from bidding at cost. To this end we examine the change in profits as the bid is changed by varying the bid parameters. It is assumed that the unit is made fully available for every hour of the week. We restrict the bid variations to one parameter at a time while each of the other bid parameters is kept constant at the values of the corresponding cost parameters. The parameters that describe the bid variable price function are the bid no-load price b^0 the bid *elbow points* ϵ^1 and ϵ^2, and the bid slopes of the linear segments η^1, η^2, η^3 in the same way the variable cost function of the unit is specified by the parameters c^0, e^1, e^2, m^1, m^2 and m^3. The bid start-up price function is specified by the parameters $b^s(0)$, $b^s(\infty)$ and τ^b. We refer to the set of values $\{b^0, \epsilon^1, \epsilon^2, \eta^1, \eta^2, \eta^3, b^s(0), b^s(\infty), \tau^b\}$ as the bid parameters of the unit. The profits made by the submission of the bid β are denoted by $\Pi(\beta; \underline{\lambda})$. In figure 5, a plot of $\Pi(\beta; \underline{\lambda})$ against variations in ϵ^2 is given.

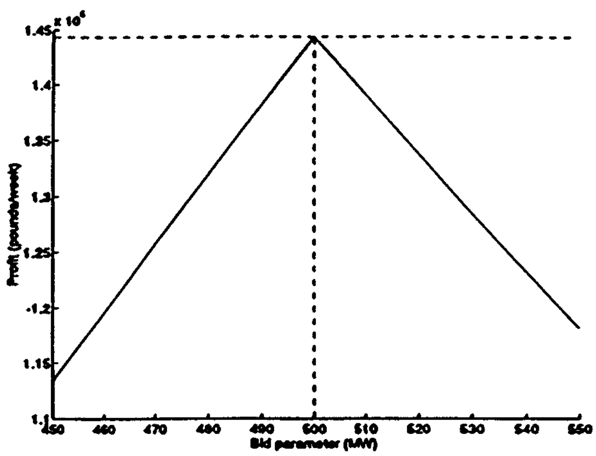

Figure 5. Plot of $\Pi(\beta; \underline{\lambda})$ versus ϵ^2.

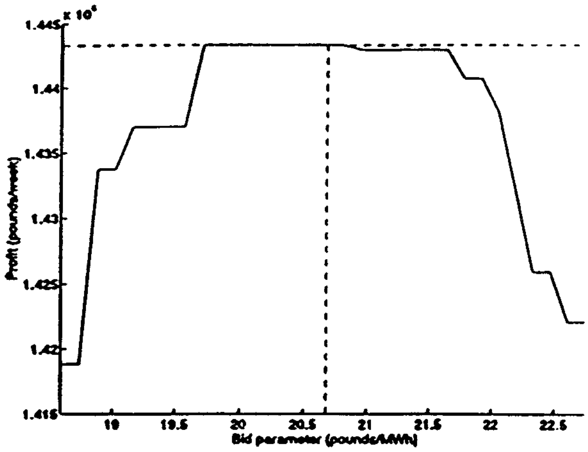

Figure 6. Plot of $\Pi(\beta; \underline{\lambda})$ versus η^2.

In figure 6 a plot is given of $\Pi(\beta; \underline{\lambda})$ against variations in η^2. Once again, it is seen that maximum profit occurs when the bid parameter is equal to its corresponding cost parameter. The "flatness" of the profit with respect to variations in η^2 deserves comment. Units that submit a piecewise linear bid variable price function are dispatched at the elbow points ϵ^1 or ϵ^2 or the maximum power point, $p^{\max}$. Clearly, small variations in η^2 do not result in the CPP dispatcher redispatching the unit to other elbow points; hence, the profit remains unchanged. Similar results are observed for changes in η^1.

5. Conclusions

This paper has reported the development of a competitive power pool (CPP) framework which incorporates the salient features of the England and Wales Power Pool (EWPP). We have applied the framework to formulate and determine the optimal bidding strategies of a bidder in the CPP under conditions of perfect competition. This paper's results are noteworthy for the explicit inclusion and detailed representation of the various considerations and constraints associated with the generation for electrical power. We have developed a globally optimal bidding strategy: regardless of generation resources, costs and constraints, a generator maximizes profits by bidding to supply generation at cost and at maximum capacity. The increasing interest in the POOLCO concept (Budhraja and Woolf, 1994) makes this work highly topical.

This paper has focused only on one aspect of CPP—the optimal bidding strategy problem for generators. The recent introduction of demand side bidding into the EWPP has introduced a problem which can be effectively solved using the CPP framework. There are several facets of the CPP that require additional work. For example, the optimal bidding strategy problem for buyers from the CPP may be formulated as a bid to optimize profits given resources, constraints and costs. The oligopoly situation in the EWPP generation markets (Green and Newbery, 1989) has thus far been resistant to analytic approaches. The construction of optimal bids by the method of this paper may provide little insight into the formulation of optimal bids under nonperfect competitive conditions. Approaches based on other concepts such as game theoretic notions need to be explored. Another extension of this work is the study of the integration of financial instruments such as contracts and futures into the CPP. An area which requires considerable work is the incorporation of transmission constraints and pricing (Wu et al., 1994) into the CPP framework. There are some fundamental difficulties in the development of appropriate schemes for the economically efficient pricing of transmission services. Research into this area is currently underway.

Appendix: Lemma for Fundamental Theorem

Let $\mathcal{C}(I^f)$ and $\mathcal{C}(I^s)$ be the spaces of continuous functions on the intervals $I^f \subseteq \mathcal{R}$ and $I^s \subseteq \mathcal{R}$ respectively, where $\mathcal{C}(I^j) \triangleq \{f(\cdot), I^j \to \mathcal{R} : f(\cdot) \text{ is continuous}\}$. Let J be a functional $J : \mathcal{C}(I^f) \times \mathcal{C}(I^s) \times \mathcal{R}^n \to \mathcal{R}$, a mapping from the vector space $\mathcal{C}(I^f) \times (I^s) \times \mathcal{R}^n$ to the real line (Luenberger, 1969). Suppose for each $\underline{a} \geq \underline{0}$, $\underline{a} \in \mathcal{R}^m$, $\Omega(\underline{a})$ is a compact nonempty set. We assume the existence of an $\underline{\hat{a}}$ such that: $\Omega(\underline{a}) \subseteq \Omega(\underline{\hat{a}})$ for all $\underline{a} \in \mathcal{R}^m$. Assume $J(b^f, b^s, \underline{x})$ is a continuous function of $\underline{x}$ $(\forall b^f \in \mathcal{C}(I_f), \forall b^s \in \mathcal{C}(I_s)$. Then,

$$\inf_{b^f, b^s, \underline{a}} \{J(c^f, c^s, x^*(b^f, b^s, \underline{a}) : b^f \in \mathcal{C}(I^f), b^s \in \mathcal{C}(I^s),\ \underline{a} \in \mathcal{R}^m, \underline{a} \geq 0\}$$

where,

$$x^*(b^f, b^s, \underline{a}) = \arg\min\{J(b^f, b^s, \underline{x}) : x \in \Omega(\underline{a})\},$$

has a global minimum given by

$$\begin{aligned} b^f(\cdot) &= c^f(\cdot) \\ b^s(\cdot) &= c^s(\cdot) \\ \underline{a} &= \underline{\hat{a}} \end{aligned}$$

The proof of this lemma as well as its application to the proof of the Fundamental Theorem is found in Finlay (1995). In words, the Lemma states that of all possible choices of $b^f \in \mathcal{C}(I^f)$ and $b^s \in \mathcal{C}(I^s)$, the optimal ones are $b^f(\cdot) = c^f(\cdot)$ and $b^s(\cdot) = c^s(\cdot)$, and the optimal choice of $\underline{a}$ is $\underline{\hat{a}}$, which specifies the largest $\Omega(\underline{a})$.

Note

1. In effect we assume no collaboration among bidders, i.e., generators behave noncooperatively and there is no cartel of generators who act together to set prices.

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