THE ROLE OF LOAD DEMAND ELASTICITY IN CONGESTION MANAGEMENT AND PRICING

Ettore Bompard, Enrico Carpaneto, Gianfranco Chicco
members

Dipartimento di Ingegneria Elettrica Industriale
Politecnico di Torino
C.so Duca degli Abruzzi 24
I-10129 Torino

George Gross
fellow

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801

Abstract: In the open access transmission regime, the *common carrier* nature of the transmission system may give rise to frequent conditions of congestion. Under such conditions, a violation of one or more physical or operational constraints in the base case or one of the contingency cases is encountered. Congestion may result in certain cases in marked price volatility and leads to price spikes. This is particularly true in competitive electricity markets that lack demand response. In this paper, we examine the role that demand responsiveness can play in competitive electricity markets.

Typically, the task of congestion management and pricing is vested in the hands of an independent grid operator (IGO). The IGO uses an optimal power flow (OPF) based tool to determine the necessary actions to relieve the system of the congestion and to determine transmission system usage charges. The actions of price responsive loads may be represented in terms of the customers' willingness to pay. From each customer's demand curve, the elasticity of the load at different prices is known and the benefit function is derived. The load at each bus ceases to be a fixed quantity and becomes a decision variable for the optimization problem of the IGO. In this way, the IGO has additional degrees of freedom in determining the necessary actions to determine congestion relief. The paper investigates the impacts of load elasticity in congestion management and pricing. We analyze the salient characteristics of the optimum determined by the IGO with elastic load demand explicitly represented. We evaluate elasticity effects on consumer, producer, merchandising and social surplus. In addition, the demand responsiveness impacts on price volatility in terms of average price and standard deviation are determined and compared to the case without load responsiveness. We present numerical results on the IEEE 30-bus system. These results illustrate the increases in efficiency attainable in the presence of load responsiveness.

Keywords: load elasticity, congestion management, Pool Model, demand side management

I. INTRODUCTION

The electricity market uses the transmission network as a common carrier for the electricity trades. Unlike the flow of other goods that have few physical-technical constraints, electricity can be operated only under very strict and rather difficult-to-control physical and technical constraints. The violation of any of the various constraints of the system (line flow limits, voltage profile limit, stability limits, etc.) causes a situation defined as *congestion* that must be handled according to some congestion management procedures. This requirement and other similar coordination needs of the interconnected power network give rise to the need for an entity, the so-called Independent Grid Operator (IGO), to coordinate the operations of the system.

In the Pool Model [1] the IGO merges the grid operator's coordination role with a centralized dispatch function. The IGO uses the offers of the suppliers and the bids of the demanders to determine the set of successful bidders, whose offers and bids are accepted. The pool determines the "optimum" solution by solving a *centralized economic dispatch* problem, taking into account the network constraints. The congestion problem is solved implicitly as part of the economic dispatch determination in the Pool Model. The OPF is the tool that is used to determine this optimum; the objective is the so-called social surplus.

In this scenario, the demand is no longer fixed but each customer expresses his willingness to pay through a bid which represents the energy price that he would pay for a given level of served demand. Load responsiveness gives to the IGO additional decision variables that are particularly important during congestion. The possibility to vary the power dispatched at each bus can alleviate the congestion and reduce the costs associated.

In an unconstrained market, where the network is not considered, the solution of the optimization problem is characterized by a certain value of social surplus and the demand prices and generator prices at each bus are equal. In this situation a *market clearing price* is defined and it is unique for the whole system. On the contrary, if the network is considered, congestion can arise causing, along with losses, a different optimum, with a lower level of social surplus. In this situation a market clearing price does no longer exist.

Congestion and losses make the prices different at each bus and could give rise to high bus volatility. Moreover, due to the different generation and demand prices at different buses, congestion causes congestion charges [2-3] that are collected by the IGO. All these effects depends on demand responsiveness [4]. In this paper, we define the load responsiveness to prices in terms of elasticity and assess the impact that different levels of elasticity have on the market, in terms of prices and main economic metrics. We use a full AC OPF, with elastic load modeled, to represent the different constraint violations, included those related to reactive power and voltage limitations.

This paper is composed by five additional Sections. In Section II a linear price elasticity demand model for loads is introduced. Section III presents the general aspects of Congestion Management in the Pool Model, while Section

IV defines some economic metrics to evaluate the impact of the load responsiveness on the market. In Section V, the load elasticity impacts are shown with some case studies on a standard IEEE 30-bus test system and some general findings are outlined. The last section is devoted to summarize the main results.

II. THE PRICE ELASTICITY DEMAND MODEL

The willingness to pay of the j-th customer can be expressed through a decreasing function that indicates the unit energy price $\rho_j^d(P_j^d)$ (*demand price*) at which the customer will pay a given amount of power. It can be seen as a marginal price, in the sense that it is the price corresponding to the last unit of power absorbed, and this price applies to all the energy bought. From the customer bid the benefit of the customer can be derived as:

$$B_j\left(P_j^d\right) = \int_{P_j^{d\,min}}^{P_j} \rho_j^d\left(P_j^d\right) dP_j^d + B_j\left(P_j^{d\,min}\right) \qquad (1)$$

The rationale for deriving a benefit function from the bid curve is that in a perfect market a customer would bid its marginal benefit to maximize its net benefit [5].

With reference to a generic bid curve $\rho_i^d(P_j^d)$, we can define the elasticity of the demand as [6]:

$$e(P_j^d) = -\frac{\rho_j^d}{P_j^d}\frac{dP_j^d}{d\rho_j^d} \qquad (2)$$

We will assume the demand bid modeled as an affine function of the demand. The generic bid curve that characterizes the demand at bus j is then expressed as:

$$\rho_j^d = b_{2j} P_j^d + b_{1j} \qquad (3)$$

The corresponding benefit function is:

$$B_j\left(P_j^d\right) = \frac{1}{2} b_{2j}\left(P_j^d\right)^2 + b_{1j} P_j^d + b_{0j} \qquad (4)$$

Under this assumption, the elasticity is given by:

$$e(P_j^d) = -\frac{b_{1j}}{b_{2j}P_j^d} - 1 \qquad (5)$$

and it is different for each demand level P_j^d. A geometrical interpretation of the load elasticity is the following: for a given point O ($P_j^{d\,*}$, $\rho_j^{d\,*}$) on the bid curve (Fig.1) the corresponding elasticity can be expressed as:

$$e = \frac{\overline{KO}}{\overline{LO}} \qquad (6)$$

For any point belonging to the segment $\overline{KM}$, demand elasticity is lower or equal to one. In the point M, middle point of the segment $\overline{KL}$, elasticity is exactly equal to one. For a point belonging to the segment $\overline{ML}$, elasticity is greater or equal one. Hence, three different behavior can be distinguished on the bid curve: elastic (e>1), inelastic (e<1) and unity elasticity (e=1).

An *elastic demand* means that the variation in the price causes a variation in the demand more than proportional to the price variation (an increase of 1% in the price causes a decrease greater than 1% in the demand).

An *inelastic demand* means that the variation in the price causes a variation in the demand less than proportional to the price variation (an increase of 1% in the price causes a decrease lower than 1% in the demand).

A *unity elasticity demand* means that the variation in the price causes a variation in the demand proportional to the price variation (an increase of 1% in the price causes a decrease exactly of 1% in the demand).

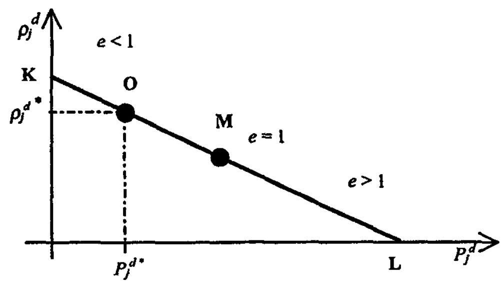

Figure 1. Geometrical interpretation of the elasticity concept

As shown in Fig. 2, two limit cases can be considered for a bid curve, with infinite elasticity and with zero elasticity. In the first case, the customer fixes the price he is willing to pay and at that price it buys any amount of power. In the second case the customer fixes the power he needs and manifest his willingness to pay that amount at any price that the market states.

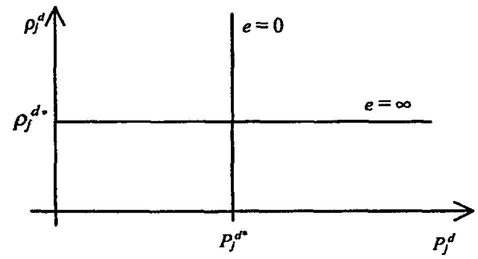

Figure 2. Load elasticity limit cases

In a market without network constraints and losses (unconstrained market), the optimum dispatch maximizes the social surplus and it is subject only to the balance between the amount of power sold and bought. In this situation a demand level, P_j^{d*}, for each load and a unique market clearing price ρ^{d*} are defined. Due to the physical characteristics of the network and, particularly, to losses and congestion, this solution may not be realized in actual electricity markets. Nevertheless, we define the linear bid curve of each load starting from the unconstrained optimum. In order to analyze the elasticity impacts, we assume that each load bid is defined by the point (ρ^{d*}, P_j^{d*}) that corresponds to the unconstrained market solution and by the elasticity at that point, which expresses its willingness to change its demand value as price varies. The bid curve can be, hence, defined through three parameters:

- a reference demand: P_j^{d*}
- a reference price: ρ^{d*}
- a load elasticity at the reference point: e^*

With these assumptions, different elasticity values mean different slopes of the linear bid curve and do not influence the unconstrained solution but play a major role when losses and congestion are taken into account.

III. CONGESTION MANAGEMENT

The congestion management in the Pool Model is performed by a one-step procedure in which the system is dispatched subject to the operational constraints of the network. The optimization problem can be stated as:

$$\min f(\underline{u},\underline{x}) \tag{7}$$

s.t.

$$\underline{h}(\underline{u},\underline{x}) = \underline{0} \tag{8}$$

$$\underline{g}(\underline{u},\underline{x}) \leq \underline{0} \tag{9}$$

where,

$\underline{u}$: the vector of m decision variables

$\underline{x}$: the vector of n state variables

and:

$f: \Re^m \times \Re^n \rightarrow \Re$ is the objective function

$\underline{h}: \Re^m \times \Re^n \rightarrow \Re^n$ is the equality constraint function

$\underline{g}: \Re^m \times \Re^n \rightarrow \Re^q$ is the inequality constraint function

Starting from the offers of generators and the bids of loads, the cost $C_i(P_i^g)$ and benefit $B_j(P_j^d)$ functions are derived and used to compute the social surplus, defined as the difference between the total benefits and the total costs. The objective that the IGO maximizes is the social surplus, that is equivalent to minimize:

$$f(\underline{u},\underline{x}) = \sum_{i\in\mathcal{G}} C_i\left(P_i^g\right) - \sum_{j\in\mathcal{D}} B_j\left(P_j^d\right) \tag{10}$$

where,

$\mathcal{G}$ is the set of buses with generators

$\mathcal{D}$ is the set of buses with loads

Different levels of detail can be considered in the model of the system. In this paper a rather general model of AC full OPF is employed to capture the main aspects related to reactive power and voltage limits. The equality constraints (8) are the real and reactive power balances at each bus of the system. The inequality constraints (9) include the bus voltage limits, the generator real and reactive power limits, the line flow limits and the real power demand limits. Reactive demand is modeled as an affine function of real demand [7].

IV. METRICS FOR THE EVALUATION OF THE DEMAND ELASTICITY ON THE MARKET

The demand elasticity, in markets with network constraints and losses, affects the optimum found by the IGO and impacts the market from an economic point of view. To compare different elasticity impacts some economic metrics are introduced.

For a dispatched demand P_j^{d*}, at bus j, with reference to the bid curve $\rho_j^d(P_j^d)$, the corresponding demand price ρ_j^{d*} is defined. The *consumer surplus* S_j^D is then expressed as:

$$S_j^D = B_j(P_j^d) - \rho_j^{d*} P_j^{d*} \qquad j\in\mathcal{D} \tag{11}$$

S_j^D increases decreasing the elasticity, tending to zero for infinite elasticity.

Analogously, for the generator at bus i, dispatched at a power level P_i^{g*} to which correspond the generator price ρ_i^{g*}, the *producer surplus* S_i^G is given by:

$$S_i^G = \rho_i^{g*} P_i^{g*} - C_i(P_i^g) \qquad i\in\mathcal{G} \tag{12}$$

We introduce some global economic metrics, useful to compare different conditions of demand elasticity.

total producer surplus S^G: $$S^G = \sum_{i\in\mathcal{G}} S_i^G \tag{13}$$

total consumer surplus S^D: $$S^D = \sum_{j\in\mathcal{D}} S_j^d \tag{14}$$

merchandise surplus S^M:

$$S^M = \sum_{j\in\mathcal{D}} \rho_j^d\left(P_j^d\right)P_j^d - \sum_{i\in\mathcal{G}} \rho_i^g\left(P_i^g\right)P_i^g \tag{15}$$

Without considering the network, demand and generator prices would be equal at each bus (market clearing price) and the total power produced by the generators would be exactly equal to that withdrawn by the loads. In this ideal condition the merchandise surplus S^M is zero. If the network is considered, the losses cause different generator and demand prices at each bus and produce a positive S^M. If a lossless system is considered with a congestion due to any kind of constraint violation, again the prices are different and S^M is different from zero and can be either positive or negative [7]. If the two effects, losses and congestion, are considered jointly, S^M is usually greater than zero. S^M can be adopted as a measure of congestion costs and is a reasonable metric to compare the congestion impact under different load elasticity.

The *social surplus* S^S is defined by the opposite of (10). When the network is considered, S^S is not greater than in the unconstrained case. Congestion impacts S^S, reducing its value differently with different load elasticity values. The reduction in social surplus can be used as a second metric to evaluate the impact of demand responsiveness. S^S can be written as:

$$\begin{aligned} S^S &= S^G + S^D + S^M = \\ &= \sum_{i\in\mathcal{G}} S_i^G + \sum_{i\in\mathcal{D}} S_i^D + \sum_{j\in\mathcal{D}} \rho_j^d\left(P_j^d\right)P_j^d - \sum_{i\in\mathcal{G}} \rho_i^g\left(P_i^g\right)P_i^g = \\ &= \sum_{j\in\mathcal{D}} B_j\left(P_j^d\right) - \sum_{i\in\mathcal{G}} C_i\left(P_i^g\right) \end{aligned} \tag{16}$$

The demand price level can be analyzed through a *weighted average demand price* defined as:

$$\overline{\rho}^d = \sum_j \rho_j^d(P_j^d)P_j^d \Big/ \sum_j P_j^d \qquad j\in\mathcal{D} \tag{17}$$

while the usual *standard deviation*, computed with reference to this average price, can be used to assess the price bus volatility.

V. CASE STUDIES AND RESULTS

The standard IEEE 30-bus has been used to assess the impact of demand elasticity on the market. The standard cost functions of the system has been scaled to give unity price for all generators in the lossless unconstrained case, at the dispatched power. For each load, a bid curve has been introduced with the same criteria (unity price for the demand that corresponds to the solution of the lossless unconstrained system). The loads have been modeled as constant power factor loads [7], with power factor corresponding to that of the standard data. All the bus voltages have been forced to lay within 0.95 and 1.05 pu. The demand elasticity impact has been considered in the following cases:

- constrained lossless system with uniform elasticity
- unconstrained lossy system with uniform elasticity
- lossless system with a bus elasticity variation

In the case of fixed loads (e=0), the demand price at each bus has been taken equal to the dual variable associated with the real power balance of the bus.

A. Constrained lossless system with uniform elasticity

We consider first the 30-bus lossless system to put into evidence the elasticity impacts in case of congestion, disregarding the effects of losses. The system is examined during congestion due to the following binding constraints:

- line flow limit on line 4-6 (0.18 pu)
- maximum voltage limit at bus 9 (0.997 pu)
- maximum reactive generation limit at bus 22 (0.393 pu)

Five different levels of elasticity are considered: $e = 0, 0.2, 1, 5, \infty$.

From Fig. 3 it is possible to see that the absolute value of S^M decreases with an increase in elasticity. For a congestion due to the maximum voltage limit, the IGO incurs in additional costs ($S^M < 0$) instead of collecting revenues. For an infinite elasticity value S^M is zero as in an unconstrained market.

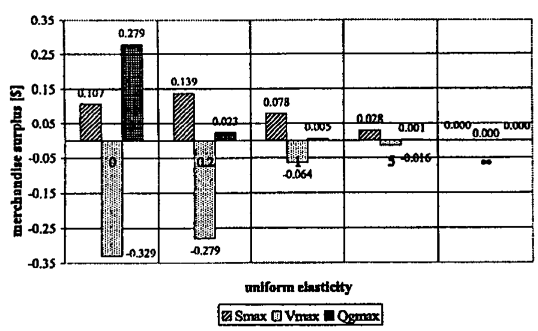

Figure 3. Merchandise surplus S^M with respect to elasticity

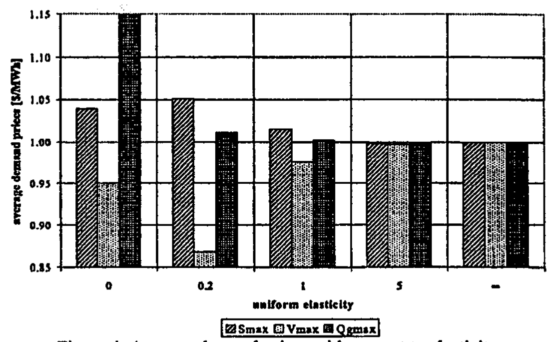

Figure 4. Average demand prices with respect to elasticity

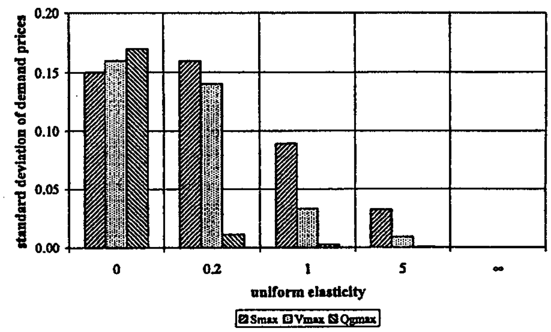

Figure 5. Standard deviation of demand prices with respect to elasticity

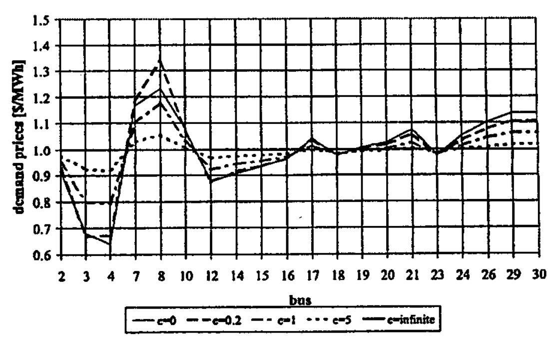

Figure 6. Nodal demand prices (line flow limit on line 4-6 binding)

Fig. 4 shows that the average demand price is decreased by an increase in elasticity and, for an uniform infinite elasticity value, the average price is equal to the market clearing price ρ^d. The standard deviation of the demand prices decreases as well and is zero for infinite elasticity (Fig.5).

Fig. 6 depicts the demand price profile in the case of line congestion. It is possible to see how the increase of elasticity decreases the bus volatility tending, for infinite elasticity, to make all demand prices equal to the market clearing price.

B. Unconstrained lossy system with uniform elasticity

To study the impacts of elasticity on the market in case of lossy systems, the 30-bus system is now considered with losses and without congestion. Fig. 7 represents the total demand and the percentage losses as the elasticity uniformly varies. It can be seen that the total demand is increased and the percentage losses are decreased by an increase in elasticity. For an infinite elasticity value the losses are very low. For elastic demand, S^M is decreased by an increase in elasticity, meaning that an increase in elasticity reduces the costs associated with losses (Fig.8).

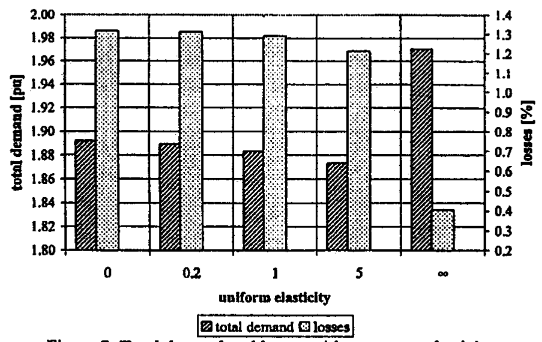

Figure 7. Total demand and losses with respect to elasticity

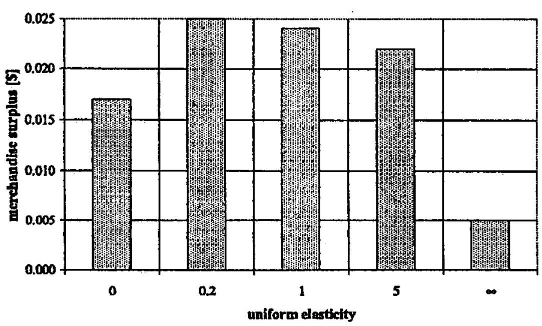

Figure 8. Merchandise surplus with respect to elasticity

C. Lossless system with a bus elasticity variation

To compare the effects of a uniform elasticity variation to the effects of a single bus variation, a line flow limit on line from bus 6 to bus 8 (0.32 pu) has been considered. Given this constraint, with a uniform elasticity e=1 for all the loads, different levels of demand elasticity (e = 0.1, 0.2, 1, 5, 10, ∞) have been considered at bus 8, receiving end of the congested line.

In Fig. 9 it is possible to see how S^M is still lowered by an elasticity increase, even if, for an infinite value of elasticity at bus 8, it is not zero, In any case, the effect of the bus elasticity increase is remarkable. The average demand prices are lowered by an increase of elasticity; a variation of elasticity at bus 8 from 0.1 to 5 causes a decrease in the average demand price of about 14% (Fig.10).

All the demand prices during congestion experience some relatively small variations except that at bus 8 which is greatly affected especially for low elasticity. The demand price at bus 8 is very high for low elasticity values (ρ^d=2.081 if e=0.1 and ρ^d=1.596 if e=0.2) and decreases with an increase in load elasticity.

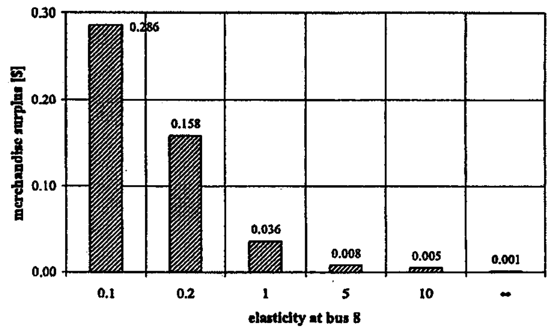

Figure 9. Merchandise surplus with respect to the variation of load elasticity at bus 8

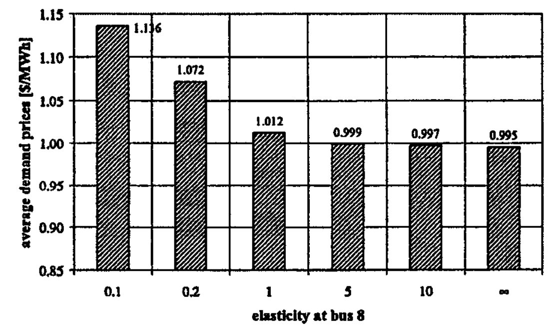

Figure 10. Average demand price with respect to the variation of load elasticity at bus 8

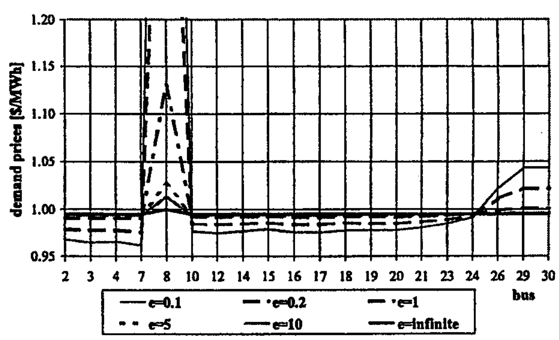

Figure 11. Nodal demand price

D. General results

Some general findings can be summarized on the basis of the preceding specific results. Since the optimization is carried on by the IGO through an OPF whose objective is maximizing social surplus, elastic loads can contribute to congestion relief or, at least, to reduce the associated costs.

Two different limit situations can be considered: uniform zero elasticity and uniform infinite elasticity. In the first case the load demands are fixed and the resulting demand prices can be very high, while in the second case the demand prices at each bus are fixed. Some general considerations can be traced on the impact of a uniform variation of demand elasticity under congestion in a Pool Model:

- *Social Surplus* S^S. As constraints are applied to an unconstrained market, the social surplus obviously decrease. S^S varies inversely with demand elasticity. For infinite elasticity, in a lossless system and for any type of constraints, the S^S has the same value that would have in the unconstrained market.
- *Merchandise Surplus* S^M. The absolute value of S^M decreased with an increase in elasticity. In a lossless system, for infinite elasticity, the S^M is zero as in an unconstrained market.
- *Demand prices*. Demand responsiveness has a great impact on demand prices. Higher values of elasticity lower the weighted average price. As elasticity increases, the demand prices decrease. In a lossless system, for infinite elasticity, all the prices are equal to the market clearing price. The average prices tend to this value, as elasticity increases, starting from higher values (if the congestion is related to an excess of reactive power) or from lower values (in other cases). Moreover, higher values of elasticity reduce the standard deviation of the demand prices. In a lossless system, for infinite elasticity, the standard deviation is zero.

In a lossy unconstrained system, the losses, computed as a percentage of the demand, are reduced by an increase in the elasticity. The merchandise surplus increases for low level of the elasticity and decreases, with respect to the fixed load case, for very high values of elasticity. An analogous behavior occurs for the average demand price and the standard deviation of the demand prices.

An increase in demand elasticity at a certain bus, especially if the bus is directly involved in the binding constraint, brings to the congestion relief even if the elasticity increase may not be able to remove the congestion. The effect of this local elasticity variation is the reduction of S^M and of the demand prices at the buses more directly involved in the congestion.

VII. CONCLUDING REMARKS

The demand responsiveness can play a major role in competitive electricity markets, particularly in the case of congestion. There are new degrees of freedom given by the introduction of load demand as an additional decision variable to be considered in the optimization problem. The demand served to each customer, along with the price that the customer pays, depend on his willingness to pay and can be associated to his demand elasticity. This load behavior is particularly important in case of congestion when at least one constraint is binding. The impacts of congestion are alleviated as the demand elasticity increases and, in the limit case of a lossless system with infinite demand elasticity, the congestion impacts tend to be removed, no matter which constraint is considered.

Demand elasticity tends to lower the congestion costs, in terms of reducing the absolute value of the merchandise surplus. The reduction of social surplus at the optimum, with reference to the unconstrained case, is mitigated by an increase in load elasticity and again, this reduction is zero if all loads have infinite elasticity.

The demand prices are greatly affected by elasticity. The average level of prices during congestion is lower for higher values of elasticity and the standard deviation of the demand prices is much lower if compared to the case without load responsiveness. In the limit to which the load elasticity tends to infinite, in a lossless system, the demand prices at each bus tend to the market clearing price and the standard deviation tends to zero.

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VIII. BIOGRAPHIES

Ettore Bompard (M'99) received his Ph.D. degree in Electrotechnical Engineering in Italy in 1994. In May 1997 he joined the Politecnico di Torino, Italy, where he is currently working as Assistant Professor. His research activities include power systems and distribution systems analysis, electricity markets restructuring and power quality.

Enrico Carpaneto (M' 85) received his Ph.D. degree in Electrotechnical Engineering in Italy in 1989. He is currently Associate Professor od Power Systems at the Politecnico di Torino, Italy. His research activities include power systems and distribution systems analysis, electricity markets restructuring and power quality.

Gianfranco Chicco (M'98) received his Ph.D. degree in Electrotechnical Engineering in Italy in 1992. In November 1995 he joined the Politecnico di Torino, Italy, where he is currently working as Assistant Professor. His research activities include power systems and distribution systems analysis, electricity markets restructuring and power quality.

George Gross (F' 88) is Professor of Electrical and Computer Engineering and Professor, Institute of Government and Public Affairs, at the University of Illinois at Urbana-Champaign. His current research and teaching activities are in the areas of power system analysis, planning, economics and operations and utility regulatory policy and industry restructuring. His undergraduate work was completed at McGill University, and he earned his graduate degrees from the University of California, Berkeley. He was previously employed by Pacific Gas and Electric Company in various technical, policy and management positions.