A Loss Allocation Mechanism for Power System Transactions

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Abstract: The proliferation of transactions coupled with the unbundling of services has created a need to evaluate the allocation of ancillary services among the transactions. The focus of this paper is on the compensation of loss service. Loss allocation is of importance in a competitive electricity marketplace for providing *a priori* information to transacting entities on the costs involved. We formulate the power flows in a network as an explicit function of the amounts of transactions between selling and buying entities. We develop a scheme based on the physical flows in the network to evaluate the losses associated with each transaction. The loss allocation scheme proposed makes detailed use of the mathematical model expressing flows in terms of transaction amounts. An important property of the proposed allocation scheme is its robustness. In addition, the mechanism can evaluate losses for any subset of transactions without requiring the complete information on *all* the transactions. Numerical tests of several networks including the 57-, 118- and 300-bus IEEE systems show that the scheme is effective in providing a physically meaningful allocation of losses. A summary of some numerical studies is given. Directions for future work are discussed.

1. INTRODUCTION

The open access transmission regime is spearheading the rapid disintegration of the well-entrenched vertically integrated structure of the electric power industry. With the entry of numerous new players, there is a steady move toward the widespread use of transactions to meet customer demands. The proliferation of transactions results in the use of the existing transmission systems for purposes very different from those for which they were originally planned. These changes are accompanied by the establishment of an independent transmission system operator. Such a function is provided by the National Grid Company in the England and Wales electricity supply industry or the various Independent System Operators already set up (ERCOT, California, PJM) or in formation (Midwest, NYPP, INDEGO). A key function of an independent transmission system operator is the provision of the necessary ancillary services and their allocations among the transactions in the system. Compensation for transmission losses, while not on the FERC list in its Order No. 888, is one such essential ancillary service. The focus of this paper is the allocation of transmission losses to the various transactions on the system.

Transmission losses represent a nontrivial cost element under the vertically integrated structure of the past. However, the limited number of third party transactions did not make loss allocation a major issue. One industry study's estimate of losses was that they represent about 4% of the total MWh generated[1]. Typically, losses were treated as an additional load in the system. Various approaches to evaluate and compensate for losses have been developed. A good survey of the schemes proposed can be found in [2]. As the industry is moving towards a transaction-based paradigm in a competitive environment, the importance of loss allocation is critical since transacting entities need *a priori* information to evaluate impacts of various transactions under consideration. Hence, the need to allocate the total system losses on an equitable and transparent basis is critical in order to facilitate a smoothly operating competitive electricity market place. While there exist well-established schemes for the evaluation of total system losses, the determination of losses associated with each individual transaction is a virtually unexplored topic. There are several complicating factors in the evaluation of the losses attributable to each transaction arising from the physical reality that losses are a function of the way in which the simultaneously occurring transactions interact. In principle, the line flows, in the presence of multiple transactions, are measurable; however, the association of flow with each particular transaction involves a good degree of arbitrariness taking into account notions of marginality and the incremental nature of flows. Moreover, in the mathematical expression for the $|I|^2 R$ losses, the total system losses are a non-separable function of all the transactions. As such, there is not a physically meaningful measurement scheme or a theoretically based evaluation methodology to determine the losses caused by each particular transaction.

Under various assumptions and approximations, several allocation schemes have been proposed. Kirschen et al.[3] introduce a basic assumption of proportionality which they use in a proposed scheme to determine the proportion of the active power flow in a transmission line contributed by each generator. They use this proportion of line use to evaluate the losses allocated to each generator. By making a similar proportionality assumption, another topology-based allocation scheme is developed by Bialek[4]. Both methods determine the share rather than the impact of a particular generator on each line flow, using assumptions that "can be neither proved nor disproved"[3]. A comparison of topology- and circuit-theory-based methods is given in [5]. These loss allocation schemes are not developed for a system with transactions since the objective is to allocate the losses to each generator. In a similar vein, the model and methodology proposed by the California ISO[6] uses a *generator meter multiplier* based on a penalty factor calculation for each generation bus. Wu and Varaiya in [7] developed a quadratic Taylor expansion of losses in terms of transactions at a given operating point. None of the schemes cited considers the possibility of negative loss allocation arising in the presence of the so-called counter flows. The objective of this paper is to develop a loss allocation mechanism for a network with multiple simultaneously occurring transactions and to effectively deal with the issue of losses associated with counter flows.

We first develop a framework for the explicit consideration of the transactions in the system. This is a far simpler construct than the structure of the system in [8]. We formulate the power flows explicitly in terms of the amounts of transactions in the system. Through the use of assumptions of the D.C. power flow, we develop a physical-flow-based allocation expression for each transaction and also one for the total system losses. Using sensitivity information and equity considerations, we develop a scheme that allocates losses in an appropriate way that is physically reasonable. The scheme deals effectively with counter flows resulting when certain transactions are in effect. This is borne out by the extensive testing that was performed with the proposed scheme.

The formulation of a transaction-based framework is given in the next section. Section 3 presents the allocation scheme and its physical interpretation. Test results of implementation on various physical systems including the IEEE 57-, 118- and 300-bus systems are presented in Section 4. In the concluding section, we discuss directions for future work. There is an Appendix presenting some details on sensitivity information used in the development of the allocation scheme.

2. TRANSACTION FRAMEWORK FORMULATION

In this section, we formulate a framework that recasts the power flow problem in a transaction-based framework. We consider a system with N+1 buses in which each load acts as a buyer to get its demands met through transactions with one or more sellers. Similarly, each generator acts as a seller and undertakes transactions with one or more buyers. Let M denote the number of system transactions. A so-called bilateral transaction is characterized by specifying the seller, the buyer and the amount of real power. We define a bilateral transaction as a set of selling buses (generators) supplying a specified amount of real power to a set of buying buses (loads). Formally, we define a transaction m by

$$\mathcal{T}^{(m)} = \{ t^{(m)}, \mathcal{S}^{(m)}, \mathcal{B}^{(m)} \} \tag{1}$$

The elements of this triplet are $t^{(m)}$, the transaction amount in MW and $\mathcal{S}^{(m)}$ ($\mathcal{B}^{(m)}$), the selling (buying) entities, where, $\mathcal{S}^{(m)}$ is the collection of 2-tuples of the $N_s^{(m)}$ selling buses

$$\mathcal{S}^{(m)} = \left\{ (s_i^{(m)}, \sigma_i^{(m)}), i = 1,2,..., N_s^{(m)} \right\} \tag{2}$$

with the selling bus $s_i^{(m)}$ supplying $\sigma_i^{(m)} t^{(m)}$ MW of the transaction amount. The fraction $\sigma_i^{(m)}$ must satisfy the conditions $\sum_{i=1}^{N_s^{(m)}} \sigma_i^{(m)} = 1$ and $\sigma_i^{(m)} \in [\,0,1\,]$, $i = 1,2,...N_s^{(m)}$.

Similarly, $\mathcal{B}^{(m)}$ is the collection of 2-tuples of the $N_b^{(m)}$ buying buses

$$\mathcal{B}^{(m)} = \left\{ (b_j^{(m)}, \beta_j^{(m)}),\ j = 1,2,..., N_b^{(m)} \right\} \tag{3}$$

where, the buying bus $b_j^{(m)}$ receives $\beta_j^{(m)} t^{(m)}$ MW of the transaction amount. The fraction $\beta_j^{(m)}$ must satisfy the conditions $\sum_{j=1}^{N_b^{(m)}} \beta_j^{(m)} = 1$ and $\beta_j^{(m)} \in [0,1]$, $j = 1,2,...N_b^{(m)}$.

In our definition of a bilateral transaction m, $t^{(m)}$ MW is injected at the $N_s^{(m)}$ selling buses and $t^{(m)}$ MW is withdrawn at the $N_b^{(m)}$ buying buses. We assume that the losses associated with each transaction are compensated at the system designated slack bus[1]. In our framework, since all load demands are met through transactions, the total system losses are caused entirely by the M transactions. Bus 0 is designated as the slack bus.

For each transaction m, we construct an injection vector $\underline{p}^{(m)}$ with components

$$p_n^{(m)} = \delta_n^{(m)} t^{(m)},\ n = 0,1,2,...N\,. \tag{4}$$

where, the components of $\underline{\delta}^{(m)}$ are

$$\delta_n^{(m)} = \begin{cases} \sigma_i^{(m)} & \text{if } n = s_i^{(m)}, i = 1,2,...N_s^{(m)} \\ -\beta_j^{(m)} & \text{if } n = b_j^{(m)},\ j = 1,2,...N_b^{(m)}. \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Then, for the system, the net real power injections at bus

[1] We may think of the independent transmission system operator as the entity that takes care of loss recovery and reimbursement from the transaction participants.

$n = 1,2,...N$ is given by the N-dimensional vector $\underline{p}^{net}$ with

$$p_n^{net} = \sum_{m=1}^{M} p_n^{(m)} = \sum_{m=1}^{M} \delta_n^{(m)} t^{(m)} \quad (6)$$

Note that in this formulation, even though the slack bus 0 may be involved in transactions as a selling bus, $\underline{p}^{net}$ does not have a component at bus 0. In our transaction-based framework, the real power flow equations at each bus except bus 0 are stated explicitly in terms of $t^{(m)}, m = 1,2,...M$:

$$\sum_{m=1}^{M} \delta_n^{(m)} t^{(m)} = V_n \sum_{k \in \mathcal{H}_n} V_k [G_{nk} \cos(\theta_n - \theta_k) + B_{nk} \sin(\theta_n - \theta_k)] + G_{nn} V_n^2, \ n = 1,2,...N \quad (7)$$

where, θ_n and V_n are the angle and voltage magnitude at bus n, $n = 1,2,...N$. At the slack bus 0, θ_0 and V_0 are set to their specified value. $\mathcal{H}_n$ is the set of buses that are directly connected to bus n, $n = 1,2,...N$ and $\underline{G} + j\underline{B}$ is the bus admittance matrix with elements $G_{ij} + jB_{ij}$. The equations specifying the reactive flows remain unchanged in this framework. Under our convention, there is no need for area control since, by definition, the net physical flow on the tie lines is equal to the net interchange specified for the inter-area transactions in effect.

Special Case: The definition of the bilateral transaction reduces to a simple node pair transaction when the transaction is between a single selling bus and a single buying bus. Then, for transaction m, $\mathcal{S}^{(m)} = \{ s^{(m)}, 1 \}$ and $\mathcal{B}^{(m)} = \{ b^{(m)}, 1 \}$, and Eq. (1) is written more simply as

$$\mathcal{T}^{(m)} = \{ t^{(m)}, s^{(m)}, b^{(m)} \} \quad (8)$$

It follows that the injection vector components $p_n^{(m)} = \delta_n^{(m)} t^{(m)}$, $n = 0,1,2,...N$ are

$$\delta_n^{(m)} = \begin{cases} 1 & \text{if } n = s^{(m)} \\ -1 & \text{if } n = b^{(m)} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Whenever the seller and the buyer are at the same bus, i.e., $s^{(m)} = b^{(m)}$, the injection vector $\underline{p}^{net} = \underline{0}$. Such a transaction is assumed to cause 0 losses.

3. LOSS ALLOCATION SCHEME

We assume that a transmission line between buses i and j is represented by its line impedance $R_{ij} + jX_{ij} = (G_{ij} + jB_{ij})^{-1}$, and shunt elements are negligibly small. Under these conditions, the real power loss l_{ij} on a line connecting buses i and j is

$$l_{ij} = \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} [V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j)] \quad (10)$$

Note that $l_{ij} = l_{ji}$. The total system losses are given by

$$l = \frac{1}{2} \sum_{i=0}^{N} \sum_{j \in \mathcal{H}_i} \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} [V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j)] \quad (11)$$

where, the division by 2 is introduced so as not to count the losses on each line twice. We next assume the D.C. power flow conditions hold:

(1) Reactive power flows maintain bus voltage magnitude close to 1.0 in p.u., i.e., $V_n \approx 1.0, n = 1,2,...N$.

(2) The bus voltage angle difference across any branch is very small so that $| \theta_i - \theta_j | \approx 0$

Then the total losses may be approximately represented by $\tilde{l}$, where,

$$\tilde{l} = \frac{1}{2} \sum_{i=0}^{N} \sum_{j \in \mathcal{H}_i} \left\{ \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} (\theta_i - \theta_j)(\theta_i - \theta_j) \right\} \quad (12)$$

Let us denote by $\underline{\hat{\theta}} = [\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_N]^T$ the voltage angle vector computed by the D.C. power flow and by $\underline{\hat{B}}$ the $N \times N$ submatrix of the (N+1)-node network susceptance matrix $\underline{B}$ [9]. Then,

$$\underline{\hat{B}}\, \underline{\hat{\theta}} = -\sum_{m=1}^{M} \underline{\delta}^{(m)} t^{(m)} \quad (13)$$

If $\underline{D} = [d_{ij}] = \underline{\hat{B}}^{-1}$, then,

$$\hat{\theta}_n = \sum_{m=1}^{M} \sum_{k=1}^{N} d_{nk} \delta_k^{(m)} t^{(m)}, \ n = 1,2,...N . \quad (14)$$

Since bus 0 is chosen as the slack bus, the value of θ_0 is specified and without any loss of generality we may assume $\theta_0 = 0$. Let us rewrite Eq. (14) as

$$\hat{\theta}_n = \sum_{m=1}^{M} \mu_n^{(m)} t^{(m)}, \ n = 1,2,...N . \quad (15)$$

where, we define for $n = 1,2,...N$,

$$\mu_n^{(m)} = \sum_{k=1}^{N} d_{nk} \delta_k^{(m)} = \sum_{i=1}^{N_s^{(m)}} d_{is_i^{(m)}} \sigma_i^{(m)} - \sum_{j=1}^{N_b^{(m)}} d_{jb_j^{(m)}} \sigma_j^{(m)} \quad (16)$$

For completeness, we define $\mu_0^{(m)} = 0, m = 1,2,...M$, $d_{n0} = 0, n = 1,2,...N.$ So we can write

$$\hat{\theta}_i - \hat{\theta}_j = \sum_{m=1}^{M} \pi_{ij}^{(m)} t^{(m)}, \ i, j = 0,1,2,...N, i \neq j . \quad (17)$$

with,

$$\pi_{ij}^{(m)} = \mu_i^{(m)} - \mu_j^{(m)}, \ m = 1,2,...M; \ i, j = 0,1,..., N . \quad (18)$$

In order to express the approximation of the system losses explicitly in terms of the M transactions, we reformulate the approximation $\tilde{l}$ in Eq. (12) as

$$\begin{aligned}\tilde{l} &\approx \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\left\{\frac{R_{ij}}{R_{ij}^2+X_{ij}^2}(\theta_i-\theta_j)(\hat{\theta}_i-\hat{\theta}_j)\right\} \\ &= \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\left\{\frac{R_{ij}}{R_{ij}^2+X_{ij}^2}(\theta_i-\theta_j)\sum_{m=1}^{M}\pi_{ij}^{(m)}t^{(m)}\right\} \\ &= \sum_{m=1}^{M}\left\{\frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{R_{ij}}{R_{ij}^2+X_{ij}^2}(\theta_i-\theta_j)\sum_{m=1}^{M}\pi_{ij}^{(m)}\right\}t^{(m)}\end{aligned} \tag{19}$$

If we define for $m = 1,2,...M$,

$$\lambda^{(m)} = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\left\{\frac{R_{ij}}{R_{ij}^2+X_{ij}^2}(\theta_i-\theta_j)\pi_{ij}^{(m)}\right\} \tag{20}$$

then, $\hat{l}$ is explicitly written in terms of $t^{(m)}$ as

$$\hat{l} = \sum_{m=1}^{M}\lambda^{(m)}t^{(m)} \tag{21}$$

Using the analogous development above with the assumption that the D.C. conditions (1) and (2) hold, we may approximate $\lambda^{(m)}$ by a linear function of $\underline{\hat{\theta}}$ as

$$\lambda^{(m)} = \frac{1}{2}\sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{R_{ij}}{R_{ij}^2+X_{ij}^2}(\hat{\theta}_i-\hat{\theta}_j)\pi_{ij}^{(m)} = \frac{1}{2}\sum_{m=1}^{M}\gamma_k^{(m)}t^{(k)} \tag{22}$$

where,

$$\gamma_k^{(m)} = \sum_{i=0}^{N}\sum_{j\in\mathcal{H}_i}\frac{R_{ij}}{R_{ij}^2+X_{ij}^2}\pi_{ij}^{(m)}\pi_{ij}^{(k)} \tag{23}$$

It follows that under the D.C. conditions (1) and (2), the total loss may be approximated by a quadratic function of $t^{(k)}, k = 1,2,...M$. Moreover, under these conditions, the sensitivity of the system losses with respect to a transaction $t^{(m)}$, i.e., $\frac{\partial \hat{l}}{\partial t^{(m)}}$ may be approximated by a linear function of all transactions $t^{(k)}, k = 1,2,...M$. Additional aspects of this approximation are discussed in more detail in the Appendix 1.

We return to the expression for $\hat{l}$ in Eq. (21). Let

$$\hat{l}_a^{(m)} = \lambda^{(m)}t^{(m)},\ m = 1,2,...M. \tag{24}$$

so that

$$\hat{l} = \sum_{m=1}^{M}\hat{l}_a^{(m)} \tag{25}$$

and we may interpret $\hat{l}_a^{(m)}$ to be an expression for the "contribution"[2] made by transaction m to the total system losses. In certain cases, however, $\lambda^{(m)}$ may be negative so that one may conclude that transaction m has a negative loss allocation. Such a conclusion would imply, however, that by using the transmission network, transaction m may in effect "injects" additional power into the system. This implication is, of course, physically incorrect and the allocation scheme must address this issue.

Physically, whether a particular transaction increases or lowers the system losses depends on the system state and the impact of the particular transaction on the system. Through the use of the D.C. power flow assumptions, we derive an approximation to the system losses as a linear expression of the transactions of the system. Some transactions cause flow in the same direction as the net flow, while others cause flow in the opposite direction. The flow in the same direction as the net flow is called a dominant flow, while the flow in the opposite direction is a counter flow. Dominant flows increase the total system losses, while counter flows lower the total system losses as the amount of the corresponding transaction is increased.

Absent the dominant flow, the counter flow cannot exist. If the dominant flow disappears, the counter flow itself becomes the dominant flow. The counter flow helps reduce the losses only in presence of the dominant flow. Thus, the reduction in the system losses is not due to a particular transaction but is an attribute of the system state. As such it should be shared by all the transactions on the system.

Therefore, we modify the loss allocation by replacing $\lambda^{(m)}$ by its absolute value of $\left|\lambda^{(m)}\right|$. However, this modification would lead to allocating more losses than are actually incurred. Consequently, we allow all transactions to benefit from the "negative" losses and will normalize the allocations to ensure that the sum of the allocated losses equals $\hat{l}$. Thus, the loss allocated to transaction m is:

$$l_a^{(m)} = \frac{\left|\lambda^{(m)}\right| t^{(m)}}{\sum_{k=1}^{M}\left|\lambda^{(k)}\right| t^{(k)}}\hat{l} \tag{26}$$

Consequently, a positive loss is associated with each transaction. Additional motivation for the normalization in the allocation formula is given in the Appendix 1. This motivation is based on sensitivity information.

An attractive property of the allocation scheme is that the allocation may be evaluated for any subset of the

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[2] The expression for $\hat{l}_a^{(m)}$ may be given a straightforward physical interpretation. The $|I|^2 R$ losses in a line joining buses i and j due to the line current resulting from transaction m is $V_{R_{ij}} I_{ij}^{(m)} \approx \frac{R_{ij}}{X_{ij}}(\theta_i-\theta_j)\frac{\pi_{ij}^{(m)}t^{(m)}}{X_{ij}} \triangleq \hat{l}_{a,ij}^{(m)}$. Here, $V_{R_{ij}}$ is the voltage drop across R_{ij} that results from all the flows due to all the M transactions in the system and $I_{ij}^{(m)}$ is the current flowing between buses i and j as a result of transaction m. Since $V_{R_{ij}}$ and $I_{ij}^{(m)}$ can have different signs, $\hat{l}_{a,ij}^{(m)}$ may be negative. It follows that $\hat{l}_a^{(m)}$, the algebraic sum of all line losses due to transaction m, may also be negative.

transactions. $\lambda^{(m)}$ is evaluated using Eq. (24) for each transaction *m* in the specified subset. For the unspecified transactions, an equivalent transaction representing the effect of all the unspecified transactions is constructed. Its corresponding $t^{(eq)}$ term is determined by subtracting from the total system load the sum of the specified transactions. Its $\lambda^{(eq)}$ term is evaluated by subtracting from $\hat{l}$ the sum of the loss allocations to the specified transactions. The allocation scheme allows the computation of the allocation for as few transactions as desired without requiring information on all the unspecified transactions. An application of this useful property is discussed in the next section.

Another physically intuitive property of the allocation scheme is the treatment of the losses for a transaction which is represented by two or more equivalent transactions. Consider a transaction *m* with $t^{(m)} = t$, $s^{(m)} = i$, and $b^{(m)} = j$, so that $\mathcal{T}^{(m)} = \{t^{(m)}, i, j\}$. Let k be an arbitrary bus $k \neq i, k \neq j$. Consider the transactions $\mathcal{T}^{(\bar{m})} = \{t, i, k\}$ and $\mathcal{T}^{(\underline{m})} = \{t, k, j\}$ each with the identical transaction amount *t*. Note that the net effect of these two independent transactions is identical to that of transaction *m*. Under our definitions, the net injections into the network for the transaction $\mathcal{T}^{(m)}$ and the two independent transactions $\mathcal{T}^{(\bar{m})}$ and $\mathcal{T}^{(\underline{m})}$ are identical. Correspondingly, the state of the system under the transaction $\mathcal{T}^{(m)}$ and under the transactions $\mathcal{T}^{(\bar{m})}$ and $\mathcal{T}^{(\underline{m})}$ is identical. The proposed allocation scheme provides allocation $l_a^{(m)}$ for the system under the transaction $\mathcal{T}^{(m)}$ and allocations $l_a^{(\bar{m})}$ and $l_a^{(\underline{m})}$ under the transactions $\mathcal{T}^{(\bar{m})}$ and $\mathcal{T}^{(\underline{m})}$, respectively. It can be shown that these allocations have the property that

$$l_a^{(m)} \le l_a^{(\bar{m})} + l_a^{(\underline{m})} \tag{27}$$

Appendix 2 provides the proof of this property of the allocation scheme. This property may be generalized for any number of independent transactions representing a single transaction. It follows that the allocation scheme provides a loss allocation for a single transaction that is always less than or equal to the sum of the loss allocations of an arbitrary number of independent transactions representing that transaction, with each transaction having the identical transaction amount.

4. NUMERICAL RESULTS

We have tested the proposed loss allocation scheme extensively on a number of different systems including the IEEE 57-, 118-, 300-bus systems. Our numerical work indicates that the scheme is not only effective in providing an allocation of the losses and but also that the allocation behaves in a physically meaningful and appropriate manner in all cases studied.

To start with, we investigated the overall performance of the estimate given by the approximation formula in Eq. (19) of the total system losses under different operating conditions. For each system, we use the generation/load data of the system to specify the base case for the transactions. We vary uniformly and simultaneously the amount of each transaction using a scaling factor $0.75 \le \omega \le 1.25$, where, $\omega = 1$ corresponds to the base case. The performance of the $\hat{l}$ approximation on the IEEE 300-bus system is shown in Fig. 1. For the selected range of η, the error magnitude of the approximation is under 15%. It is interesting to note that in this range of η, the total system losses l may be approximated by a linear function of the total volume of the transactions. The approximation

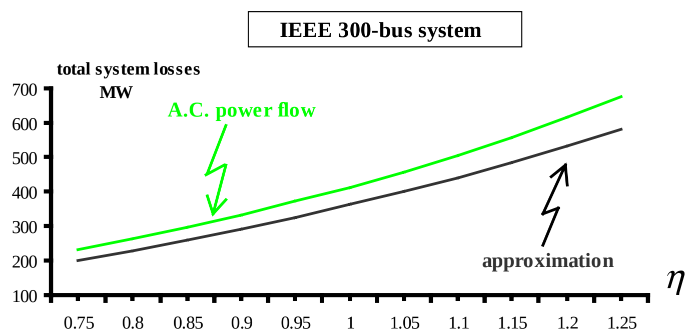


Figure 1. Comparison of the total system losses computed by A.C. power flow and the approximation formula in Eq. (19)

$\hat{l}$ tracks closely l over the range. Such close tracking was observed in all cases studied for the various systems.

We first illustrate how the allocation scheme evaluates losses associated with specified transactions. We give as an example the IEEE 57-bus system with four specified transactions. We consider the case where no counter flow results from the six specified transactions in the system. The allocation mechanism produces allocations that behave in a physically meaningful way. As the amount of each transaction increases with all other transactions remaining fixed, the corresponding loss allocation also increases. Fig. 2 provides a plot of the behavior of the total system losses approximation and the loss allocation as a function of the amount of one of the four transactions. The amount of transaction 3 is varied around its base case value within the range of $\pm 25\%$ of the base case value. The base case corresponds to the value 0. The result shown in Fig. 2 is representative of the behavior of the allocation mechanism on systems where transactions produce no counter flows.

Table 1. Transaction profile and allocation results in MW in the IEEE 118-bus system

m	1	2	3	4	5	6
$t^{(m)}$	755	744	784	566	621	773
$\hat{l}_a^{(m)}$	53.47	24.35	12.49	-13.85	15.14	28.22
$l_a^{(m)}$	43.43	19.28	10.14	11.25	12.29	22.92

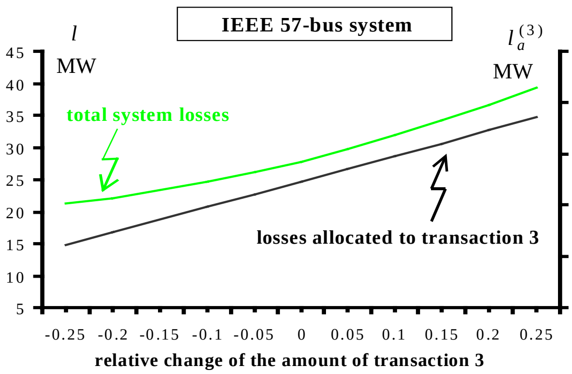
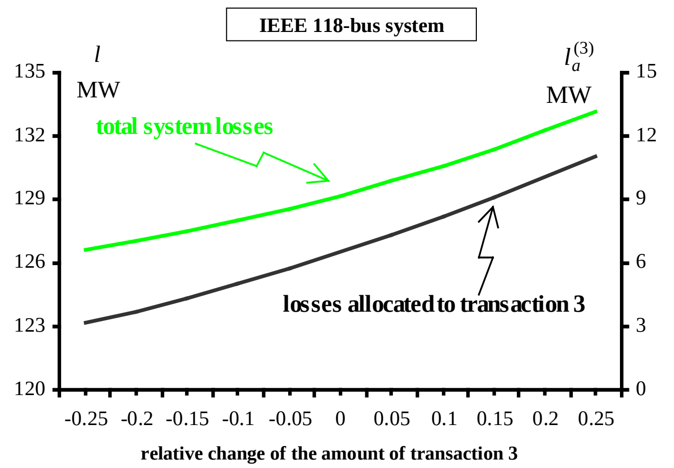


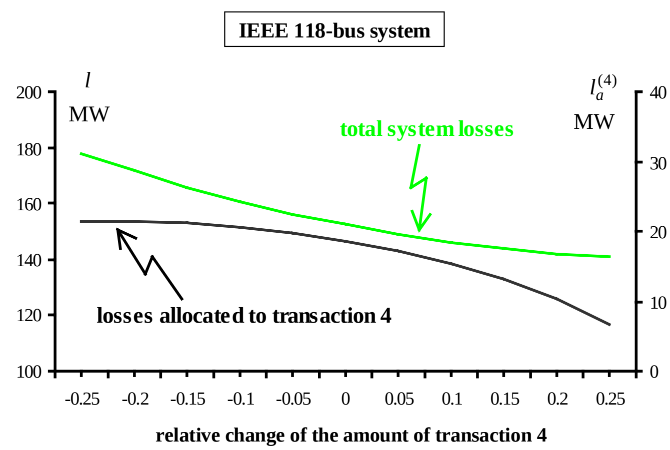
Figure 2. Loss allocation with no counter flow

We next illustrate the capability of the allocation scheme to evaluate losses associated with transactions in the presence of counter flow. Consider the IEEE 118-bus system that has been partitioned into four areas with six specified transactions. Each area has its own generation and loads and represents an entity that may undertake transactions with one or more other areas. Each transaction may involve multiple generation sources and multiple load centers. Table 1 summarizes the transaction profiles and the loss allocation results. The total amount of the six transactions is 4242MW. The total system losses evaluated by A.C. power flow are 132.75MW, and the approximation $\hat{l}$ gives a value of 119.82MW.

We focus on transactions 3 and 4 to illustrate the allocation mechanism when a particular transaction results in counter flow. The amount of transaction 3 is changed around its base value with all the other transactions being kept fixed. We investigate the impact on the total system losses. Fig. 3a shows that as the amount of transaction 3 increases, the total system losses also increase. The same is true for $l_a^{(3)}$, the losses allocated to transaction 3. If, on the other hand, we vary the amount of transaction 4 as all other transactions are kept fixed, we obtain the plots in Fig. 3b. These plots show that both the system losses and $l_a^{(4)}$, the losses allocated to transaction 4, actually decrease as the amount of transaction 4 increases. The results indicate that transaction 3 produces a dominant flow while transaction 4 results in a counter flow. The plots in Fig. 3 show that while the total system losses move in the opposite direction as the amounts of transactions 3 and 4 increase, the losses allocated to each transaction capture appropriately the impacts of the transactions on the system. In both cases, the scheme gives a physically reasonable loss allocation.



3a. dominant flow



3b. counter flow

Figure 3. Variation of the total system losses and loss allocation as a function of the transaction amount

Now let us consider the case when transaction 3 is canceled. With the absence of transaction 3, we vary the amount of transaction 4. Fig. 4 shows that in the absence of the dominant flow created by transaction 3, the previous counter flow caused by transaction 4 becomes a dominant flow. Correspondingly, the system losses increase as transaction 4 increases The allocation scheme once again captures the appropriate movement of the system losses as a function of the amount of the transaction and gives an appropriate loss allocation to transaction 4.

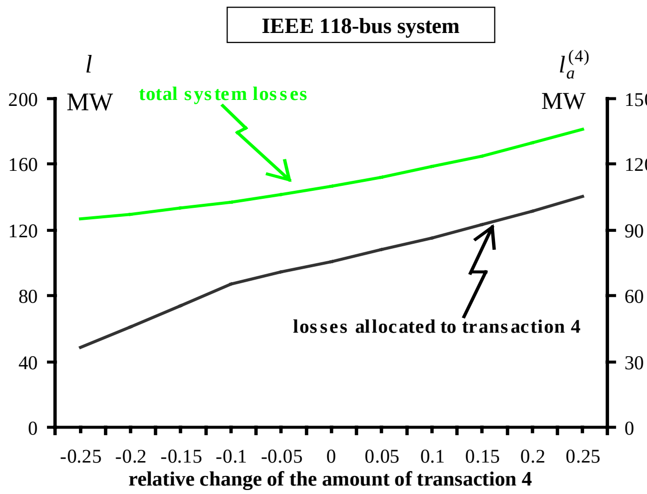


Figure 4. Plots of the total system losses and loss allocation as a function of the amount of transaction 4

We have tested the property of the allocation scheme that allows the evaluation of loss allocation for a specific subset of transactions and found it quite useful. For an example, consider the case of the IEEE 118-bus system where we are interested in the evaluation of loss allocation for the transactions 1,2, and 3. The use of Eq. (19) and (24) provides the identical results to those in Table 1 without the use of information on the transactions 4, 5 and 6.

Finally, we use an example of the IEEE 57-bus system to illustrate the property of Eq. (27). Suppose that a broker is considering arranging a trade that buys 40MW of the power produced at bus 3 and sells to the load at bus 16. Due to some reason, the broker is unable to arrange a direct transaction from the selling bus 3 to the buying bus 16. Instead he sets up two transactions using the intermediate bus 12. He thus replaces the single transaction $\{40,3,16\}$ by the two transactions $\{40,3,12\}$ and $\{40,12,16\}$. Since these two transactions involve different entities, they are considered as two independent transactions. According to our allocation scheme, the sum of losses allocated to these two transactions are always no less than the losses allocated to the direct transaction [3] from bus 3 to bus 16. The losses allocated to the single transaction $\{40,3,16\}$ are 0.34MW; the losses allocated to the two transactions $\{40,3,12\}$ and $\{40,12,16\}$ are 0.62MW and 0.25MW, respectively. The sum of the losses allocated to these two transactions is 0.87MW, which is greater than the losses for the single transaction.

[3] Only in the case when all of them result in dominant or counter flows at the same time, the loss allocation results are equivalent.

5. CONCLUSION

The issue of allocating losses in a multiple-transaction network is of concern as the number of transactions is steadily growing. Given that there is a good degree of arbitrariness in the evaluation and measurement of losses and, consequently, in their allocation, the need is critical for an effective scheme for reasonably allocating losses that is acceptable to the players in the competitive marketplace. This paper has presented a scheme that uses physical flows to allocate losses to all the transactions in a system. The scheme provides allocations that are appropriate and behave in a physically reasonable manner and produce *ex ante* information to enable transacting entities to undertake various transactions. The scheme is particularly effective in dealing with the situation of counter flows.

There are several extensions of the work presented here. A natural question to address is the issue of direct compensation by each transaction for the allocated losses. Rather than using the centrally managed loss compensation service of the independent grid operator, each transaction may be able to inject additional generation at a designated bus. On a more general basis, the development of allocation mechanisms for other ancillary services is a topic of active interest. The adaptation and extension of the transaction framework presented here to other network based ancillary services such as reactive power and voltage control is one specific area. Work is underway on the application of the transaction framework for the allocation of reactive power services.

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APPENDIX 1: SENSITIVITY INFORMATION

The loss allocation formula of Eq. (24) uses the absolute values of $\lambda^{(m)}$ defined in Eq. (20) so as to assign nonnegative losses to all transactions. This appendix provides additional motivation for associating nonnegative losses with each transaction.

We examine the sensitivity information in the unnormalized allocation $\hat{l}_a^{(m)}$ of Eq. (22). From Eq. (23) it follows that $\gamma_m^{(m)} \geq 0$. However, $\gamma_k^{(m)}$ may be either positive or negative for $k \neq m$. Consider the evaluation of the change $\Delta\hat{l}$ in the total system losses corresponding to a change $\Delta t^{(m)}$ in the value of transaction *m* with the values of all other transactions $k \neq m$ remaining unchanged. Using a first order approximation

$$\Delta\hat{l} \approx \frac{\partial \hat{l}}{\partial t^{(m)}} \Delta t^{(m)} = \left[\gamma_m^{(m)} t^{(m)} + \sum_{k \neq m} \gamma_k^{(m)} t^{(k)} \right] \Delta t^{(m)} \qquad (28)$$

We interpret the expression for the sensitivity of $\hat{l}$ to $t^{(m)}$ as the sum of two distinct terms: a "self contribution" $\gamma_m^{(m)} t^{(m)}$ and a " contribution " of all the other transactions. The first term is only dependent on transaction *m* and is always nonnegative. In fact, if $t^{(k)} = 0$ for all $k \neq m$, the total system losses increase as $t^{(m)}$ increases. However, if any $k \neq m, t^{(k)} \neq 0$, the change in the total system losses due to $\Delta t^{(m)}$ may be non-positive. The magnitude of the second term may be greater or equal or lesser than, that of the first term. It is not possible to analytically determine the relative contribution of the two terms. However, good insight may be obtained from numerical studies.

We studied the sensitivity for the various systems discussed in Section 4. We examine the results obtained on the IEEE 118-bus system for the case with the transaction profile given in Table 1. The terms corresponding to transactions 3 and 4 are given in Table 2. As discussed in Section 4, transaction 3 corresponds to the dominant flow with transaction 4 being the counter flow. As Table 2 shows, for the dominant flow the "self contribution" term outweighs the negative contribution of the other transactions. However, for the counter flow situation of transaction 4, the opposite is the case. For transaction 4, the term representing the effect of all other transactions is negative with magnitude greater than that of the "self contribution" term. Note that the negative value of sensitivity, however, is not due to the transaction itself, but to its interaction with all the other transactions. This situation for transaction 4 is true in general, whenever a counter flow arises in a network. It is not appropriate to associate "negative" losses with a counter flow producing transaction. Rather, the decrease in the losses is due to the impact of all the other transactions in the system. Consequently, the reduction in the total system losses should be shared equitably by all the transactions. Hence, we have the motivation for the normalized allocation used in Eq. (24).

Table 2. The sensitivity results for transactions 3 and 4.

m	$\gamma_m^{(m)} t^{(m)}$	$\sum_{k \neq m} \gamma_k^{(m)} t^{(k)}$
3	0.032	-0.0001
4	0.2289	-0.2605

APPENDIX 2: PROOF OF THE PROPERTY IN EQ. (27)

We discuss in detail the property given in Eq. (27). Consider an arbitrary network. Let case 1, denoted by the subscript [1], correspond to the network with *M* transactions. Without any loss of generality, we re-label transaction *M* to be $\mathcal{T}^{(M)} = \{t, i, j\}$. We call case 2, denoted by the subscript [2], the system where we replace the transaction *M* of case 1 by the equivalent two independent transactions $\mathcal{T}^{(M)} = \{t, i, k\}$ and $\mathcal{T}^{(M+1)} = \{t, k, j\}$, where k is an arbitrary bus $k \neq i, k \neq j$. For cases 1 and 2, transactions 1,2,…, *M*-1 are identical. Note that for cases 1 and 2 the quantities $\underline{p}_{[1]}^{net} = \underline{p}_{[2]}^{net} = \underline{p}^{net}$ using Eq. (6). It follows that the A.C. and D.C. power flow results are identical:

$$\theta_{n,[1]} = \theta_{n,[2]} = \theta_n, \hat{\theta}_{n,[1]} = \hat{\theta}_{n,[2]} = \hat{\theta}_n, n = 0,1,2,..., N \qquad (29)$$

Consequently,

$$l_{[1]} = l_{[2]} = l = \frac{1}{2} \sum_{\nu=0}^{N} \sum_{\eta \in \mathcal{H}_\nu} \frac{R_{\nu\eta}^2}{R_{\nu\eta}^2 + X_{\nu\eta}^2} [V_\nu^2 + V_\eta^2 - 2V_\nu V_\eta \cos(\theta_\nu - \theta_\eta)] \qquad (30)$$

so that

$$\hat{l}_{[1]} = \hat{l}_{[2]} = \hat{l} = \frac{1}{2} \sum_{\nu=0}^{N} \sum_{\eta \in \mathcal{H}_\nu} \frac{R_{\nu\eta}^2}{R_{\nu\eta}^2 + X_{\nu\eta}^2} [(\theta_\nu - \theta_\eta)(\hat{\theta}_\nu - \hat{\theta}_\eta)] \qquad (31)$$

Since for cases 1 and 2 the state of the network is identical, it follows that $\underline{D} = \underline{\hat{B}}^{-1}$ is the same. In other words, $d_{\nu\eta,[1]} = d_{\nu\eta,[2]}, \nu, \eta = 1,2,...N$ and furthermore, by definition, $d_{n0,[1]} = d_{n0,[2]} = 0, n = 1,2,...N$. Using the relations in Section 2, it follows that for transactions m=1,2,…,M-1 that are identical for cases 1 and 2 that

$$\delta_{n,[1]}^{(m)} = \delta_{n,[2]}^{(m)}, n = 0,1,..., N, m = 1,2,... M-1 \quad (32)$$

From Section 3, we have

$$\pi_{\nu\eta,[1]}^{(m)} = \pi_{\nu\eta,[2]}^{(m)}, m = 1,2,....M-1; \nu, \eta = 0,1,..., N \quad (33)$$

Consequently

$$\lambda_{[1]}^{(m)} = \lambda_{[2]}^{(m)}, m = 1,2,..., M-1 \quad (34)$$

Next, consider the factors corresponding to the transaction M in case 1, and the transactions M and M+1 in case 2. From Section 2 it follows that for n=0,1,2,…,N

$$\delta_{n,[1]}^{(M)} = \begin{cases} 1 & n = i \\ -1 & n = j \\ 0 & \text{otherwise} \end{cases}$$

and

$$\delta_{n,[2]}^{(M)} = \begin{cases} 1 & n = i \\ -1 & n = k \\ 0 & \text{otherwise} \end{cases} \quad \delta_{n,[2]}^{(M+1)} = \begin{cases} 1 & n = k \\ -1 & n = j \\ 0 & \text{otherwise} \end{cases} \quad (35)$$

Therefore,

$$\delta_{n,[1]}^{(M)} = \delta_{n,[2]}^{(M)} + \delta_{n,[2]}^{(M+1)}, n = 0,1,2..., N \quad (36)$$

Using the relations in Section 3 we have

$$\begin{aligned} \mu_{n,[1]}^{(M)} &= \sum_{k=1}^{N} d_{nk} \delta_{k,[1]}^{(M)} = \sum_{k=1}^{N} d_{nk} [\delta_{k,[2]}^{(M)} + \delta_{k,[2]}^{(M+1)}] \\ &= \mu_{n,[2]}^{(M)} + \mu_{n,[2]}^{(M+1)}, n = 0,1,2,..., N \end{aligned} \quad (37)$$

Therefore,

$$\begin{aligned} \pi_{\nu\eta,[1]}^{(M)} &= \mu_{\nu,[1]}^{(M)} - \mu_{\eta,[1]}^{(M+1)} \\ &= [\mu_{\nu,[2]}^{(M)} - \mu_{\eta,[2]}^{(M)}] + [\mu_{\nu,[2]}^{(M+1)} - \mu_{\eta,[2]}^{(M+1)}] \\ &= \pi_{\nu\eta,[2]}^{(M)} + \pi_{\nu\eta,[2]}^{(M+1)} \end{aligned} \quad (38)$$

so that

$$\lambda_{[1]}^{(M)} = \lambda_{[2]}^{(M)} + \lambda_{[2]}^{(M+1)}. \quad (39)$$

Now, we consider the allocations for the two cases. They are given by

$$l_{a,[1]}^{(M)} = \frac{\left|\lambda_{[1]}^{(M)}\right| t}{\sum_{m=1}^{M-1} \left|\lambda_{[1]}^{(m)}\right| t^{(m)} + \left|\lambda_{[1]}^{(M)}\right| t} \quad (40)$$

and

$$l_{a,[2]}^{(M)} + l_{a,[2]}^{(M+1)} = \frac{\left|\lambda_{[2]}^{(M)}\right| t + \left|\lambda_{[2]}^{(M+1)}\right| t}{\sum_{m=1}^{M-1} \left|\lambda_{[2]}^{(m)}\right| t^{(m)} + \left|\lambda_{[2]}^{(M)}\right| t + \left|\lambda_{[2]}^{(M+1)}\right| t} \quad (41)$$

From Eq. (34), the first M-1 transactions in the two cases result in $\lambda_{[1]}^{(m)} = \lambda_{[2]}^{(m)}, m = 1,2,..., M-1$. In these two cases

$$l_{a,[1]}^{(M)} = \frac{\left|\lambda_{[1]}^{(M)}\right| t}{\sum_{m=1}^{M-1} \left|\lambda_{[1]}^{(m)}\right| t^{(m)} + \left|\lambda_{[1]}^{(M)}\right| t} \hat{l} = \frac{1}{1 + \sum_{m=1}^{M-1} \frac{\left|\lambda_{[1]}^{(m)}\right|}{\left|\lambda_{[1]}^{(M)}\right|} \frac{t^{(m)}}{t}} \hat{l} \quad (42)$$

and

$$\begin{aligned} l_{a,[2]}^{(M)} + l_{a,[2]}^{(M+1)} &= \frac{[\left|\lambda_{[2]}^{(M)}\right| + \left|\lambda_{[2]}^{(M+1)}\right|] t}{\sum_{m=1}^{M-1} \left|\lambda_{[2]}^{(m)}\right| t^{(m)} + \left|\lambda_{[2]}^{(M)}\right| t + \left|\lambda_{[2]}^{(M+1)}\right| t} \hat{l} \\ &= \frac{1}{1 + \sum_{m=1}^{M-1} \frac{\left|\lambda_{[2]}^{(m)}\right|}{[\left|\lambda_{[2]}^{(M)}\right| + \left|\lambda_{[2]}^{(M+1)}\right|]} \frac{t^{(m)}}{t}} \hat{l} \end{aligned} \quad (43)$$

It follows from Eq. (39) that

$$\left|\lambda_{[1]}^{(M)}\right| = \left|\lambda_{[2]}^{(M)} + \lambda_{[2]}^{(M+1)}\right| \le \left|\lambda_{[2]}^{(M)}\right| + \left|\lambda_{[2]}^{(M+1)}\right| \quad (44)$$

so that

$$l_{a,[1]}^{(M)} \le l_{a,[2]}^{(M)} + l_{a,[2]}^{(M+1)} \quad (45)$$

The equality holds only if $\lambda_{[1]}^{(M)}, \lambda_{[2]}^{(M)}$ and $\lambda_{[2]}^{(M+1)}$ have the same sign.