REACTIVE LOAD MODELING IMPACTS ON NODAL PRICES IN POOL MODEL ELECTRICITY MARKETS

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Abstract. This paper is concerned with the interpretation of the nodal prices in competitive electricity markets based on the Pool paradigm. Such prices are the byproducts of the optimization performed by the independent grid operator (IGO) to determine the centralized economic dispatch taking into account all transmission network and physical/operations constraints. The IGO implicitly takes into account congestion considerations in determining the centralized economic dispatch. Under the Pool paradigm, a system marginal price no longer exists and each bus may have a different real and reactive power nodal price due to line losses and congestion avoidance considerations that can arise when the limit of one or more constraints is reached. The objective is to explore the economic signals provided by these prices and effectively apply them in the design of markets and the *rules of the road* for these markets.

The main focus of the paper is on the explicit evaluation of the impacts of the reactive load on the nodal real and reactive prices. We adopt a rather general model for reactive load model in which the reactive power at each node is represented as an affine function of the real power at that node, i.e., the reactive load is the sum of a constant and a constant power factor component. This model includes as special cases the constant reactive load and the constant power factor load including the case of purely real load corresponding to unity power factor. We investigate the relationship between the real and reactive nodal prices and evaluate the impacts on them of the dual variables due to the various other physical/operations constraints in the system. We discuss the significance of the nodal price observations and the effective utilization in developing appropriate price signals in the Pool paradigm.

Keywords: nodal prices, pool paradigm, congestion management, reactive load model.

I. INTRODUCTION

The open access to transmission represents the basis for the establishment of a free market of electricity. The physical and operational constraints inherent to power systems require the introduction of an independent authority charged for the managing of the transmission network; its main task is to facilitate as many transactions as possible while keeping the system secure and free of congestion. Such a situation arises whenever the current or a postulated contingency case leads to one or more violations of any physical or operational constraints. We refer to this entity as the Independent Grid Operator (IGO). The different implementations of the various IGO's have different levels of authority. The *Pool Model* is the paradigm in which the IGO has a vast amount of authority and decision making power [1]. The Pool model merges the grid operator's coordination role with a centralized economic dispatch function. All the power traded goes through the pool as the pool is the sole buyer and seller of electricity. The pool determines the "optimum" solution by solving a *centralized economic dispatch* problem taking into account the network constraints. The Optimal Power Flow (OPF) tool may be used to determine this optimum according to some metric. In effect this entails the determination of all suppliers who will inject power into the network and all loads that will withdraw power from the network with all network constraints being met [2]. As such, the *Pool Model* IGO fulfills both the market and the coordination functions simultaneously. The PJM (*Pennsylvania-Jersey-Maryland*) Interconnection is an example of the implementation of the Pool model [3].

The objective of the optimization problem is the maximization of social surplus. If each generator (load) set its price equal to its marginal costs (marginal benefits) it maximizes its profit (net benefit) [4]. Under this hypothesis the costs and benefits, which are used to compute social surplus, can be derived from the offer and bid curves.

An important byproduct of the optimum of the OPF is the economic information obtained from the dual variables and with direct applications in electricity markets. The dual variables associated with the real and reactive power nodal balances are interpreted as sensitivities of the objective to a small injection of real or reactive power at that node and are defined as the nodal real and reactive power Nodal Prices (NP's) at each bus. The offer price ρ^g is defined as the unit energy price at which the power is sold by a generator; analogously, the bid price ρ^d defines the price at which an energy unit is paid by the load. Both of these quantities are determined directly from the offer and bid curves of generators and loads. These prices may also be expressed in terms of dual variables associated with the equality and inequality constraints of the optimization problem.

If we consider a market without the network constraints, the optimum gives what is known as the market clearing price and the corresponding market clearing quantity. The market equilibrium point determined by the market clearing price and the market clearing quantity maximizes the social surplus. Once network considerations are brought in, the losses and the various constraints move the optimum away from this economic equilibrium and a market clearing price no longer exists. Instead, there are different NP's at each bus.

The reactive power plays a major role in the power system operations and its effects must be taken into account [5]. Depending principally on the real and reactive load level of the system, situations with lack or excess of reactive power may arise. Such situations lead to congestion due to the violation of one or more voltage and/or reactive power limits. In such cases, the realistic representation of the reactive

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behavior of the loads is of the paramount importance in order to send the appropriate economic signals to both generators and loads.

Different approaches have been proposed [6-8]. In this paper we assume a rather general model for the reactive load, in the context of elastic loads, in which the power demanded depends on its price. We represent explicitly the relationship between the nodal prices and the bid prices paid by the loads and the offer prices paid to the generators at the optimum. The focus of the paper is the scope of impacts the load reactive model has on these prices. The numerical examples illustrate the impacts of various constraints for the general reactive load model proposed on the nodal and the optimum bid and offer prices for congestion due to different type of limit violations. The role of the representation of losses is also investigated. We use a set of appropriate economic metrics to evaluate these aspects. We use a nonlinear AC OPF for the analysis and the illustrative examples.

This paper consists of five additional sections. Section II describes the reactive model adopted. We illustrate the key aspects of the Pool Model dispatch in Section III. Section IV is devoted to the analysis of the NP's in terms of the optimization results. In Section V the economic metrics used to assess the impacts are introduced. We present a set of case studies in Section VI. We summarize the basic results and general findings in the last section.

II. REACTIVE LOAD MODEL

The load behavior in terms of reactive power has a significant impact on the transmission system, mainly, for the voltage profile. Moreover, the reactive power consumption affects the optimum point reached in the optimization problem used to dispatch the system due to the losses it induces. From this point of view, the impact of reactive power on the grid must be considered.

In general the load reactive power Q_j^d at bus j can be expressed as a function of the real power P_j^d:

$$Q_j^d = \psi_j(P_j^d) \quad (1)$$

Modelling $\psi_j(P_j^d)$ as an affine function, we will use:

$$Q_j^d = Q_j^{d0} + w_j \cdot P_j^d \quad (2)$$

with: $P_j^{d\,min} \le P_j^d \le P_j^{d\,max}$ (3)

For a purely real load, $Q_j^{d0} = w_j = 0$. For a constant power factor load, $Q_j^{d0} = 0$ and $w_j = \tan \varphi_j$.

III. DISPATCH IN THE POOL MODEL

In the usual OPF, the loads are fixed and the problem solution gives the optimal operating point that minimizes the total generation costs. In the Pool Model, the load active power is a decision variable that is determined through the solution of an optimization problem whose objective is the maximization of the social surplus.

The basic economic information needed by the IGO to perform the optimization are given by generators and loads through offer and bid curves. The offer $\rho_i^g(P_i^g)$ is the price function for the generator at bus i (equal to its incremental cost at different generation levels). Analogously, the load bid $\rho_i^d(P_j^d)$ expresses the *willingness* to pay of the customer and is defined as the price function for the j-th load (equal to the incremental benefit at certain power withdrawal level).

The offers and bids are supposed to be related to the costs C_i and to the benefits B_j:

$$\frac{dC_i}{dP_i^g} = \rho_i^g\left(P_i^g\right) \qquad \frac{dB_j}{dP_j^d} = \rho_j^d\left(P_j^d\right) \quad (4)$$

The social surplus S^S is given by:

$$S^S = \sum_{j \in \mathcal{D}} B_j\left(P_j^d\right) - \sum_{i \in \mathcal{G}} C_i\left(P_i^g\right) \quad (5)$$

The optimization problem of maximizing the social surplus is then stated in terms of minimizing $f(z) = -S^S$:

$$\min f(z) \quad (6)$$

$$\text{s.t. } h(z) = 0 \quad (7)$$

$$g(z) \le 0 \quad (8)$$

The set of variables z is:

$$z^T = \left[v^T,\ \theta^T,\ p^{gT},\ q^{gT},\ p^{dT},\ q^{dT}\right]$$

Let's define:

$\mathcal{N}$	set of the N buses of the system
$\mathcal{G}$	set of the N_g buses with generators
$\mathcal{D}$	set of the N_d buses with loads
$\mathcal{N}_L$	set of the N_l lines
v	bus voltages (size N)
θ	bus voltage angles (size N, θ_1=0 slack)
p^g	real generators power (size N)
q^g	reactive generators power (size N)
p^d	real loads power (size N)
q^d	reactive loads power (size N)
s	apparent line power (size N_L)
ρ_i^g	generation offer price at bus i
ρ_j^d	load bid price at bus j
$C_i(P_i^g)$	cost function of generator at bus i
$B_j(P_j^d)$	benefit function of load at bus j

In this paper, the generator offers and the loads bids are assumed as:

$$\rho_i^g = c_{2i} P_i^g + c_{1i} \qquad i \in \mathcal{G} \quad (9)$$

$$\rho_j^d = b_{2j} P_j^d + b_{1j} \qquad j \in \mathcal{D} \quad (10)$$

where c_{2i} is positive and b_{2j} is negative. Hence, the generation cost C_i and the load benefit B_j are written as:

$$C_i\left(P_i^g\right) = \frac{1}{2} c_{2i} \left(P_i^g\right)^2 + c_{1i} P_i^g + c_{oi} \qquad i \in \mathcal{G} \quad (11)$$

$$B_j\left(P_j^d\right) = \frac{1}{2} b_{2j} \left(P_j^d\right)^2 + b_{1j} P_j^d + b_{oj} \qquad j \in \mathcal{D} \quad (12)$$

The equality (7) and inequality (8) constraints of the optimization problem (6) are:

active power balances (dual variables λ_k^a):

$$-P_k^g + P_k^d + \sum_{m=1}^{N} V_k V_m Y_{km} \cos\left(\theta_k - \theta_m - \phi_{km}\right) = 0 \qquad k \in \mathcal{N} \quad (13)$$

reactive power balances (dual variables λ^r_k)

$$-Q_k^g+Q_k^d+\sum_{m=1}^{N}V_kV_mY_{km}\sin(\theta_k-\theta_m-\phi_{km})=0 \qquad k\in\mathcal{N} \quad (14)$$

reactive load model (dual variables γ_j):

$$Q^d_j=\psi_j(P^d_j) \quad \leftrightarrow \quad \gamma_j \qquad j\in\mathcal{D} \quad (15)$$

bus voltage limits (dual variables μ_k^v,η_k^v)

$$V_k-V_k^{max}\le 0 \quad \leftrightarrow \quad \mu_k^v\ge 0$$
$$-V_k+V_k^{min}\le 0 \quad \leftrightarrow \quad \eta_k^v\ge 0 \qquad k\in\mathcal{N}$$

generation limits (dual variables $\mu_i^{pg},\eta_i^{pg},\mu_i^{pg},\eta_i^{pg}$)

$$P_i^g-P_i^{g\,max}\le 0 \quad \leftrightarrow \quad \mu_i^{pg}\ge 0$$
$$-P_i^g+P_i^{g\,min}\le 0 \quad \leftrightarrow \quad \eta_i^{pg}\ge 0 \qquad i\in\mathcal{G}$$
$$Q_i^g-Q_i^{g\,max}\le 0 \quad \leftrightarrow \quad \mu_i^{qg}\ge 0$$
$$-Q_i^g+Q_i^{g\,min}\le 0 \quad \leftrightarrow \quad \eta_i^{qg}\ge 0 \qquad i\in\mathcal{G}$$

line current limits (dual variables μ^s_l)

$$-h_l(v,\underline{\theta})+S_{l\,max}\le 0 \quad \leftrightarrow \quad \mu^s_l\ge 0 \qquad l\in\mathcal{L}$$

real demand limits (dual variables μ_j^{pd},η_j^{pd})

$$P_j^d-P_j^{d\,max}\le 0 \quad \leftrightarrow \quad \mu_j^{pd}\ge 0$$
$$-P_j^d+P_j^{d\,min}\le 0 \quad \leftrightarrow \quad \eta_j^{pd}\ge 0 \qquad j\in\mathcal{D}$$

IV. ANALYSIS OF NODAL PRICES

The offer and bid prices can be expressed as a function of the dual variables obtained from the optimization problem. Applying the Kuhn-Tucker conditions:

$$\frac{\partial\mathcal{L}}{\partial P_i^g}=\rho_i^g-\lambda^a_i-\eta^{pg}+\mu^{pg}=0 \qquad i\in\mathcal{G} \quad (16)$$

$$\frac{\partial\mathcal{L}}{\partial P_j^d}=-\rho_j^d+\lambda^a_j-\eta^{pd}+\mu^{pd}+\lambda^r_j\frac{dQ_j^d}{dP_j^d}=0 \quad j\in\mathcal{D} \quad (17)$$

$$\frac{\partial\mathcal{L}}{\partial Q_i^d}=\lambda^r_j-\gamma_j=0 \qquad j\in\mathcal{D} \quad (18)$$

The dual variables λ^a_i and λ^r_i are, respectively, the real power nodal price (real NP) and the reactive power nodal price (reactive NP). The generation price and the demand price become then:

$$\rho_i^g=\lambda^a_i+\eta^{pg}-\mu^{pg} \qquad i\in\mathcal{G} \quad (19)$$

$$\rho_j^d=\lambda^a_j+\lambda^r_j\frac{dQ_j^d}{dP_j^d}-\eta^d+\mu^d_j \qquad j\in\mathcal{D} \quad (20)$$

Hence, the generator price ρ_i^g is equal to the real NP only if no active generation limit is reached. Analogously, the demand price ρ_j^d is equal to real NP only if the active demand limit is not reached and the reactive power does not depend on the active one or the reactive NP is zero (as it happens in a lossless system without congestion).

V. METRICS FOR THE EVALUATION OF IMPACTS

To evaluate the effect on the nodal prices, the following metrics have been used:

producer surplus S_i^G of the generator at bus i:

$$S_i^G=\rho_i^gP_i^g-C_i(P_i^g) \quad (21)$$

consumer surplus S_j^D of the demand at bus j:

$$S_j^D=B_j(P_j^d)-\rho_j^dP_j^d \quad (22)$$

average generation price:

$$\overline{\rho}^g=\sum_i\rho_i^g(P_i^g)P_i^g\Big/\sum_i P_i^g \qquad i\in\mathcal{G} \quad (23)$$

average demand price:

$$\overline{\rho}^d=\sum_j\rho_j^d(P_j^d)P_j^d\Big/\sum_j P_j^d \qquad j\in\mathcal{D} \quad (24)$$

total producer surplus S^G: $\quad S^G=\sum_{i\in\mathcal{G}}S_i^G \quad (25)$

total consumer surplus S^D: $\quad S^D=\sum_{j\in\mathcal{D}}S_j^D \quad (26)$

merchandise surplus S^M:

$$S^M=\sum_{j\in\mathcal{D}}\rho_j^d(P_j^d)P_j^d-\sum_{i\in\mathcal{G}}\rho_i^g(P_i^g)P_i^g \quad (27)$$

Note that the S^M is different from zero if the system is lossy and/or it is affected by a congestion.

social surplus S^S: $\quad S^S=S^G+S^D+S^M \quad (28)$

VI. CASE STUDIES AND RESULTS

The nodal prices incorporate the effects of sensitivity of the objective function with respect to the different system constraints. The objective is to show the impacts of the different reactive demand models on the behavior of the nodal prices and some key economic metrics with a set of different operating conditions. Two systems are considered: a 10-bus system, whose data are given in the Appendix, and the IEEE 30-bus system. The loads at each bus is modeled with constant power factor ($Q_j^{d0}=0$, $w_j=0.5$, $j\in\mathcal{D}$). A case of constant reactive power load is also presented. The 10-bus system has been considered firstly lossless to permit to put into evidence the effects of the congestion decoupled from the effects of losses. Then, the unconstrained case has been studied with losses to assess the losses impact.

A. Base case – lossless and unconstrained

The base case does not consider any physical or operational constraints binding. This case corresponds to the solution found by the market without any network consideration.

Table 1a. Decision and dual variables in the base case

Bus	λ^a	λ^r	ρ^g	ρ^d	P^g	P^d
1	1.000	0.000	1.000		3.200	
2	1.000	0.000		1.000		1.800
3	1.000	0.000		1.000		2.000
4	1.000	0.000	1.000		4.300	
5	1.000	0.000	1.000		2.600	
6	1.000	0.000	1.000		1.900	
7	1.000	0.000		1.000		1.400
8	1.000	0.000		1.000		3.000
9	1.000	0.000		1.000		1.800
10	1.000	0.000		1.000		2.000

Table 1b. Value of the key economic indexes

$\overline{\rho^g}$	$\overline{\rho^d}$	S^G	S^D	S^M	S^S	*Total demand*
1.000	1.000	3.401	6.000	0.000	9.401	12.000

From Tab. 1a it is possible to see that demand (ρ_j^d) and generator (ρ_i^g) prices are equal at each bus and hence a market clearing price exists for the system. The generation and demand prices correspond exactly to the real NP's λ^a_k, at each bus. All the reactive NP's λ^r_k are null; the load reactive model has no impact in this case and the reactive power is "free" for generators and loads. Tab. 1b shows that the power demand that would be traded by the market without any constraint is 12 pu. In this case S^M is equal to zero, meaning that the IGO does not collect money for the system operation.

B. Line flow limit impact - Line flow limit (apparent power) on line 6-7 (2.1 pu)

A line flow limit activation (Tab. 2a) causes non uniform values of λ^a at each bus. The reactive NP's λ^r becomes different from zero at the load buses, causing an increase of ρ_j^d with respect to the corresponding real NP and showing that, due to congestion, the reactive power affects the prices.

Table 2a . Decision and dual variables of case B

Bus	λ^a	λ^r	ρ^g	ρ^d	P^g	P^d
1	1.044	0.000	1.044		3.453	
2	1.027	0.012		1.033		1.740
3	1.001	0.008		1.005		1.991
4	0.984	0.000	0.984		4.187	
5	0.945	0.000	0.945		2.321	
6	0.888	0.000	0.888		1.507	
7	1.093	0.062		1.123		1.227
8	1.058	0.027		1.071		2.786
9	1.027	0.025		1.039		1.730
10	0.996	0.015		1.003		1.994

Table 2b. Value of the key economic indexes

$\overline{\rho^g}$	$\overline{\rho^d}$	S^G	S^D	S^M	S^S	*Total demand*
0.981	1.043	3.152	5.489	0.703	9.344	11.468

For all the loads ρ_j^d is higher than the real NP, and the difference is due to the reactive/active sensitivity term. The congestion (Tab.2b) causes a reduction in the power traded. The working point of system is less efficient from an economic point of view (reduction of S^S) and the IGO can collect money from the system (S^M>0).

C. Minimum voltage limit impacts - Minimum Voltage limit at bus 3 (0.965 pu)

a) constant power factor loads

A minimum voltage limit activation is considered. The results are reported in Tab.3a and 3b. The situation is analogous at that of case B. The NP is greater at the bus 3, in which the voltage limit is binding.

The activation of a minimum voltage limit puts into evidence a reactive power lack in the network. In this case the consumption of reactive power at load buses causes an increase in the demand prices (λ^r_j>0); this increase corresponds to a decrease in the load demand. The price signal obtained strives for the direction of congestion removal.

Table 3a. Decision and dual variables of case C-a

Bus	λ^a	λ^r	ρ^g	ρ^d	P^g	P^d
1	0.989	0.000	0.989		3.139	
2	1.014	0.137		1.082		1.652
3	1.019	0.398		1.218		1.564
4	0.963	0.000	0.963		4.046	
5	0.970	0.000	0.970		2.445	
6	0.973	0.000	0.973		1.807	
7	0.983	0.020		0.993		1.409
8	0.990	0.019		1.000		3.001
9	0.987	0.046		1.011		1.781
10	0.974	0.022		0.985		2.030

Table 3b. Value of the key economic indexes

$\overline{\rho^g}$	$\overline{\rho^d}$	S^G	S^D	S^M	S^S	*Total demand*
0.973	1.040	3.089	5.491	0.759	9.338	11.437

b) constant reactive power loads

The same case was run with a constant reactive load (Q_j^{do} equal to the values of case A, $w_j = 0.5, j\in \mathcal{D}$). The results are shown in Tab. 4a and Tab. 4b.

Table 4a. Decision and dual variables of case C-b

Bus	λ^a	λ^r	ρ^g	ρ^d	P^g	P^d
1	0.843	0.000	0.843		2.296	
2	1.824	18.163		1.824		0.316
3	1.861	53.984		1.861		0.277
4	0.589	0.000	0.589		1.448	
5	0.750	0.000	0.750		1.322	
6	0.834	0.000	0.834		1.318	
7	1.251	2.419		1.251		1.048
8	1.321	2.391		1.321		2.038
9	1.533	5.738		1.533		0.840
10	1.067	2.662		1.067		1.866

Table 4b. Value of the key economic indexes

$\overline{\rho^g}$	$\overline{\rho^d}$	S^G	S^D	S^M	S^S	*Total demand*
0.764	1.312	1.029	2.197	3.496	6.723	6.384

At load buses the reactive NP is even greater than the real NP. The system is forced to work at a very inefficient point, as it can be seen by the value of S^S that is greatly reduced. The total demand is greatly decreased; a significant smaller amount of power is served at a much higher price. Nevertheless, the demand prices are equal to the active dual variables due to the independence of load reactive power on real power. The average generator offer price is lower and the average demand bid price is higher explaining the increase of merchandise surplus S^M with respect to case of the constant power factor load.

The case of the proposed model in which the power factor is constant, brings to a solution with higher total demand and lower demand prices, if compared with the solution obtained with constant reactive power demand. In the latter case, the constancy of reactive demand causes higher demand prices, due to higher values of real NP's, and a remarkable reduction in the total demand. So one has to question the use of a constant reactive power load model in a market context.

D. Maximum voltage limit impacts - Maximum Voltage limit at bus 10 (0.97 pu)

The activation of a maximum voltage constraint leads to the situation depicted in Tab. 5a and Tab. 5b.

Table 5a. Decision and dual variables of case D

Bus	λ^a	λ^r	ρ^g	ρ^d	P^g	P^d
1	0.998	0.000		0.998		3.191
2	0.988	-0.076	0.950		1.889	
3	1.017	-0.036	0.999		2.002	
4	1.039	0.000	1.039		4.568	
5	1.041	0.000	1.041		2.810	
6	1.033	0.000	1.033		2.015	
7	1.005	-0.065		0.973		1.438
8	0.988	-0.062		0.957		3.128
9	0.987	-0.145		0.915		1.953
10	1.015	-0.202		0.914		2.172

Table 5b. Value of the key economic indexes

$\overline{\rho^g}$	$\overline{\rho^d}$	S^G	S^D	S^M	S^S	Total demand
1.028	0.951	3.742	6.602	-0.974	9.370	12.583

As in case C, there is not a unique value of demand price for all loads. The real NP is different at each bus while the reactive NP is non-zero and negative, causing a decrease in the demand price with respect to the real NP. The activation of a maximum voltage limit puts into evidence a reactive power surplus in the network. The negative value of λ^r gives the right signal to the market, reducing the demand bid price and so increasing the real and reactive power consumption and striving for the direction of congestion removal. The average generator price is higher than the average demand price, causing a negative value of S^M; in this case, the IGO collects from the load less money than it pays to the generators.

E. Losses impacts

Line losses are introduced in the unconstrained system. The results are reported in Tab. 6a and Tab. 6b.

Table 6a . Decision and dual variables of case E

Bus	λ^a	λ^r	ρ^g	ρ^d	P^g	P^d
1	1.009	0.000		1.009		3.252
2	1.061	0.040	1.081		1.655	
3	1.047	0.043	1.069		1.862	
4	0.978	0.000	0.978		4.150	
5	0.964	0.000	0.964		2.415	
6	0.970	0.000	0.970		1.795	
7	1.034	0.035		1.052		1.328
8	1.055	0.037		1.074		2.779
9	1.045	0.038		1.064		1.685
10	1.015	0.026		1.028		1.944

Table 6b. Value of the key economic indexes

$\overline{\rho^g}$	$\overline{\rho^d}$	S^G	S^D	S^M	S^S	Total demand
0.983	1.062	3.192	5.277	0.540	9.009	11.612

The situation is very similar to that found for a congestion due to a line or minimum voltage constraint (case B and C). Due to losses, demand prices are different at each bus both for the real NP diversity and for the non-zero reactive NP. S^M is positive.

F. Line flow limits with losses

The standard IEEE 30-bus system has been used to verify the results obtained from the proposed 10-bus system. The test system has been modified, scaling the cost function of the generators (offers) to give unity price, at the dispatched power, for all generators, in the lossless unconstrained case. A bid curve has been introduced for each load with the same criteria (unity price for the demand that corresponds to the solution of the lossless unconstrained system). The loads have been modelled as constant power factor loads with power factors at the PQ nodes given by the standard configuration. All the bus voltages have been forced to lay within 0.95 and 1.05 pu. The results obtained confirm the consideration traced for the 10-bus system. Let's consider the 30-bus system under three different conditions:

a) lossless unconstrained system
b) lossy system with maximum voltage binding at buses 5,13,27
c) lossy system with line flow limits on lines 2-6 (0.18 pu) and 25-27 (0.095 pu)

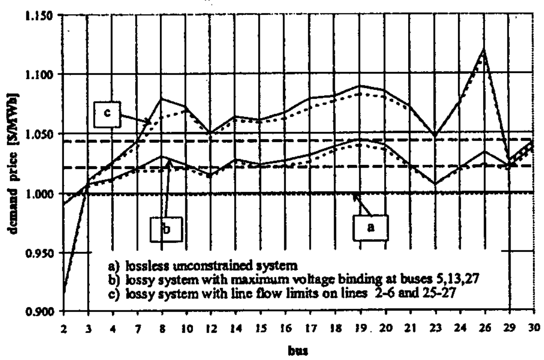

Figure 1 – Demand prices, average prices and real NP's

Fig. 1 depicts the spatial demand price distribution for the three cases along with their average values. Below the NP curves is possible to see the real NP plot for cases b and c. The difference between demand price curve and λ^a curve is due to the reactive load model.

A market clearing price exists in the market only with no losses and no congestion. The effects of both losses and congestion is that of making different NP's at each bus. The demand price is affected by any non-zero reactive NP, due to losses and constraints, as a result of the model adopted.

VI. CONCLUDING REMARKS

In this paper, we proposed a reactive load model in which the reactive demand is an affine function of the real demand. The proposed model is able to capture the interaction between real and reactive power consumption in term of their impact in determining the demand level of each load. This model has direct impact on the demand prices that depend on both real and reactive NP's.

For a lossless system without congestion, the reactive NP's are zero and the demand price is due only to the real NP. If losses are taken into account and congestion is considered, the demand price and the demand level, that are related by the bid curve, depend, in our model, both on real and reactive NP's.

The proposed model gives the right signals during congestion, causing a decrease or increase of the real and reactive demand, with a consequent demand price increase or decrease, that strives for the direction of alleviating congestion.

In a lossless system, the merchandise surplus S^M may be negative under congestion related to constraints violation due to a network surplus of reactive power. In such a situation, the IGO incurs in additional expenses as opposed to the revenues expected. If losses are represented, the S^M becomes positive but it may be reduced due to this type of constraint violation. In these cases, the economic signals that the IGO receives from the market are in the right direction, in the sense that, as opposite for other constraints violation, congestion causes a reduction of its revenues, encouraging the investments for the appropriate corrective actions.

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VIII. BIOGRAPHIES

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APPENDIX

The 10-bus system used as test system is depicted in Fig. A1. Tab. A1 reports the line parameters. Tab A2 and Tab. A3 list the offer and bid price parameters for generators and loads respectively.

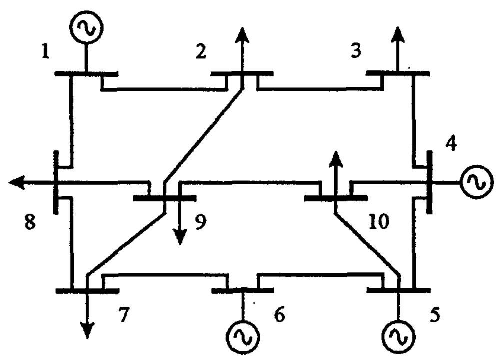

Figure A1. 10 bus system

Table A1. 10 bus system – line parameters

Bus a	*Bus b*	*Resistance*	*Reactance*	*Charging*
1	2	0.0180	0.1400	0.2220
2	3	0.0210	0.1600	0.2530
3	4	0.0130	0.1000	0.1580
4	5	0.0160	0.1200	0.1900
5	6	0.0110	0.0800	0.1270
6	7	0.0130	0.1000	0.1580
7	8	0.0130	0.1000	0.1580
8	1	0.0090	0.0700	0.1100
8	9	0.0160	0.1200	0.1900
9	10	0.0050	0.0400	0.0630
2	9	0.0140	0.1050	0.1660
7	9	0.0200	0.1500	0.2380
5	10	0.0145	0.1100	0.1740
4	10	0.0065	0.0500	0.0790

Table A2. 10 bus system – offer price parameter

Bus	c_2	c_1	c_0
1	0.1740	0.4432	0
4	0.1440	0.3808	0
5	0.1960	0.4904	0
6	0.2860	0.4566	0

Table A3. 10 bus system – bid price parameter

Bus	b_2	b_1	b_0
2	-0.5556	2	0
3	-0.5000	2	0
7	-0.7142	2	0
8	-0.3334	2	0
9	-0.5556	2	0
10	-0.5000	2	0