Transmission Loss Compensation in Multiple Transaction Networks

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***Abstract*—We develop and apply the equivalent loss compensation concept to construct flexible and effective procedures for compensating losses in a multi-transaction network. The procedures are developed in the multiple transaction framework and are based on the physical-flow allocation of losses among the transactions. The proposed procedures provide transactions the choice of selecting self-acquisition of loss compensation at designated bus(es) or to purchase the loss compensation service from a central independent grid operator (IGO). The IGO can provide loss compensation as a value-added service to its transmission customers. IGO-acquisition of loss compensation uses a linear program formulation in which network constraints are explicitly represented to determine the solution which gives the least-price at which the IGO can acquire the service. The self-acquisition service may coexist side-by-side with the IGO-acquisition and any physically feasible combination of these acquisition schemes is possible. The effectiveness and flexibility of the procedures are illustrated with numerical results using the IEEE 118- and 300-bus systems.**

***Index Terms*—Ancillary services, equivalent loss compensation, loss compensation, transactions, transmission losses.**

I. Introduction

THE OPEN access transmission regime is spearheading the rapid disintegration of the well-entrenched vertically integrated structure of the electric power industry. Under the new regime, new players and *new rules of the road* result in the proliferation of transactions that use the transmission network increasingly in a *common carrier* mode. As the use of the transmission grid diverges further away from the purposes for which it was originally designed, there is a move in the industry to establish an independent grid operator (IGO) whose mission is to facilitate access to the grid and ensure comparable transmission services to the increasing number of transmission customers. The responsibilities of the IGO may be broad such as those of the ISO in California and PJM or more narrow such as those in the Midwest ISO. Typically, one of the responsibilities of the IGO is the provision of the necessary ancillary services and the allocation of the costs among the various users of the grid. The focus of this paper is on the development of procedures for the recovery of costs in the provision of loss compensation services in a multi-transaction network.

Under the previous vertically integrated structure, the loss allocation and compensation was not an issue since in the limited number of third party use transactions, losses were stipulated in the contractual agreement between the various parties. As the industry is moving toward a transaction-based paradigm, the issue of allocating and compensating the losses needs to be addressed. Since transmission losses represent a nontrivial component of the operating costs, the transacting entities need *a priori* information about loss allocation and compensation service to evaluate various transactions under consideration. Physically, the losses are caused by all the transactions present in the system, and it is very difficult to attribute a certain amount of losses to each individual transaction. Consequently, there is considerable arbitrariness involved in the loss allocation. This arbitrariness contributes to the difficulty of compensating losses allocated to each transaction. While the so-called *real power loss service* [1] may be acquired from a variety of sources, such as the power supplier, transmission customer, a third party, or the transmission provider, the actual specification of loss compensation service is not prescribed by either NERC [1] or FERC [2].

We have recently developed a general framework for the consideration of multiple transactions in a network and a physical-flow-based allocation scheme to determine the losses attributable to each transaction [3]. A salient characteristic of the proposed mechanism is that it is independent of the ordering of the simultaneously occurring transactions. We make use of this allocation scheme to develop flexible and efficient loss compensation procedures in a multiple transaction network. We develop the equivalent loss compensation concept and apply it to such a network to develop the basis for compensating losses at any bus in the system. Loss compensation at multiple buses using fixed pre-specified factors has been proposed in [4] and is used in California [5]. In the former, the so-called participation factors at selected locations are computed taking into consideration economic and reliability criteria. The factors in the latter are the *generation meter multipliers* computed by the California ISO using the penalty factor calculation at the generation buses. The objective of these schemes is to ensure that the total system losses are covered through the fixed pre-specified factors. The schemes do not focus on the amount of compensation to be provided by each individual transaction. In this paper, we construct a loss compensation procedure that provides each transaction with the choice of the compensation buses and the respective amounts of compensation. The compensation scheme allows transactions much broader choice to compensate losses than the scheme proposed in [6], where, each transaction in a multilateral trade framework can choose either the injection or the withdrawal bus for compensation. We also develop a procedure to allow transactions to have their loss allocation covered by the IGO. The IGO may elect to provide its loss compensation

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as a value-added service to transmission customers so as to take advantage of the information the IGO has available and to take into consideration grid constraints. The IGO may acquire such loss compensation by determining the least-price solution of the problem. The IGO's least-price loss compensation is formulated as a linear program. The two compensation mechanisms may coexist and any physical feasible combination of them is possible. The flexibility and cost effectiveness of the procedures have been extensively tested on various systems.

The development of the equivalent loss compensation concept is given and applied to evaluate a transaction's self-acquisition of loss compensation in the next section. This concept is further extended and is used to propose the formulation of the IGO-acquired loss compensation as a linear programming problem in Section III. Test results of the implementation and comparison of the compensation procedures on the IEEE 118- and 300-bus system are presented in Section IV. In the concluding section, we list some promising directions for future work.

II. Equivalent Loss Compensation

We have developed an allocation mechanism in a network with multiple transactions [1]. For this purpose we constructed a multi-transaction framework in which we define a transaction as a triplet consisting of a set of selling buses (generators) supplying a specified amount of real power to a set of buying buses (loads). Formally, a transaction m is defined to be

$$\mathcal{T}^{(m)} = \left\{ t^{(m)}, \mathcal{S}^{(m)}, \mathcal{B}^{(m)} \right\} \tag{1}$$

with $t^{(m)}$ being the MW amount of the transaction; $\mathcal{S}^{(m)}$ the set consisting of the collection of 2-tuples of the $N_s^{(m)}$ selling buses

$$\mathcal{S}^{(m)} = \left\{ \left(s_i^{(m)}, \sigma_i^{(m)} \right) i = 1, 2, \cdots, N_s^{(m)} \right\} \tag{2}$$

and, $\mathcal{B}^{(m)}$ the set consisting of the collection of 2-tuples of the $N_b^{(m)}$ buying buses

$$\mathcal{B}^{(m)} = \left\{ \left(b_j^{(m)}, \beta_j^{(m)} \right), j = 1, 2, \cdots, N_b^{(m)} \right\}. \tag{3}$$

Each selling bus $s_i^{(m)}$ supplies $\sigma_i^{(m)}\, t^{(m)}$ MW of the transaction. The nonnegative fractions $\sigma_i^{(m)}$ must add to 1. Each buying bus $b_j^{(m)}$ receives $\beta^{(m)} t^{(m)}$ MW of the transaction and the nonnegative fractions $\beta_j^{(m)}$ must also add to 1. In our definition of a bilateral transaction m, $t^{(m)}$ MW is injected at the $N_s^{(m)}$ selling buses and $t^{(m)}$ MW is withdrawn from the $N_b^{(m)}$ buying buses. Thus, the definition does not account for the transaction's losses.

Under the assumption that the losses are covered by the supplemental generation at a single bus designated as the slack bus, a physical flow-based allocation scheme [1] was developed to evaluate the losses allocated to each transaction, $l_a^{(m)}$, $m = 1, 2, \cdots, M$. The grid operator of the a interconnected network can receive reimbursement from each transaction for providing the loss compensation $l_a^{(m)}$ at the slack bus. However, a transaction may wish to exercise choice by self-acquiring loss compensation from a third party at buses other than the slack bus. Then, the problem arises of how to compute such loss compensation. This problem is addressed in this paper.

Consider a system of $N+1$ buses and M transactions with bus 0 selected as the slack bus. The total system losses are compensated at the slack bus. Let p_n denote the net power injection at each bus $n = 1, 2, \cdots N$ under the M transactions:

$$p_n = \sum_{m=1}^{M} \delta_n^{(m)} t^{(m)}, \quad n = 1, 2, \cdots, N \tag{4}$$

where,

$$\delta_n^{(m)} = \begin{cases} \sigma_i^{(m)} & \text{if } n = s_i^{(m)}, \quad i = 1, 2, \cdots N_s^{(m)} \\ -\beta_j^{(m)} & \text{if } n = b_j^{(m)}, \quad j = 1, 2, \cdots N_b^{(m)} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

At bus 0 additional power is produced to cover the total system losses l so that

$$p_0 = l + \sum_{m=1}^{M} \delta_0^{(m)} t^{(m)} \tag{6}$$

We define at an arbitrary bus k the equivalent loss compensation for Δp_0 MW generated at the slack bus to be the injection of real power Δp_k at bus k with the property that such injection results in Δp_0 MW decrease in the injection at the slack bus with all transactions on the system remaining unchanged. At a load bus the real power injection is equivalent to decreasing the load by the same amount of injection.

The power balance equation before injection of Δp_k is

$$p_0 + \sum_{n=1,\, n \neq k}^{N} p_n + p_k = l \tag{7}$$

With the additional amount of power Δp_k injected at bus k into the system to offload the loss compensation at the slack bus, the power balance equation becomes

$$p_0 - \Delta p_0 = \sum_{n=1,\, n \neq k}^{N} p_n + p_k + \Delta p_k = l + \Delta l \tag{8}$$

where Δl is the change in the total system losses due to the additional injection Δp_k at bus k. Thus,

$$\Delta l = \Delta p_k - \Delta p_0 \tag{9}$$

Typically, the transmission losses are a small percent of the total generation in a system. One study estimates that losses represent about 4% of the total MWh generated [7]. Since $\Delta l < l$, it follows that Δl is indeed a small quantity. We use linear analysis for the *small* signal model for Δl. In Eqs. (7) and (8) we may view l as a function of $p_1, p_2, \cdots, p_N$, and since the only change in these variables is Δp_k, it follows that

$$\Delta l \approx \frac{\partial l}{\partial p_k} \Delta p_k \tag{10}$$

From Eqs. (9) and (10), we have

$$\Delta p_k \approx \Delta p_0 \left[1 - \frac{\partial l}{\partial p_k} \right]^{-1} \tag{11}$$

We introduce further approximation by replacing $\partial l/\partial p_k$ by the value Ω_k obtained under the D.C. power flow conditions:

$$\frac{\partial l}{\partial p_k} \approx \Omega_k = \sum_{i=0}^{N} \sum_{j \in \mathcal{H}_i} \frac{r_{ij}}{r_{ij}^2 + x_{ij}^2} [(\theta_i^0 - \theta_j^0)(d_{jk} - d_{ik})] \quad (12)$$

Here, $r_{ij} + jx_{ij}$ is the line impedance of the line connecting buses i and j, $\mathcal{H}_i$ is the set of buses that are directly connected to bus i, d_{ik} are components of the matrix $\underline{D} = \hat{\underline{B}}^{-1}$, where $\hat{\underline{B}}$ is the $N \times N$ submatrix of the $(N+1)$-node network susceptance matrix $\underline{B}$ used in the D.C. power flow [8]. The angles θ_n^0 are computed using the A.C. power flow and correspond to the case before the additional injection Δp_k. We call Ω_k the D.C. loss sensitivity factor at bus k. This quantity Ω_k is already computed for the loss allocation. The derivation of the approximation of $\partial l/\partial p_k$ by Ω_k is shown in the Appendix. For the small signal analysis, we drop the approximation and replace by equality. This leads to the expression

$$\Delta p_k = \zeta_k \Delta p_0 \quad (13)$$

where,

$$\zeta_k = [1 - \Omega_k]^{-1} \quad (14)$$

We call ζ_k the bus k loss compensation multiplier. The physical interpretation of ζ_k is that ζ_k MW injected at bus k is equivalent to reducing loss compensation by 1 MW at the slack bus. In fact, ζ_k has the form of the penalty factor of the generator at bus k in the classical economic dispatch problem formulation [8]. Note that at the slack bus, $\zeta_0 = 1$.

The linearity of the small signal model in Eq. (13) allows us to extend it to the more general case where a set of generation buses is used to compensate the loss coverage by the slack bus. Let the set of compensating buses be $\mathcal{K}$. Let the generator (load) at bus $k \in \mathcal{K}$ inject Δp_k to provide equivalent loss compensation for $\alpha_k \Delta p_0$ MW, with $0 \leq \alpha_k \leq 1$ and $\sum_{k \in \mathcal{K}} \alpha_k = 1$. Since superposition applies to the small signal model,

$$\sum_{k \in \mathcal{K}} \frac{\Delta p_k}{\zeta_k} = \sum_{k \in \mathcal{K}} \alpha_k \Delta p_0 = \Delta p_0 \quad (15)$$

Next, let us consider the special case where $\Delta p_0 = l_a^{(m)}$. The general problem becomes: if transaction m wants to self-acquire at bus k the equivalent compensation for the losses $l_a^{(m)}$ allocated to it, how many MW of power does it a need to inject at bus k? Since $l_a^{(m)}$ is a small quantity a compared with the total generation on the system, the small signal analysis of Eq. (10) holds for this case. The equivalent loss compensation at bus k is

$$\Delta p_k = \zeta_k l_a^{(m)} \quad (16)$$

If a compensating bus k is selected from the set $\mathcal{S}^{(m)}$, the seller of transaction m needs to inject an additional Δp_k MW above the amount $\sigma_k^{(m)} t^{(m)}$. If the selected compensating bus k is from the set $\mathcal{B}^{(m)}$ then Δp_k is the effective reduction in the delivered power $\beta_k^{(m)} t^{(m)}$ at the buying bus k. This particular choice is, in fact, the compensation scheme proposed for the multilateral trade structure in [6]. Thus, that scheme is a specific case of the general compensation framework developed here.

More generally, the compensation can be exercised at a set of buses $\mathcal{K}$. Suppose that transaction m selects to compensate $\alpha_k l_a^{(m)}$ by the generator at bus k in the set. From Eq. (16) we have

$$\Delta p_k = \zeta_k \alpha_k l_a^{(m)}, \qquad k \in \mathcal{K} \quad (17)$$

Note the choice of α_k is completely under the control of each transaction.

The self-acquisition of loss compensation is flexible and can be tailored to satisfy each individual transaction's needs. However, some transactions may lack the capabilities to self-acquire real power injection or the compensation from third-party cannot be realized due to some constraints such as those due to transmission that are not under the control of the transaction. On the other hand, an independent grid operator (IGO) with sufficient information about the system may be in a better position to provide the loss compensation service to those transactions. We consider this option in the following section.

III. IGO-Acquired Loss Compensation

Consider the situation in which transactions $m \in \tilde{\mathcal{M}}$ choose to purchase the loss compensation services from the IGO; transactions $m \in \tilde{\mathcal{M}}^c$ select to acquire loss compensation on their own, where $\tilde{\mathcal{M}}$ is a subset of $\mathcal{M} \triangleq \{1, 2, \cdots, M\}$. The self-acquisition of loss compensation is carried out along the lines presented above. To provide the loss compensation service, the IGO can solicit generation supplies or load reductions from any player willing to do so. A player who has generation at bus k will bid an incremental price of c_k \$/MWh for each MW increase in output for an hour. A player who has a demand at bus k' will bid a decremental price $c_{k'}$ \$/MWh for each MW decrease in the load for an hour. Let $\mathcal{K}$ be the set of players that bid to provide the incremental or decremental power to the IGO. Let the maximum capacity that the player at bus k is willing or able to provide be $\Delta p_k^{\max}$ MW. Without any loss of generality, the incremental/decremental cost is assumed constant for the interval $[0, \Delta p_k^{\max}]$. The IGO may provide loss compensation as a value added service to the transactions. The IGO uses the bids of the players to determine the least-price loss compensation acquisition for the transactions of the subset $\tilde{\mathcal{M}}$. Under the assumptions introduced, the determination of the optimal compensation levels at each bus $k \in \mathcal{K}$ for the $\tilde{\mathcal{M}}$ transactions is established from the solution of a linear programming (LP) problem. The least-price loss compensation acquisition is the one in which the total costs incurred by the IGO for acquiring the compensation service are minimized while all the necessary physical limits are satisfied.

The decision variables are $\Delta p_k^{(m)}$, $k \in \mathcal{K}$, $m \in \tilde{\mathcal{M}}$ representing the amount of compensation acquired from the player at bus k for transaction m. The objective function is

$$\min \sum_{k \in \mathcal{K}} c_k \sum_{m \in \tilde{\mathcal{M}}} \Delta p_k^{(m)} \quad (18)$$

The constraints we are considering include:

- For each transaction $m \in \tilde{\mathcal{M}}$, the sum of the compensation acquired from the buses $k \in \mathcal{K}$ covers the losses allocated to it, $l_a^{(m)}$. From Eq. (16) we have

$$\sum_{k \in \mathcal{K}} \frac{\Delta p_k^{(m)}}{\zeta_k} = l_a^{(m)}, \qquad \forall m \in \tilde{\mathcal{M}} \tag{19}$$

- The sum of the compensation supplied by the player at bus k for the transactions of $\tilde{\mathcal{M}}$ cannot be larger than the maximum capacity that it is willing/able to provide

$$\sum_{m \in \tilde{\mathcal{M}}} \Delta p_k^{(m)} \leq \Delta p_k^{\max}, \qquad \forall k \in \mathcal{K} \tag{20}$$

- The additional injections or load decreases for loss compensation cannot violate the transmission line limits,

$$\underline{F}\underline{\Delta p} \leq \underline{f}^{\max} \tag{21}$$

where, $\underline{\Delta p}$ is the $|\mathcal{K}|$-dimensional vector with components

$$\Delta p_k = \sum_{m \in \tilde{\mathcal{M}}} \Delta p_k^{(m)}, \qquad \forall k \in \mathcal{K} \tag{22}$$

H is the total number of lines. $\underline{F}$ is the $H \times |\mathcal{K}|$ matrix of the generation shift factors [8]. The component of F_{hk} represents the sensitivity of the line flow on the transmission line h with respect to the injection at bus k. $\underline{f}^{\max}$ is an H vector with components $f_h^{\max}$ as the unused line capacity of transmission line h by the line flows caused by the existing transactions and the loss compensation of transactions $m \in \tilde{\mathcal{M}}^c$.

- By definition, the loss compensation is nonnegative,

$$\Delta p_k^{(m)} \geq 0, \qquad \forall k \in \mathcal{K}, \qquad \forall m \in \tilde{\mathcal{M}} \tag{23}$$

The LP formulation of the least-price loss compensation problem is given by Eqs. (18)–(23)

Let us denote by $\rho^{(m)}$, $\tau_k \geq 0$, and $\eta_h \geq 0$ the dual variables associated with constraints in (19)–(21), respectively. Let $\Delta p_k^{*(m)}$ be the optimal solution of the LP problem and let the optimal dual variables be $\rho^{*(m)}$, τ_k^*, and η_h^*. Then, $\rho^{*(m)}$ has an important economic interpretation: it is the sensitivity of the total costs of loss compensation paid by the IGO with respect to the losses allocated to transaction m. This sensitivity information can be very useful in that it provides to the dispatcher a basis for pricing the compensation service supplied to transaction m. It follow that the dispatcher's costs for providing the loss compensation service to transaction m are $\rho^{*(m)} l_a^{(m)}$. From the complementary slackness condition at the optimal solution, we can obtain

$$\left[\frac{\rho^{*(m)}}{\zeta_k} + \tau_k^* + \sum_{h=1}^{H} f_{hk}\eta_h^* - c_k\right] \Delta p_k^{*(m)} = 0, \qquad \forall k \in \mathcal{K}, \quad \forall m \in \tilde{\mathcal{M}} \tag{24}$$

For each bus k, we define a set $\mathcal{M}_k = \{m\colon \Delta p_k^{*(m)} > 0\}$, i.e., the subset of transactions for which loss compensation is acquired from bus k. Then, it follows from Eq. (24) that

$$\rho^{*(m)} = \left[c_k - \tau_k^* - \sum_{h=1}^{H} f_{hk}\eta_h^*\right] \zeta_k, \qquad m \in \mathcal{M}_k \tag{25}$$

The right hand side of Eq. (25) is independent of m so that $\rho^{*(m)} = \rho*$, for each $m \in \mathcal{M}_k$. In other words, whenever a player provides loss compensation for more than one transaction, the sensitivity of the total charges for loss compensation with respect to each of those transactions is identical. For the important special case when there exists a certain bus k such that $\Delta p_k^{(m)} > 0$, $\forall m \in \tilde{\mathcal{M}}$, then all the sensitivities are identical, i.e.,

$$\rho^{*(m)} = \rho* \qquad \forall m \in \tilde{\mathcal{M}} \tag{26}$$

Physically this result is intuitive since the problem we are solving involves the compensation of losses at a single bus-the slack bus. The impact of 1 MW increase of the losses allocated to any one transaction on the total compensation payment is clearly independent of the transaction. Therefore, the IGO incurs a payment of $\rho^{*(m)}$ (\$/MWh) for each transaction m but this payment is identical for all the transactions.

The IGO can provide this least-price loss compensation service to the interested transactions at "cost" or at slightly above cost. This option may allow certain transactions to cover their allocated losses at a lower cost than they otherwise would incur.

The loss compensation on a self-acquisition basis and that provided by the IGO can easily coexist. The transactions can do either self-acquisition or the IGO least-price compensation acquisition or a mixture of the two.

IV. Numerical Results

We have tested the proposed loss compensation methodology on a number of systems including the IEEE 118- and 300-bus systems. The numerical results indicate that the self-acquisition and the IGO-provided compensation mechanisms developed in the paper are effective and provide good flexibility in arranging for the loss compensation service in a multi-transaction framework.

For the IEEE 118-bus system, we use the six-transaction configuration described in [1] with the specified generation/load data. The loss allocation results determined by the scheme in [1] and the data for loss compensation are summarized in the tableau of Table I. Transactions 2 and 3 undertake self-acquisition for loss compensation. The remaining four transactions make use of the least-price loss compensation service acquired by the IGO. The locations, compensating capacities and the prices of the players that bid to provide loss compensation services are shown in the tableau.

The self-acquisition option is exercised in a straight-forward manner by a transaction as shown in the tableau. The IGO-acquired least-price loss compensation scheme for the remaining four transactions is the solution of the LP problem formulated with the specified data. Since at the optimal solution, the IGO acquires loss compensation for each of the transactions from

TABLE I
THE IEEE 118-BUS SYSTEM LOSS COMPENSATION TABLEAU

TRANSACTION m	$l_a^{(m)}$ (MW)	COMPEN-SATION CHOICE	COMPENSATING BUS (MW)								TOTAL COSTS ($)
			54	87	46	89	66	69 (SLACK BUS)	49	61	
2	19.28	SELF			(40%) 7.7		(60%) 11.6				535.1
3	10.14	SELF								(100%) 11.6	294
1	43.43	IGO	19.33	7.97	9.40	4.73					946.77
4	11.25	IGO	0	4.36	5.41	1.31					245.25
5	12.29	IGO	0	4.70	5.79	1.64					267.92
6	22.92	IGO	0.67	7.97	9.40	4.73					499.66
IGO's TOTAL COMPENSATION			20	25	20	12.41					1959.6
COMPENSATING BUS DATA	ζ_k		0.91	1.03	0.93	1.09	0.99	1	0.95	0.99	
	c_k ($/MWh)		12.5	15	19	20	22.5	25	27	30	
	$\Delta p_k^{\max}$ (MW)		20	25	30	60	35	45	50	50	

bus 46, the sensitivity of the total costs to the IGO to a change in the loss allocated to one transaction, i.e., the marginal cost ρ^* of the IGO-acquired compensation is uniform and is equal to $21.8/MWh. If, on the other hand, these four transactions were able to obtain their loss compensation at the designated slack bus, bus 69, their costs would be $25/MWh. This is over 14% higher than under the IGO acquisition cost. In fact, however, compensation at the slack bus is not available since it is willing/able to provide no more than 45 MW for the purposes of loss compensation while 89 MW are required to be injected into the slack bus. Note that at the optimal solution, none of the line limits is active.

Next, we investigate the use of the IGO-acquisition by all six transactions including transactions 2 and 3. The comparison of the IGO's least-price loss compensation service obtained from the solution of the new LP problem with the participation of the two additional transactions and the self-acquisition results are shown in Table II. The solution of the LP is such that ρ^* remains unchanged, leading to significant savings for each transaction.

For the IEEE 300-bus system with the thirteen transactions specified in [1], we consider the IGO-acquired loss compensation for all but transactions 1, 4, 7, and 13, which acquire the loss compensation service on their own. The focus of our study is on the effect of the transmission constraints of the IGO-acquired least-price compensation. At the optimal solution of the LP with given values for the line limits, no limit is binding. The results are summarized in Table III with the results being the first entries of the upper part of the tableau. Since bus 7166 provides nonzero compensation for all the participating transactions, ρ^* is uniform and its value is $22.8/MWh. We next resolve the LP with a 10% reduction on the line flow capacity of the line between buses 7049 and 49. This line limit becomes active, and the results are shown as the second entries of the upper part of the tableau of Table III. We compare the IGO-acquired compensation solutions with and without the 10% decrease in the line limit. The marginal compensation cost ρ^* increases from $22.8 to $23.3/MWh. The results also indicate that since the active line limit, the compensation from bus 147 has to be replaced by the more expensive compensation at bus 7071, resulting in a 2% increase in the total compensation costs.

TABLE II
COMPARISON OF SELF-ACQUISITION AND LEAST-PRICE LOSS COMPENSATION IN THE IEEE 118-BUS SYSTEM

TRANS-ACTION	SELF-ACQUISI-TION	IGO-ACQUISI-TION	% ABOVE IGO-ACQUISITION
2	535.1	420.3	27.3
3	294	221	33.0

A second study considered the impacts of four arbitrarily chosen lines whose limits are continuously reduced simultaneously. Fig. 1 shows that the IGO's least-price acquisition becomes increasingly more expensive when the line limits of these four lines become restrictive and the system becomes more constrained correspondingly. This illustrates the capability of the IGO-acquired procedure to adapt effectively to the physical constraints of the network.

V. CONCLUSION

This paper has presented an efficient and flexible compensation scheme in a multi-transaction network. By exploiting the characteristics of the equivalent loss compensation concept we constructed loss compensation schemes that provide transactions with choice and flexibility. With the proposed scheme, a transaction can self-acquire to compensate its losses at some selected locations or purchase the loss compensation services provided by the IGO or any combination of the two. The scheme produces *a priori* information. The IGO-provided compensation

TABLE III
THE IEEE 300-BUS SYSTEM LOSS COMPENSATION TABLEAU

TRAN-SACTION m	$l_a^{(m)}$ (MW)	COMPENSATING BUS (MW) (WITHOUT/ WITH 10% REDUCTION IN THE LINE LIMIT)									
		125	7002	7166	176	147	7049 SLACK	7071	185	227	TOTAL COST ($)
2	23.3	0/0.2	0.2/0.5	19.5/19.3	4.6/4.3						533/543
3	64.9	0/0.9	0/1.1	15.2/19.9	0/4.9	50/16.3		0.7/19.1			1481/1511
5	47.9	1.0/4.2	1.3/4.0	20.5/22.7	5.6/7.7	20.7/0		0/8.5			1093/1152
6	26.2	0.7/1.0	0.9/1.2	20.3/19.9	5.3/5.0						599/611
8	48.2	6.8/4.2	6.4/4.0	25.6/22.7	10.5/7.7			0/8.8			1100/1123
9	33.1	2.6/2.9	2.7/2.9	21.9/21.6	6.9/6.6						756/772
10	9.2					9.3/9.3					210/214
11	59.3	6.8/4.2	6.4/4.0	25.6/22.7	10.6/7.7			9.0/17.9			1353/1380
12	30.9	2.0/2.3	2.1/2.3	21.4/21.1	6.4/6.1						706/720
IGO's TOTAL COMPENSATION		20/20	20/20	170/170	50/50	80/25.6		9.7/54.4			7989/7831
COMPEN-SATING BUS DATA	ζ_k	0.91	1.02	1.04	1.06	1.01	1	0.83	0.95	0.87	
	c_k ($/MWh)	15	18	20	20	22.5	25	27.5	35	40	
	$\Delta p_k^{\max}$ (MW)	20	20	170	50	80	100	150	200	250	

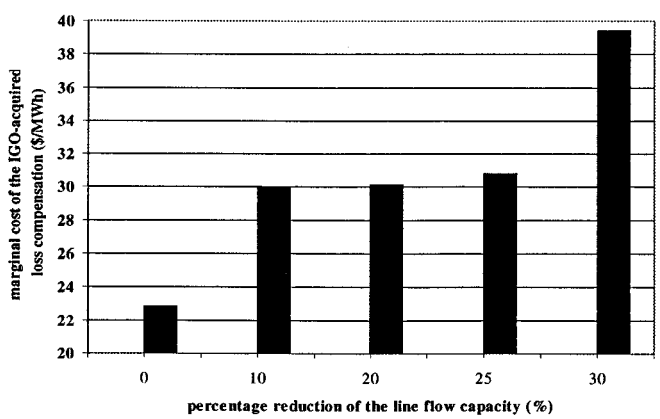

Fig. 1. The impact of the transmission system on the solution of the IGO-acquired least-price loss compensation.

is a value-added service to transmission customers and is based on a least-price optimization.

The extension of the work presented here to the development of allocation and acquisition mechanisms for other ancillary services is a logical step. We are currently studying the adaptation of the transaction framework to reactive power and voltage control.

APPENDIX
LOSS SENSITIVITY FACTOR Ω_k

This derivation uses the notation and allocation results of [1]. The starting point of the derivation of the loss approximation $\hat{l}$ was

$$l = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{r_{ij}}{r_{ij}^2+x_{ij}^2}\left[V_i^{0^2}+V_j^{0^2}-2V_i^0V_j^0 \cdot\cos(\theta_i^0-\theta_j^0)\right] \tag{27}$$

Here, V_n^0 and θ_n^0 denote the bus voltage magnitude and angle at bus n determined with the aid of an A.C. power flow. For the system with the M transactions we obtained using the D.C. power flow conditions the approximation

$$\hat{l} = \sum_{m=1}^{M}\lambda^{(m)}t^{(m)} \tag{28}$$

where,

$$\lambda(m) = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\left\{\frac{r_{ij}}{r_{ij}^2+x_{ij}^2}(\theta_i^0-\theta_j^0)\pi_{ij}^{(m)}\right\} \tag{29}$$

with

$$\pi_{ij}^{(m)} = \mu_i^{(m)}-\mu_j^{(m)}, \quad m=1,2,\cdots M, \quad i,j=0,1,\cdots,N \tag{30}$$

and

$$\mu_n^{(m)} = \sum_{v=1}^{N_b^{(m)}} d_{nb_v^{(m)}}\beta_v^{(m)} - \sum_{r=1}^{N_s^{(m)}} d_{ns_i^{(m)}}\sigma_r^{(m)} \tag{31}$$

It follows from Eqs. (29)–(31) that

$$\lambda^{(m)} = \frac{1}{2}\sum_{r=1}^{N_s^{(m)}}\sigma_r^{(m)}\Omega_{s_r^{(m)}} - \frac{1}{2}\sum_{v=1}^{N_b^{(m)}}\beta_v^{(m)}\Omega_{b_v^{(m)}} \tag{32}$$

where,

$$\Omega_k = \sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{r_{ij}}{r_{ij}^2+x_{ij}^2}(\theta_i^0-\theta_j^0)(d_{jk}-d_{ik})$$
$$k = s_r^{(m)} \text{ or } b_v^{(m)}, \quad r=1,2,\cdots,N_s^{(m)},$$

and

$$v = 1,2,\cdots,N_b^{(m)} \tag{33}$$

We next consider the expression for Ω_k. Let k be an arbitrary bus and we introduce a small injection Δp_k that is much

smaller than the net injection at bus k corresponding to the M transactions. All the M transactions remain fixed at their original values. This small disturbance will change the angle from $\underline{\theta}^0$ to $\underline{\theta}^0 + \underline{\Delta\theta}$; the corresponding change in the voltage magnitudes is negligibly small. We can show using the D.C. power flow approximation that for any pair of nodes i and j,

$$\Delta\theta_i - \Delta\theta_j \approx (d_{jk} - d_{ik})\Delta p_k \tag{34}$$

We evaluate the change Δl in the losses corresponding to the disturbance Δp_k starting at the expression for l in the Eq. (27). Under the D.C. power flow assumptions,

$$l \approx \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}(\theta_i^0 - \theta_j^0)^2 \tag{35}$$

so that the first order Taylor series approximation obtains

$$\Delta l \approx \sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}[(\theta_i^0 - \theta_j^0)(\Delta\theta_i - \Delta\theta_j)] \tag{36}$$

It follows using Eq. (34) that

$$\Delta l \approx \left[\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}[(\theta_i^0 - \theta_j^0)(d_{jk} - d_{ik})]\right]\Delta p_k \tag{37}$$

Thus

$$\begin{aligned}\frac{\partial l}{\partial p_k} &= \lim_{\Delta p_k \to 0}\frac{\Delta l}{\Delta p_k}\\ &\approx \sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{r_{ij}}{r_{ij}^2 + x_{ij}^2}[\theta_i^0 - \theta_j^0)(d_{jk} - d_{ik})]\end{aligned} \tag{38}$$

The expression in Eq. (38) is identical to the expression for Ω_k at the buses participating in a transaction as given in Equation (33). Consequently Ω_k at a bus participating in a transaction is the approximation to the sensitivity of the system losses to the net injection at that bus. As such, the Ω_k term is already available for the evaluation of the loss allocation to the transaction. Since it is most natural for a transaction to use one of the participating selling or buying buses for compensation of its losses, the Ω_k term is known from the allocation calculations.

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