APPLICATION OF MICROECONOMIC METRICS IN COMPETITIVE ELECTRICITY MARKETS

Pedro Correia and George Gross
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801

Ettore Bompard, Enrico Carpaneto and Gianfranco Chicco
Dipartimento di Ingeneria Elettrica Industriale
Politecnico di Torino
I-10129 Torino

Abstract – The proliferation of wholesale electricity markets in various jurisdictions around the world is a salient characteristic of the restructuring of the electricity business. The establishment of the various markets, typically in the form of commercial as opposed to reliability power pools and electricity exchanges, has resulted in structured short-term electricity markets in which the MWh is being traded like many other commodities on established exchanges. The various implementations of electricity markets use some form of auctions as the principal competitive mechanism. The focus of this paper is on the providing the basis for understanding how competitive mechanisms in wholesale electricity markets function and how their performance is assessed taking explicit account of the specific characteristics and constraints of electrical generation and consumption. The objectives of this paper are to provide a well-balanced explanation of the basics of the trading mechanism typically in use in structured electricity markets; to define appropriate metrics taking explicitly into account both considerations of the nature of market participants and of the unique constraints of the electricity system and to illustrate their usefulness in the analysis of the performance of electricity markets.

1. – Introduction

The proliferation of wholesale electricity markets in various jurisdictions around the world is a salient characteristic of the restructuring of the electricity business. The establishment of the various markets, typically, in the form of commercial as opposed to reliability power pools and electricity exchanges have resulted in structured short-term electricity markets in which the MWh is being traded like many other commodities on established exchanges. The various implementations of electricity markets use some form of auctions as the principal competitive mechanism. The focus of this paper is on the providing the basis for understanding how competitive mechanisms in wholesale electricity markets function and how their performance is assessed taking explicit account of the specific characteristics and constraints of electrical generation and consumption.

The principal players in electricity markets are the trading participants – producers, generators, sellers, consumers, loads, buyers, and brokers – the market operator and an organization that is unique to electricity and called the Independent Grid Operator or IGO. The IGO is the entity that is responsible for managing the grid usage. The commercial transactions take place using the electric network or grid, a fact that makes the trading of electricity quite complex. In addition to the set of physical and operational constraints that characterize the individual components of the electricity system that are involved with the generation and consumption of power, the numerous transmission or grid considerations must be taken into account. These lead to the specification of grid constraints, which must be obeyed in the operation of the interconnected system. In particular, the design of the trading mechanisms or institutions and the specification of the rules must consider the impacts of these constraints to allow the smooth and efficient trading of electricity.

The IGO, an independent entity that operates and controls the transmission system in a region, has as its principal objective to encourage the smooth functioning of competitive markets while ensuring that the reliability of the interconnected system is maintained. The setting up of an IGO may be in addition to an electricity exchange – as is the case in California – or may be combined with the power exchange, as is the case in the PJM Interconnection. Due to the network nature of electricity, all transactions need to be first approved by the IGO before they can be implemented.

The IGO coordinates trade among the participants by ensuring the feasibility of the trading solutions. A trading solution is the operational point of the system when all the agreed upon transactions are put in place simultaneously. The feasibility means the satisfaction of all the system constraints. The IGO receives the preferred schedules, i.e., the winning *bids* and *offers* submitted by the buyers and sellers that are selected to be the trade solution.

The unbundling in electricity in the competitive environment has created new markets in ancillary services. The new structures under competition have created a number of distinct markets that are strongly interrelated. Typical examples of electricity markets include the day-ahead, the hour-ahead, and the real-time balancing energy markets and the capacity markets in ancillary services. Those electricity markets as the regulation, or the ancillary services, share these same characteristics up to a certain extent.

It is the structured market operator whose responsibility is to choose the trading mechanism or institution for establishing the trade among the participants. Typically, the aim is to implement a trading mechanism that results in *efficient short-term* solutions. By efficient we mean that the electricity commodity that is sold and bought minimizes the *costs* incurred in its purchase and maximizes the *benefits* gained from its use. Because we are looking at market outcomes that last only for the market horizon, the focus is on short-term solutions. A key consideration in electricity markets is the establishment of the system operations within the physical, engineering and operating limits of the electricity system. The market mechanism adopted needs to consequently incorporate procedures to ensure feasible solutions.

In order to evaluate the performance of electricity markets in terms of assessing the attained trading solutions appropriate measures or metrics must be introduced. Such metrics assume particular importance in the comparison of different market mechanisms. Different system constraints may result in very different values of the metrics of the trading solutions. In addition, the metrics are particularly useful in evaluating the impacts of particular rules that may entail money transfers among the players. This paper aims to make clear the definition and use of metrics that are meaningful in the analysis of electricity markets.

The objectives of this paper are to provide a well-balanced explanation of the basics of the trading mechanism typically in use in structured electricity markets; to define appropriate metrics taking explicitly into account both considerations of the nature of market participants and of the unique constraints of the electricity system; and to illustrate their usefulness in the analysis of the performance of electricity markets. This paper has five additional sections and two appendices. Section 2 presents an overview of the trading mechanism customarily employed in structured markets and provides the rationale for its use. The definition and interpretation of some appropriate and meaningful metrics are given in Section 3. In Section 4 we illustrate using numerical examples the application of the metrics introduced. The formulation of the optimization problem is given in Appendix A and the data used for the numerical examples are summarized in Appendix B.

2. – The Electricity Markets

In the well established vertically integrated utility (VIU) industry the entire grid was operated by a single entity. The central operator would dispatch the system having full knowledge of all the information about each unit's physical capabilities and limitations, and its costs, including operational and investment components, the loadings on the system, and the various technical, physical, engineering and policy grid constraints. The dispatch problem when formulated as an OPF had a solution that provided the least cost dispatch under the system constraints that were considered.

In the new competitive environment, we no longer have a single decision maker that has either full knowledge of the costs of all units or direct control of them. Moreover, loads may also become decision variables since the demand can be determined so as to be responsive to prices in the market. The market now includes many participants whose costs constitute *private information*. So, in effect, one cannot talk about costs with any certainty. The information that is available to market players is the prices at which the sellers are willing to offer their electricity and at which buyers are willing to bid. Moreover, the participants may game the market in an attempt to maximize their profits by bidding and offering in a way that does not reflect their true costs or benefits [1]. In this paper we use the terms sellers, producers, and generators, interchangeably; in addition the terms buyers, demands, and loads are interchangeable.

In order to participate in electricity markets, consumers' bids and producers' offers are submitted in the form of *value* and *price schedules*, respectively, one each time period of the market. The term *schedule* is used to indicate that the participants submit a set of values/prices for the set of corresponding consumed/produced quantities of the electricity commodity as opposed to a single pair of value/price and the corresponding commodity amount. We assume that the supply and the demand are constant for the duration of the smallest time period of such a market – typically an hour for electricity markets – and therefore the schedules correspond to a snapshot in time. Each market has its own trading protocols, which prescribe, among other rules, the formats in which bid and offer schedules are submitted. Fig. 1 illustrates a producer's offer curve, indicating the price and quantity pairs at which the producer is willing to sell the commodity, including the quantity limits for which the offer is valid. We will consider the meaning of these limits in more detail later. For the sake of simplicity we adopt step functions for purposes of illustration in this section.

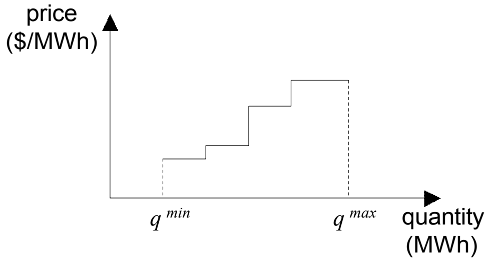


Fig. 1: *A seller's price schedule*

By ordering the offers of all the sellers according to increasing price, with the lowest price first and the highest price last, we may construct the so-called *supply curve* [2]. The ordering of the bids of all the buyers in decreasing order results in the *demand curve*. Each step of the supply curve is defined by a specific offer price of a particular supplier and the quantity given by the sum of all the quantities offered at a price lower or equal by it and the other competing suppliers. Similarly, each step of the demand curve is defined by a specific value that a particular buyer is willing to pay and the sum of all the quantities it and the other buyers are willing to pay a price greater or equal to this price. In Fig. 2 we show an example of a supply and a demand curve for an electricity market.

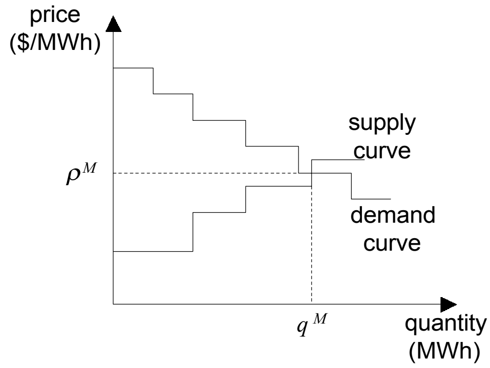


Fig. 2: *Supply and demand curves*

A commonly used trading *institution* or *mechanism* for organized electricity markets is the *uniform-price double auction* [3]. In the absence of any constraints in such an auction, the intersection of the demand curve and the supply curve determines the *market-clearing price* or ρ^M and the *market-clearing quantity* q^M that *mediate the trade*. These are shown in Fig. 2 by the dotted lines. By *mediate* we mean that, once ρ^M is established, every participant is assigned its corresponding participation in the sale/purchase of the commodity as established by its value/price schedule.

The market solution $\left(\rho^M, q^M\right)$ is usually called the *equilibrium* by economists [4]. An equilibrium is characterized by fixed supply, demand, and price. The notion of an economic equilibrium has a strong analog with that of a stability equilibrium point. For example, any price increase would entail an increase in supply and a decrease in demand. This imbalance leads the price to decrease, thereby returning to the equilibrium. Similarly, any price decrease would lead to a decrease in supply and an increase in demand. This excess of demand with respect to supply would force the price to increase, therefore returning to the equilibrium.

By analyzing Fig. 2 we may conclude that, because of the way the supply and the demand curves order the bids and offers, the uniform-price double auction matches the highest value bids with the lowest price offers. In this way, this trading institution maximizes the total value of the transactions between the participants by allowing each buyer to maximize its savings and each seller to maximize its gains. The matching of the pairs of buyers and sellers in this manner is seen to maximize the area between the supply and demand curves. We say that the market-clearing price is established at the level at which all the possible trade among the participants has been realized. None of the participants to the right of the intersection of the supply and demand curves are involved in trade since their offer prices are too high and their willingness to pay is too low. In other words, this institution maximizes the total gains of the participants or *social surplus*, as defined below, based on their submitted bids and offers.

The nature of the electric system requires that the grid usage maintain the operation within system constraints. Therefore, the trading mechanism may include rules to account for some or all of those system constraints. One example of such a rule may be the partitioning of the market into grid zones to take care of power flow limits between those zones. The application of such a rule is illustrated in the numerical studies of section 4.

Once the trade becomes restricted due to operation at one or more of the system limits, we may no longer have a unique ρ^M, as we shall see in more detail below. The trading mechanism is still the double auction, but constrained by the physical or the operational feasibility limits. In such cases we adopt the maximization of the gains of the participants as the general objective of the trading mechanism. We keep this objective although we no longer have a single price that defines the trade under the auction mechanism.

3. – Metrics

Independent of the structure and form of the particular mechanism employed for trading, there is always the need to evaluate the market solutions that are attained by the market participants. It is for this evaluation of the solutions that the application of metrics is particularly useful.

3.1 – Definition of the Metrics

One critically important aspect in electricity markets is the requirement that power system operations must be undertaken within physical and operational limits of the network and its components. As such, in the submission of each schedule, the limits associated with the constraints of each buyer/seller have to be considered. At the least, for each buyer and seller there is a minimum and maximum quantity level; for an energy schedule these are represented as a maximum and a minimum power output or demand. Another important aspect to keep in mind about the evaluation of the metrics is that the costs and benefits of the participants are private information. Therefore, the metrics are based on the submitted schedules and not on actual costs and benefits. Moreover, the price and value schedules are represented in terms of *marginal* values. They specify the unit price/value for an additional unit of the commodity at a given total quantity output/consumption. As such, the metrics' evaluation is performed from the point of view of a market observer as distinct from that of a participant, by integrating the participants' schedules. Because the market is studied as a snapshot in time for each market time increment, the metrics will be defined for a specific time increment.

The *producer surplus* S_i^g for producer is defined as the difference between its revenues R_i obtained from selling the commodity and the costs C_i to produce it. Similarly, the *consumer surplus* S_i^d for consumer i, is the difference between the benefits B_i from utilizing the commodity and the payments T_i for acquiring it. From the point of view of a market observer these two metrics are mathematically defined by

$$\begin{aligned} S_i^g\left(\rho_i^g, q_i^g\right) &= R_i\left(\rho_i^g, q_i^g\right) - C_i\left(\rho_i^g, q_i^g\right) \\ &= \rho_i^g \cdot q_i^g - \left(\int_{q_i^{g,\min}}^{q_i^g} p_i^g(q) \cdot dq + K_i^g \right), \quad i \in \mathrm{G} \qquad (1) \end{aligned}$$

$$\begin{aligned} S_i^d\left(\rho_i^d, q_i^d\right) &= B_i\left(\rho_i^d, q_i^d\right) - T_i\left(\rho_i^d, q_i^d\right) \\ &= \left(\int_{q_i^{d,\min}}^{q_i^d} v_i^d(q) \cdot dq + K_i^d \right) - \rho_i^d \cdot q_i^d, \quad i \in \mathrm{D} \qquad (2) \end{aligned}$$

Here we use the following notation for the functions:

$S_i^g(\cdot)$ = surplus of producer i

$S_i^d(\cdot)$ = surplus of consumer i

$R_i(\cdot)$ = revenues of producer i

$C_i(\cdot)$ = costs of producer i

$B_i(\cdot)$ = benefits of consumer i

$T_i(\cdot)$ = payments of consumer i

$p_i^g(\cdot)$ = price schedule of producer i

$v_i^d(\cdot)$ = value schedule of consumer i

We denote the market-clearing quantities by

ρ_i^g = market-clearing price for producer i

$q_i^g = \max\left(q_i^{g,\min}, \min\left\{q_i^{g,\max}, q_i^{g,M}\right\}\right)$

ρ_i^d = market-clearing price for consumer i

$q_i^d = \max\left(q_i^{d,\min}, \min\left\{q_i^{d,\max}, q_i^{d,M}\right\}\right)$

with $q_i^{g,M}$ defined by

$$\rho_i^g \equiv p_i^g\left(q_i^{g,M}\right)$$

and $q_i^{d,M}$ defined by

$$\rho_i^d \equiv v_i^d\left(q_i^{d,M}\right).$$

The limiting values are given by

$q_i^{g,\min}$ = minimum output of producer i

$q_i^{g,\max}$ = maximum output of producer i

$q_i^{d,\min}$ = minimum demand of consumer i

$q_i^{d,\max}$ = maximum demand of consumer i

with the values of the constants being given by

$$K_i^g = C_i\left(q_i^{g,\min}\right)$$

$$K_i^d = B_i\left(q_i^{d,\min}\right).$$

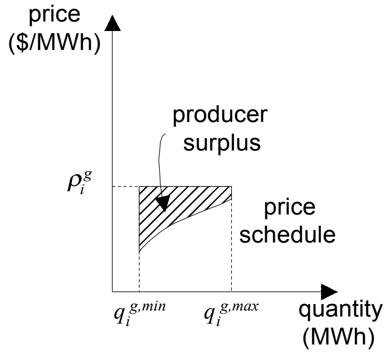
We use the notation

$\mathscr{G}$ = the set of producers

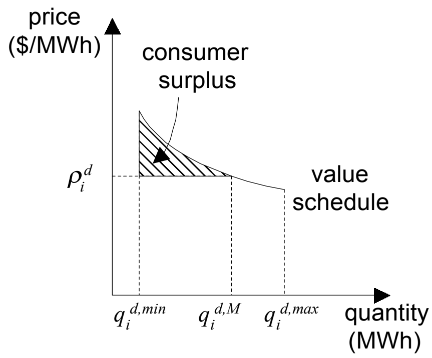
$\mathscr{D}$ = the set of consumers.

These definitions reflect the participants' limits and also the unavailability of information on true costs and benefits. The producer costs and the consumer benefits are only known up to the constants K_i^g and K_i^d, respectively. The constants depend on the boundary conditions. However, the specification of the boundary condition is not critical because, for comparing different market outcomes, only the differences in the metric values come into play and so we are indifferent to the value of the particular constant.

Fig. 3 presents the price schedule for a producer and the value schedule for a consumer as submitted in terms of an offer and a bid, respectively. The metrics just defined are indicated by the shaded areas in the diagram with the participants' limits, which define the intervals over which the schedules are valid. The unknown constants are of course not taken into account. Please note that in these illustrations we are no longer using piecewise functions such as step functions but rather more general continuous functions.



(a) producer surplus



(b) consumer surplus

Fig. 3: *Producer and consumer surplus*

The defined surpluses provide individual metrics for each participant. For evaluating the performance of markets we define the following composite metrics:

$$S^G \equiv \sum_{i \in \mathscr{G}} S_i^g \tag{3}$$

$$S^D \equiv \sum_{i \in \mathscr{D}} S_i^d \tag{4}$$

We call S^G the *total producer surplus* and S^D the *total consumer surplus*. The social surplus S^S is defined as the difference of the sum of the benefits and the sum of the costs of all participants:

$$S^S \equiv \sum_{i \in \mathscr{D}} B_i\left(q_i^d\right) - \sum_{i \in \mathscr{G}} C_i\left(q_i^g\right) \tag{5}$$

3.2 – The Unconstrained Market

We call the unconstrained market the one for which the market solution is not attained at any of the system constraints. Only the participants' limits are considered. Regardless of the participants' limits, the market-clearing price ρ^M is unique for the unconstrained market. However, it may no longer correspond to the intersection of the demand and supply curves and, in fact, such an intersection may no longer exist. The definition of the unique ρ^M is due to the satisfaction of the unique supply/demand balance constraint of all the participants. The mathematical expression for ρ^M is discussed in detail in Appendix A.

We examine S^S. For the unconstrained case in the uniform price market, all the participants are paid and charged the unique market-clearing price ρ^M. The social surplus may be shown to be equal to the sum of the total producer surplus and the total consumer surplus. We begin with the definition given in Eq. (5) and substitute from Eqs. (1) and (2) for the benefits and costs terms respectively:

$$
\begin{aligned}
S^S &= \sum_{i\in\mathcal{D}} B_i\left(q_i^d\right) - \sum_{i\in\mathcal{G}} C_i\left(q_i^g\right) \\
&= \sum_{i\in\mathcal{D}} \left[S_i^d\left(q_i^d\right) + T_i\left(q_i^d\right)\right] - \sum_{i\in\mathcal{G}} \left[R_i\left(q_i^g\right) - S_i^g\left(q_i^g\right)\right] \\
&= S^D + \sum_{i\in\mathcal{D}} \rho_i^d \cdot q_i^d - \sum_{i\in\mathcal{G}} \rho_i^g \cdot q_i^g + S^G \\
&= S^D + S^G + \rho^M \cdot \left(\sum_{i\in\mathcal{D}} q_i^d - \sum_{i\in\mathcal{G}} q_i^g \right) \\
&= S^D + S^G
\end{aligned}
\qquad (6)
$$

Here we used the fact that $\rho_i^d = \rho_i^g = \rho^M$ and that total supply equals total demand. The situation in the constrained market is somewhat more complicated and is discussed next.

3.3 – The Constrained Market

Now we consider the case in which, due to system constraints, the unconstrained solution cannot be maintained. The explicit consideration of these system constraints has vastly different implications for the market outcomes than the simple quantity level limits of the participants. In particular, such constraints have significant impacts on the prices charged from and paid to each buying and selling participant, respectively.

When the grid constraints are taken into account there is no longer a unique ρ^M. In this case the optimum of S^S may provide us with a different clearing price for each participant. The mathematical formulation that clarifies this issue is presented in Appendix A. One example is, when in the presence of line flow limits, the power flow balance at each bus is enforced; in this case, a different market-clearing price may result for each participant at each bus [5].

There are two important consequences from the unequal prices. First, there is an efficiency forfeit – a so-called dead-weight loss [6]. This loss reduces the social surplus from that in the unconstrained solution. This reduction in S^S is a direct function of the price difference. Second, in light of the presence of this loss in efficiency, the relation in Eq. (6) no longer holds. The definition of S^S implies, following the steps used to derive Eq. (6), that

$$
\begin{aligned}
S^S &= S^D + \sum_{i\in\mathcal{D}} \rho_i^d \cdot q_i^d - \sum_{i\in\mathcal{G}} \rho_i^g \cdot q_i^g + S^G \\
&= S^D + S^G + \left(\sum_{i\in\mathcal{D}} \rho_i^d \cdot q_i^d - \sum_{i\in\mathcal{G}} \rho_i^g \cdot q_i^g \right)
\end{aligned}
\qquad (7)
$$

The bracketed term is associated with the efficiency decrease and is termed the *merchandise surplus* S^M, i.e.,

$$
S^M \equiv \sum_{i\in\mathcal{D}} \rho_i^d \cdot q_i^d - \sum_{i\in\mathcal{G}} \rho_i^g \cdot q_i^g \qquad (8)
$$

Thus

$$
S^S = S^G + S^D + S^M \qquad (9)
$$

is the general expression for S^S. The term S^M gives an indication of the manner in which the trade among the participants is mediated: the commodity is bought from the producers at ρ_i^g and it is sold to the consumers at ρ_i^d, resulting in the mediation costs of S^M.

In general, the diversity of the grid constraints may result in either positive or negative S^M. A positive S^M corresponds to the case in which the market operator or the IGO collects a surplus. On the other hand, a negative S^M means that the optimum solution is achieved by a subsidy from the market operator or the IGO.

An important consideration is that the S^M does not diminish by itself the social surplus of the solution, but it is part of it, as defined in Eq. (9). It represents merely a transfer of money between the participants and the IGO or market operator. Moreover, the reduction in surplus need not be equal for the buyer and the seller. We illustrate these concepts by considering a simple energy market with a single seller and a single buyer. The power system in this case is the 2-bus system shown in Fig. 4. The generator selling the power is at one bus, and the load, which is buying the power, is at the other bus. The buses are connected by a line with a power flow limit of P^C.

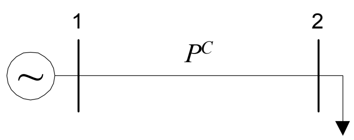
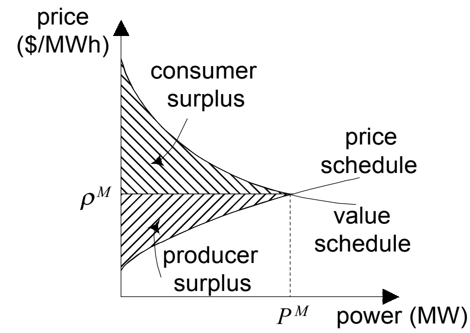


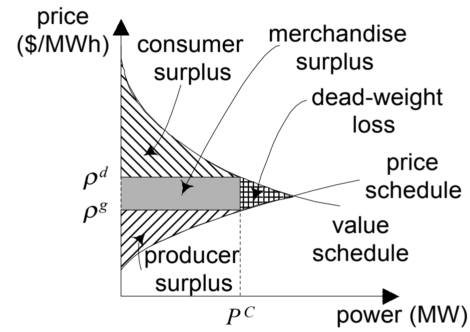
Fig. 4: The simple t*wo-bus system*

When the line constraint is ignored, the solution optimizing the social surplus corresponds to the intersection of the offer curve of the generator at bus 1 and the bid curve of the load at bus 2. As shown in Fig. 5 (a), this solution specifies uniquely the market-clearing price ρ^M and the market-clearing power P^M. In Fig. 5 we have represented the schedules as a function of power. In power systems we are used to balance power instead of energy while in energy markets the prices and values are given for energy. But in this context power and energy are equivalent if the market period is one hour with the implicit assumption that the output and the demand remain fixed for that hour.

Once the limit is taken into account and the flow on the line is exactly at the limiting value of the line, there is no longer a unique ρ^M. The effect of the line limit is to introduce an additional constraint and, in this case, the optimum of S^S provides us with a different clearing price at each bus: a producer price ρ^g and a consumer price ρ^d that are unequal. We get the solution illustrated on Fig. 5 (b), where the S^M is given by the shaded area. Note that S^M corresponds to a portion of the surplus removed from the buyer and seller's respective surpluses. It is positive in the case of Fig. 5 (b).



(a) unconstrained solution



(b) constrained solution

Fig. 5: *The effects of system constraints on the market solution*

Consider next the case with the same participants, for which the market solution, due to a particular operational constraint, was constrained to a value above P^M. Then the resultant S^M is negative as shown in Fig. 6 (b).

We have seen that for the general market solution every participant may face a distinct price and the IGO may collect or pay the merchandise surplus S^M arising from the price differences. We can no longer talk about a market-clearing price; however, we may define two additional metrics using weighted average prices.

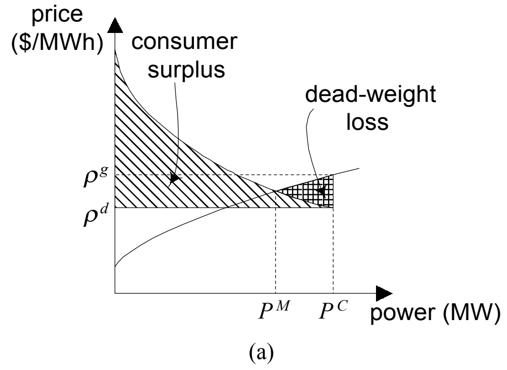


Fig.6: *Negative merchandise surplus*

The weighting in each average price is by the amount associated with the particular price. We denote the weighted average price of the producers by $\overline{\rho}^G$ and by $\overline{\rho}^D$ the weighted average price of the consumers and use the defining relations:

$$\overline{\rho}^G \equiv \sum_{i \in \mathscr{G}} \rho_i^g \cdot q_i^g \Big/ \sum_{i \in \mathscr{G}} q_i^g \tag{9}$$

$$\overline{\rho}^D \equiv \sum_{i \in \mathscr{D}} \rho_i^d \cdot q_i^d \Big/ \sum_{i \in \mathscr{D}} q_i^d \tag{10}$$

These weighted average prices are useful proxies for the ρ^g and the ρ^d quantities, which are no longer defined. The difference between these weighted averages serves as an indicator of the impact of the system constraints on the market solution. In particular, the sign of the difference indicates the nature of the merchandise surplus.

4. - Illustrative Example

In order to illustrate the application of the metrics, and their behavior in presence of different grid considerations and different levels of details in grid modeling, we provide two examples using a simple 4-bus system. These examples are designed to assess the behavior of various metrics under different consideration. The S^S maximizing solution is obtained in each case using a full AC optimal power flow [7]. The optimization model used is summarized in Appendix A. The data for the examples, including the participants bids and offers, are given in Appendix B.

Example 1 – Operation of the system as a single zone

We evaluate the metrics of Section 3 for the 4-bus system in Fig. B1. We consider 3 different cases. The reference case is ***A*** and is without any constraints. Then, change cases with two different types of constraints are introduced: in case ***B*** we include the physical constraint of the ma*ximum flow limit on line 1-4* set at 2.5 p.u. and in case ***C*** we introduce the operational constraint of *maximum voltage limit on bus 2* specified at 0.96 p.u. These two cases illustrate the impacts of these grid constraints on the market solution. We report the values of the metrics and nodal prices at each bus of the system in Table 1. In addition, the total power traded in the market for each case is also reported.

Table 1. *Values of the key economic metrics for Example 1*

Variable	*CASE*		
	A	***B***	***C***
ρ_1^g	1.000	0.850	1.038
ρ_2^d	1.000	1.033	0.836
ρ_3^g	1.000	1.064	1.019
ρ_4^d	1.000	1.179	1.039
$\overline{\rho}^G$	1.000	0.932	1.032
$\overline{\rho}^D$	1.000	1.111	0.942
S^G	2.353	1.850	2.585
S^D	3.500	2.748	3.878
S^M	0.000	1.104	-0.659
S^S	5.853	5.701	5.804
Total power traded	6.998	6.180	7.332

In the unconstrained reference case ***A*** the price faced by each market participant is uniform and equal to the market-clearing price. Because of the existence of this single market-clearing price, the S^M is equal to zero. Since there is no dead-weight loss for this, the S^S is the highest of all the cases.

In case ***B***, the physical constraint causes a reduction of the traded power with reference to the unconstrained solution. The efficiency of the equilibrium point of the system is reduced leading to a lower value of S^S. However, the reduction in surplus is not equally shared between the consumers and the producers. The participants' prices are no longer equal and, on the average, the generators are paid a lower price than that paid by the loads. The difference between $\overline{\rho}^D$ and $\overline{\rho}^G$ results in a positive S^M.

In case ***C***, a single market-clearing price also does not exist, but in this case the generators' average price is higher than the loads' average price. The difference between $\overline{\rho}^D$ and $\overline{\rho}^G$ causes a negative S^M. In this case, the IGO subsidizes the market players to undertake more power trade than in the unconstrained case. In other words, the solution for maximizing S^S is achieved by increasing the level of production and consumption with reference to the base case. Because of this subsidy the producer and the consumer surplus are this time also increased with respect to case *A* but in unequal amounts. The S^S is still lower than in the unconstrained case.

The three cases serve to illustrate the possible market outcomes in this small system. The cases show the usefulness of each of the metrics introduced and provide concrete application of their use.

Example 2 – Two-zone representation

The 4-bus system is considered to consist of two well-defined zones. Within each zone there is no constraint binding. However, there is a major interzonal constraint limiting the amount of flow between the two zones. Fig. B1 indicates the definition of the zones and Fig. B2 shows the equivalent two-zone representation. The rule of operation of this system is to split the system into the two zones whenever the interzonal constraint limit is hit or violated. The objective of this example is to illustrate the impact of the rule of operation on the assessment of the market behavior.

The reference case of the system is precisely the same as case A in Example 1. As long as there are no flow constraints between the two zones, the network operates as in Example. The equivalence of the reference case is restated as case D in Table 2. We introduce the zonal prices ρ_{Z1} and ρ_{Z2} for the zone $Z_1$and the zone Z_2 prices , which are equal to the nodal price at each bus in the respective zone. Each tie-line between Z_1 and Z_2 has flow limits. When one of them hits a limit the operation of the system is split into the two zones. The system operated as the two zones results in the solution shown as case E in Table 2. In both cases *D* and *E* the prices in a zone are equal for all generators and loads due to the fact that no constraints are hit within a zone. Note that with respect to case D the S^S decreases as the S^G and S^D.

This example illustrates the impact on market performance of the *rules of the road* of the system.

Table 2. *Values of the key economic metrics for Example 2*

Variable	*CASE*	
	D	***E***
ρ_1^g	1.000	0.966
ρ_2^d	1.000	1.045
ρ_3^g	1.000	1.045
ρ_4^d	1.000	0.966
$\overline{\rho}_{Z1}$	1.000	0.966
$\overline{\rho}_{Z2}$	1.000	1.045
S^G	2.353	2.293
S^D	3.500	3.428
S^M	0.000	0.119
S^S	5.853	5.840
Total power traded	6.998	6.920

5. – Summary

We discussed the effective deployment of microeconomic concepts and metrics for the analysis of competitive electricity markets. The paper combines a better understanding of the trading mechanism incurred in electricity markets with the consideration of the physical nature of the electricity system and the constraints of the market participants as well as the formulation of the rules of the road. The application of the concepts and metrics were illustrated with two simple numerical examples.

6. – References

[1] C.A. Berry, B.F. Hobbs, W.A. Meroney, R.P. O'Neil and W.R. Steward, "Analyzing Strategic Bidding Behavior in Transmission Networks", Proc. of the IEEE Power Engineering Society Winter Meeting, Singapore, January 23-27, 2000.

[2] P.D. Klemperer and M.A. Meyer, "Supply Function Equilibria in Oligopoly Under Uncertainty", Econometrica, vol. 57, pages 1243-1277, 1989.

[3] D. Friedman and J. Rust, "The Double Auction Market", SFI Studies in the Sciences of Complexity, Proc. Vol. XIV, Addison-Wesley, 1993.

[4] P.A. Samuelson, W.D. Nordhaus, "Economics", McGraw-Hill, New York, 1985, chapter 3.

[5] F.C. Scheppe, M. Caramanis, R. Tabors and R. Bohn, "Spot Pricing of Electricity", Kluwer Academic Publishers, Boston, 1998.

[6] J. Tirole, "The Theory of Industrial Organization", MIT Press, Cambridge, Massachusetts, 1998, pp. 65-69.

[7] G.Gross, E. Bompard. P. Marannino and G.Demartini, "The Uses and Misuses of Optimal Power Flow in Unbundled Electricity Markets", 23° Corso di Aggiornamento su Apparecchi, Macchine e Impianti Elettrici: "Il Nuovo Assetto dell'Energia Electtrica in Italia a Seguito dell'Attuazione della Direttiva Comunitaria 92/1996 per la Liberalizzazione del Mercato Elettrico", Pavia, Italy, 19-21 October 1999.

[8] E. Bompard, E. Carpaneto, G. Chicco, R. Napoli and G. Gross, "Reactive Load Modeling Impacts on Nodal Prices in Pool Model Electricity Markets", Proc. of the IEEE Power Engineering Society Summer Meeting, Seattle, 16-20 July 2000.

7. – Acknowledgment

The authors thankfully acknowledge the support and encouragement of Prof. Roberto Napoli in writing this paper.

Appendix A: The optimization framework

The market solution corresponds to the following optimization problem:

$$\begin{aligned} min \quad & -S^S(\underline{x},\underline{u}) \\ s.t. \quad & \underline{g}(\underline{x},\underline{u}) = \underline{0} \\ & \underline{h}(\underline{x},\underline{u}) \le \underline{0} \end{aligned} \qquad (A1)$$

We use the following notation:

$\underline{g}(\underline{x},\underline{u}) \in \Re^k$ = vector of system and participants' equality constraints

$\underline{h}(\underline{x},\underline{u}) \in \Re^l$ = vector of system and participants' inequality constraints

$\underline{u} \in \Re^m$ = vector of decision variables

$\underline{x} \in \Re^n$ = vector of state variables

Both the choice of the constraints and variables depend on the specific market rules and considerations. The commodity balance equations may be separated from the other equality constraints. In effect, g is partitioned into g_1 and g_2 where the former correspond to the commodity balance equations that represents the active power flow equations for each bus of the grid. The latter are the remaining reactive equality constraints. The Lagrange function of this optimization problem may be written as follows:

$$\mathscr{L}(\underline{u},\underline{x},\underline{\rho},\underline{\lambda},\underline{\mu}) = -S^S(\underline{x},\underline{u}) + \underline{\rho}^T \cdot \underline{g_1}(\underline{x},\underline{u}) + \underline{\lambda}^T \cdot \underline{g_2}(\underline{x},\underline{u}) + \underline{\mu}^T \cdot \underline{h}(\underline{x},\underline{u}) \qquad (A2)$$

where

$\underline{g_1}(\underline{x},\underline{u}) \in \Re^{k_1}$ = vector of commodity balance constraints

$\underline{g_2}(\underline{x},\underline{u}) \in \Re^{k_2}$ = vector of other equality constraints

with $k_1 + k_2 = k$.

$\underline{\rho} \in \Re^{k_1}$ = vector of Langrangian multipliers of the commodity balance constraints, or vector of market-clearing prices

$\underline{\lambda} \in \Re^{k_2}$ = vector of Langrangian multipliers of the other equality constraints

$\underline{\mu} \in \Re^{l}$ = vector of Lagrangian multipliers of the inequality constraints

The Kuhn-Tucker (K-T) conditions are given by the expressions

$$\left.\frac{\partial \mathscr{L}}{\partial \underline{u}}\right|_{(\underline{x}^*,\underline{u}^*)} = \underline{0}, \qquad \left.\frac{\partial \mathscr{L}}{\partial \underline{x}}\right|_{(\underline{x}^*,\underline{u}^*)} = \underline{0},$$
$$\left.\frac{\partial \mathscr{L}}{\partial \underline{\rho}}\right|_{(\underline{x}^*,\underline{u}^*)} = \underline{0}, \qquad \left.\frac{\partial \mathscr{L}}{\partial \underline{\rho}}\right|_{(\underline{x}^*,\underline{u}^*)} = \underline{0} \qquad (A3)$$
$$and \quad \underline{\mu}^T \cdot \underline{h}(\underline{x},\underline{u}) = \underline{0}$$

At the optimum $(\underline{x}^*,\underline{u}^*)$, which corresponds to the *market equilibrium*, the K-T conditions are assumed to hold.

We next consider the special case of the small system of Fig. 4. For the two participants, the optimization problem becomes the following:

$$x = q_{12}; \quad \underline{u} = \left(q^g, q^d\right)^T$$

$$\begin{aligned} & S^S = C\left(q^g\right) - B\left(q^d\right) \\ & g_1\left(q_{12}, q^g, q^d\right) = \left(-q^g + q_{12}, \quad q^d - q_{12}\right)^T \\ & h(q_{12}) = q_{12} - q_{12}^{max} \end{aligned}$$

$$\begin{aligned} & \mathscr{L}\left(q_{12}, q^g, q^d, \rho^g, \rho^d, \mu\right) = C\left(q^g\right) - B\left(q^d\right) \\ & \rho^d \cdot \left(-q^g + q_{12}\right) + \rho^g \cdot \left(q^d - q_{12}\right) + \\ & \mu \cdot \left(q_{12} - q_{12}^{max}\right) \end{aligned} \qquad (A4)$$

$$\left.\frac{\partial C(q^g)}{\partial q^g}\right|_{q^{g^*}} = \rho^{g^*}, \qquad \left.\frac{\partial B(q^d)}{\partial q^d}\right|_{q^{d^*}} = \rho^{d^*},$$
$$\rho^{d^*} - \rho^{g^*} + \mu^* = 0$$
$$and \quad \mu^* \cdot \left(q_{12}^{\ *} - q_{12}^{max}\right) = 0$$

Suppose that $q_{12}^{\ *} = q_{12}^{max}$. Since K-T complementary slackness condition $\mu^* \ge 0$, consequently ρ^{d^*} need not be equal to ρ^{g^*}.

On the other hand, if we consider that the generator is operating at its maximum output limit, we modify the above formulation as follows. Assume that the line flow limit is ignored and instead we have a new inequality constraint.

$$h'\left(q^g\right) = q^g - q^{g,max} \leq 0 \quad \text{(A5)}$$

The equality constraint simplifies in

$$g_1\left(q^g, q^d\right) = -q^g + q^d = 0 \quad \text{(A6)}$$

Let the corresponding dual multipliers be denoted by μ and ρ^M, respectively. Then the Lagrangian and the corresponding K-T conditions are

$$\mathscr{L}\left(q^g, q^d, \rho^M, \mu\right) = C\left(q^g\right) - B\left(q^d\right) + \rho^M \cdot \left(-q^g + q^d\right) + \mu \cdot \left(q^g - q^{g,max}\right)$$

$$\left.\frac{\partial C\left(q^g\right)}{\partial q^g}\right|_{q^{g^*}} = \rho^{M^*} - \mu^*, \qquad \left.\frac{\partial B\left(q^d\right)}{\partial q^d}\right|_{q^{d^*}} = \rho^{M^*}, \quad \text{(A7)}$$

$$\text{and} \quad \mu^* \cdot \left(q^{g^*} - q^{g,max}\right) = 0$$

Clearly, the market-clearing price ρ^{M^*} is unique.

Appendix B: Numerical Data

The lossless 4-bus system used for the numerical examples is depicted in Fig. B1. Table B1 reports the line parameters.

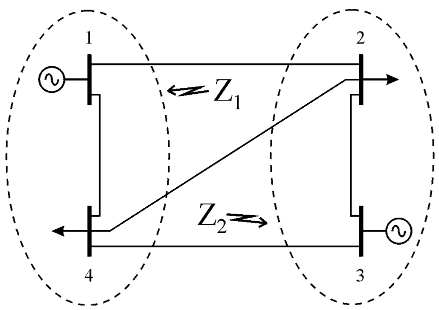


Fig. B1: The *4-bus system used in the numerical examples*

Table B1: *The line parameters of the 4-bus system*

Line	*Reactance* (p.u.)	*Charging* (p.u.)
1-2	0.1200	0.1900
2-3	0.0700	0.1100
3-4	0.1000	0.1580
1-4	0.0800	0.1270
2-4	0.1600	0.2530

Different levels of detail may be considered in the model of the system. In the illustrative examples a general full A.C. OPF is employed. The equality constraints of the general model described in (A1) considered in the examples are the real and reactive power balances at each bus of the system. The reactive power load at each bus Q_i^d was modeled as a function of the active power load [8].

$$Q_i^d = 0.5 \cdot P_i^d \quad \text{(B1)}$$

The inequality constraints considered are the bus voltage limits, the generator real and reactive power limits, the apparent power line flow limits, and the real power demand limits. We adopted an affine offer/bid function for each generator/load with the parameters in Tables B2 and B3, respectively.

$$p_i^g = c_{2,i} \cdot P_i^g + c_{1,i} \quad \text{(B2)}$$

$$v_i^d = b_{2,i} \cdot P_i^d + b_{1,i} \quad \text{(B3)}$$

Table B2: *The offer parameters for the 4-bus system*

Bus	c_2	c_1	C_0
1	0.036	0.3031	0
3	0.0715	0.3817	0

Table B3: *The bid parameters for the 4-bus system*

Bus	b_2	b_1	B_0
2	0.0834	2	0
4	0.0626	2	0

The two-zone representation of the 4-bus system is shown in Fig. B2. Z_1 includes the generator at bus 1 and the load at bus 4, while Z_2 includes the load at bus 2 and the generator at bus 3. In this zonal representation we assume no intrazonal congestion. The zones are connected by a so-called *interface* that is an equivalent representation of all the lines connecting the two zones.

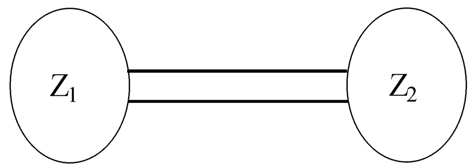


Fig. B2: *Two-zone system model*

In the example referred above, we modeled this interface as an equivalent tie-line. The reactance of this equivalent is derived by neglecting the lines charging capacitances and computing the equivalent (parallel) reactance. The maximum line flow limit of the equivalent interface depends on the maximum limits of each line and the actual flows upon the onset of interzonal congestion. For the simple case treated the parameters used are a reactance of 0.0407 and a maximum flow of 1.55 p.u.