EFFICIENT LARGE-SCALE HYDRO SYSTEM SCHEDULING WITH FORCED SPILL CONDITIONS

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Abstract - We present a general framework for the formulation and solution of large-scale hydro system scheduling problems (h.s.s.p.). We use a nonlinear programming formulation that permits the representation of virtually all types of constraints imposed on a hydroelectric system: the physical, operational, legislative or contractual constraints. The problem formulation explicitly represents the nonlinear relationship between spillage and the reservoir storage level. Such constraints are called forced spill conditions and are modeled by nonlinear equalities. In the proposed method, the nonlinear constraints representing the forced spill conditions are treated by the exact penalty technique. The resulting problem has a nonlinear objective function and only linear constraints. The solution scheme makes detailed use of the structural characteristics of the h.s.s.p. The underlying network structure of the h.s.s.p. is exploited to determine a good starting point via the application of an efficient network flow algorithm. The sparsity of the linear constraints is exploited by the nonlinear optimization algorithm. The proposed method is computationally efficient for determining optimal schedules for large river systems. Results on several cases including one with 3300 decision variables, 2200 linear equalities, 2700 linear inequalities and 200 nonlinear equality constraints, are presented.

INTRODUCTION

The efficient utilization of hydro resources is of paramount importance in the planning and operation of a power system where the hydroelectric generation plants constitute a significant portion of the installed capacity. A key task in water resource mangement is the determination of optimal schedules for hydro generation. The so-called hydro system scheduling problem (h.s.s.p.) aims to determine the water releases from each reservoir and through each power house so as to optimize the total benefit of the hydro generated energy, while the various environmental, physical, legal and contractual constraints are satisfied. Consider a river system for which the schedules are to be determined for a specified study period. The initial contents of each reservoir and for each subperiod of the study period, the forecasted natural inflows from each tributary stream and the planned outages of power houses and canals, are given. The h.s.s.p. is to determine the water releases from the reservoirs and the flows through the power houses so as to maximize the value of the total energy generation subject to a number of constraints. The latter include environmental considerations, irrigational demands, storage index decrees, flood control storage limitations, operational practices, contractual obligations and other water-use legislation.

The determination of optimal schedules entails the solution of a nonlinear programming problem. For river systems of practical interest and for typical study periods of one to two years, large-scale optimization problems must be solved. A wide variety of techniques for solving such problems have been reported in the literature. Comprehensive surveys are presented in [7] and [12]. For the multi-reservoir, multi-period, and deterministic inflow problem, representative methods include linear [13], nonlinear [4,8,10], and dynamic programming [3,11].

The approach in [4] is to transform a general nonlinear program formulation of the h.s.s.p. into an optimization problem with a nonlinear objective and only linear constraints. All the nonlinear constraints are expressed as penalty terms and are added to the original objective function. A modified conjugate gradient method that takes advantage of the sparsity of the linear constraint matrix is used. This approach can solve very large h.s.s.p.'s: cases involving a nonlinear objective, 6000 variables, 4000 linear constraints and 11,000 nonlinear and linear inequalities have been optimized. The approach of [4] represents the first successful attempt at solving such large h.s.s.p.'s by nonlinear optimization techniques. However, the computational times reported are rather lengthy. Another noteworthy application of the large scale nonlinear programming techniques is the methodology developed in [8]. A simpler model without any nonlinear constraints is employed. The solution algorithm is based on a reduced gradient technique. Large-scale scheduling problems are solved within reasonable times.

The dynamic programming formulation of the h.s.s.p. is given in [3,11]. This solution scheme uses dynamic programming by successive approximations to convert the h.s.s.p. with a high dimensional state space into a sequence of problems with lower dimensional state spaces. An efficient implementation of this methodology allows the optimization of a simplified representation of the TVA system [11].

The solution methods of [10] and [13] make good use of the underlying network structure of the h.s.s.p. The formulation of [10] has a general nonlinear objective with linear network flow constraints. A nonlinear network algorithm is based on a modified reduced gradient technique and on a primal linear network flow algorithm that exploits the underlying network structure. The algorithm uses the solution of an integer programming problem to determine the search direction. The formulation of the work in [13] is similar to that of [10]. The authors use the Ford-Fulkerson network flow algorithm [1] successively to accommodate the nonlinear objective function. This simple approach is efficient for the solution of the h.s.s.p. formulation in [13].

This paper reports the development of a new solution methodology for the h.s.s.p. A general formulation that allows the incorporation of virtually any physical, environmental or legal constraint is employed. The h.s.s.p. statement is given in terms of a nonlinear objective, linear and nonlinear equalities and inequalities. One of the major contributions of the present work is that it explicitly considers forced spill

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conditions. Forced spills occur only when the storage in a reservoir reaches its upper limit. The nonlinear constraints representing the forced spill conditions are treated by the exact penalty method [6]. All other constraints involving nonlinear relationships are represented by piece-wise linear functions.

The proposed solution methodology makes extensive use of the structural characteristics of the h.s.s.p. The sparsity of the linear constraint matrix is exploited. The overall solution time is reduced by using a good starting point determined from the application of a network flow algorithm. The resulting method is computationally efficient. Numerical results illustrating the application of the proposed methodology for actual river systems are presented.

The main body of the paper consists of four additional sections. The next section gives a brief overview of the hydro system and its components. Then, the formulation and statement of the h.s.s.p. are given. The following section presents the proposed solution methodology. The paper concludes with a section giving computational results for existing river systems.

DESCRIPTION OF HYDRO SYSTEM

Components of a Hydroelectric Generation System

A hydroelectric generation system consists of rivers, tributaries, reservoirs (such as lakes or ponds), power houses and additional hydro facilities for power generation such as feeders and gates. Pipes, canals and rivers interconnect reservoirs and power houses. The natural inflows into the system are stored in the reservoirs or routed through a chain of hydro facilities. The flow of water from one facility to another is one-directional with the single exception of pumped storage plants. Ultimately the water flows either into another river or into a lake where the water is used for a specific purpose such as flood control, or storage. A schematic of a medium scale hydroelectric system is given in Fig. 1.

Conventional hydro power units are classified as either controllable or run-of-river plants. Because of insufficient pondage, the run-of-river power house must take the water as it comes. It operates at a nearly uniform output throughout a period. On the other hand, a controllable hydro plant can exercise control on both the level of output and times of generation by manipulating the pondage -- storing water at night and weekends and generating at maximum output at peak demand times.

The primary function of reservoirs is to store the supply of water from the run-off in each stream for subsequent releases through power houses. In general, each reservoir has another reservoir downstream from it. Each reservoir has maximum and minimum capacities of storage levels. Whenever an inflow into a reservoir causes the storage to reach its maximum capacity level, an uncontrollable spill of water occurs. Since the spill bypasses the power house immediately downstream from the filled reservoir, it is not available for power generation.

A hydro unit ususally has two associated reservoirs: the forebay located in the upstream and the afterbay located in the downstream. The water flowing out from the power house is called the tailwater, and it is released to the afterbay through the tailrace. The difference in the water surface elevation of the forebay and the elevation of the tailwater is called the head of the power house (see Fig. 2). If the tailrace is submerged in the afterbay, as is the case at a power house with a reaction turbine, the elevation of the tailwater is the surface elevation of the afterbay. On the other hand, if the tailrace is located upstream of the afterbay, as is the case at a power house with an impulse turbine, the tailwater elevation remains constant.

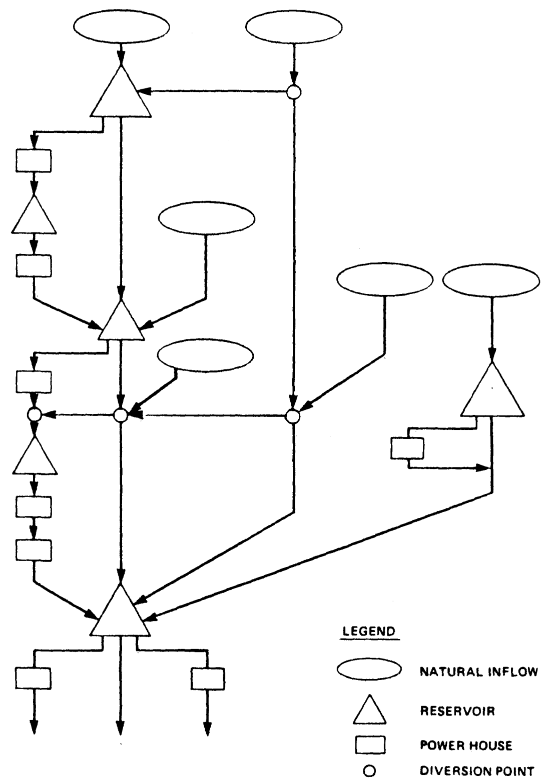

Fig. 1. Schematic of a hydro system with 6 reservoirs and 8 power houses.

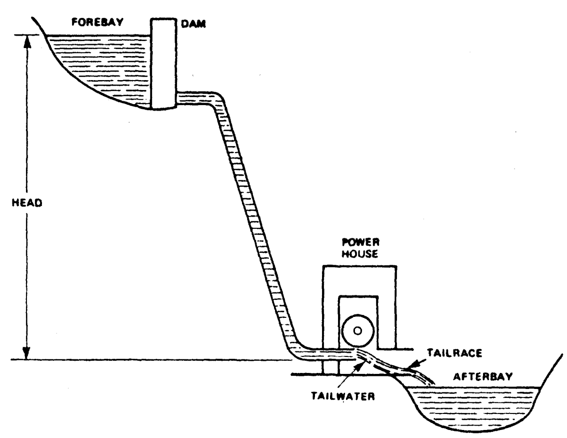

Fig. 2. Schematic of a power house with an impulse turbine.

At some power houses, changes in the surface elevations of the forebay and the afterbay may lead to significant head level variations. In such cases, the head is a function of the storage levels of the forebay and the afterbay. An example of the dependence of the head on the storage level of the forebay with a fixed tailwater elevation is shown in Fig. 3. In cases where the forebay is located above the power house at such a high altitude that the surface level changes are small compared to the head, the changes in the forebay storage level may be neglected. At a run-of-river power plant, the forebay storage level is, in general, assumed to be constant.

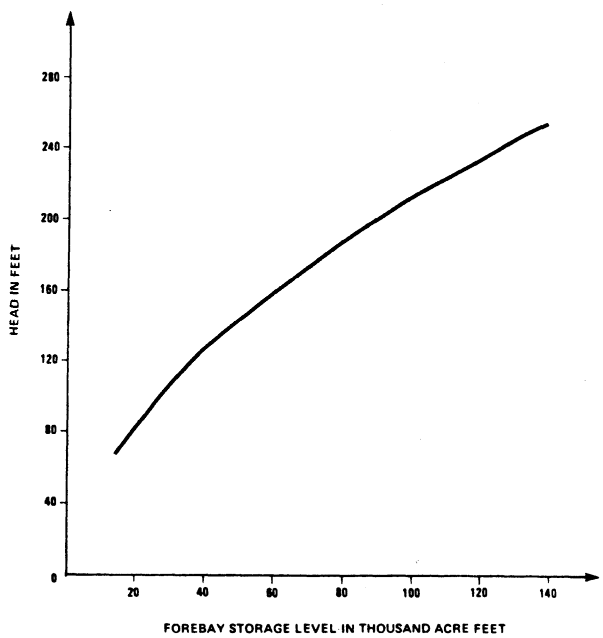

Fig. 3. Head as a function of forebay storage at a reaction type low head unit.

The MW capacity of a hydro unit depends on the head and the flow through the power house. Typical curves showing the hydro generation capacity as a function of the water flow through the power house for different values of head for a reaction type unit are shown in Fig. 4. The maximum flow through a power house is also a function of the head; therefore, the maximum MW output of a unit is, in fact, a function of the head only. This is indicated in Fig. 4 by the broken curve labeled maximum flow.

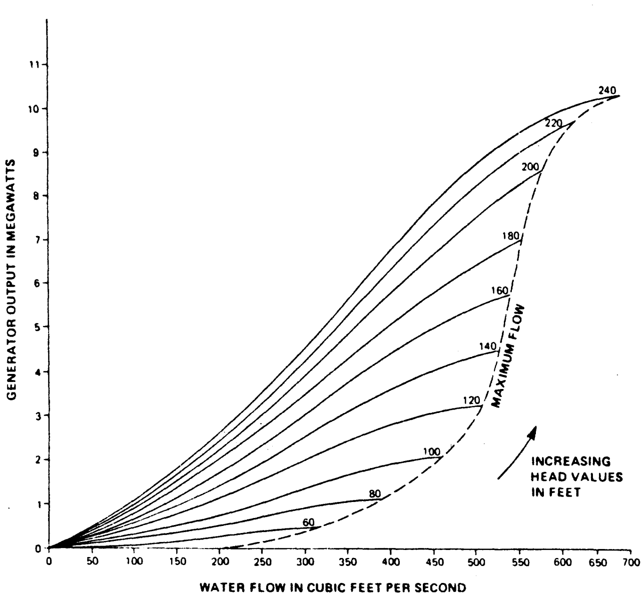

Fig. 4. Hydro generation capacity as a function of water flow through the power house for different values of head.

Operational Constraints

A river system with hydrogeneration facilities is a multi-purpose development [12]. The operation of a private hydroelectric generation system is subject to numerous governmental controls and regulations. For example, state and federal agencies specify the minimum flows to be maintained dowstream from the dam for environmental protection and safety. Due to safety considerations, constraints on storage levels of reservoirs may also be imposed. The contractual commitments with irrigation districts and water agencies often constrain the amounts of draft from various reservoirs to be fixed. In addition, there may be local decrees that specify the minimum storage levels in a specified set of reservoirs in a certain region.

In addition to the operational and contractual constraints, there are physical constraints which are in effect at each hydro facility. For example, each penstock, pipe or canal has a maximum capacity. Each reservoir has maximum and minimum storage levels. All the legal and contractual obligations, the physical constraints and the specific operating policies of the system have to be satisfied in the planning as well as the actual operation of the hydroelectric generation system, regardless of their economic impact.

THE H.S.S.P. PROBLEM

Problem Description

The primary objective in the operation of a hydroelectric generation system is to use the water in the most efficient way, while satisfying all the constraints imposed by legal and contractual obligations, physical characteristics and operating policies. The optimal water release schedules are determined at each reservoir so as to meet this goal. Due to the large-scale network structure of hydro systems of practical interest, determination of such schedules entails the solution of rather complex problems. This paper presents a deterministic model of the hydroelectric generation system and a computationally efficient method to solve the h.s.s.p. The intended application of the scheduler is to the planning of operating strategy over a specified period as well as to the day-to-day operation of hydro systems. In addition, it is useful for water resource development, for new facility planning, and for determining maintenance schedules of the hydro units.

We refer to the period for which the schedule of the water releases is determined as the study period. For example, for mid-term scheduling, the study period may be one year and for short-term scheduling, it may be one week. The study period may be divided into subperiods. In each subperiod, the water release at each hydro facility and the carry-over storage to the next subperiod at each reservoir are determined. The schedule is set up so as to optimize a specific objective function and at the same time satisfy all the contractual, regulatory, operational and physical constraints of interest.

The initial amount of storage in each reservoir and the natural inflows into each stream during each subperiod are assumed to be known. The forecasts of natural inflows are obtained using historical rainfall, river measurements and snow survey data. The demand for water, typically for irrigational purposes, is also assumed to be known at each location in each subperiod. The travel time from one hydro facility to another must be taken into account if the transportation delays are longer than the duration of a subperiod.

The h.s.s.p. is formulated as a large-scale nonlinear programming problem. The formulation given here is very general. An efficient solution methodology is developed by making detailed use of the problem structure.

The Hydro Network

The structure of a hydroelectric generation system is described in terms of the associated network $G=(N,A)$. The set N of nodes $\{v_1,v_2,\dots,v_n\}$ represents all the hydro facilities, such as reservoirs, power houses, and diversion points in the system. The set A of arcs $\{(v_i,v_j)|v_i,v_j \in N\}$ represents the hydro conduits, such as pipes, canals, and rivers. Each arc (v_i,v_j) corresponds to a physical element that connects hydro facility i to hydro facility j. Each arc has an associated direction so that the flow on each arc is one-directional.

The set of nodes is partitioned into three disjoint subsets: S the subset of sources, T the subset of sinks, and C the subset of intermediate nodes. Thus, $N=S\cup T\cup C$. A source node corresponds to a component of the river system where a natural inflow originates. A sink node corresponds to the final destination of the released water. We distinguish between two distinct classes of sink nodes. A type I sink node corresponds to the final destination of the released water for hydro generation. A type II sink node represents a location where a demand for water, other than electric energy generation, such as irrigation or drinking water, exists. In addition, we introduce a fictitious node v_n connected to all the type I sink nodes. The sum of the flows into v_n represents the total amount of water released and used for hydro generation. All the other members of the set N are the intermediate nodes. These are characterized by the property that the conservation of flows is observed: the sum of all the inflows equals the sum of all the outflows. An intermediate node may correspond to a reservoir where water may be stored from one subperiod to the next, a power house or a facility such as a feeder, diversion point or connector. We denote the subset of the r reservoir nodes by R, and the subset of the power house nodes by P. All the other intermediate nodes constitute the subset Q. Since these subsets are disjoint, $C=R\cup P\cup Q$.

Let the subscript t denote quantities associated with the t-th subperiod, $t=1,2,\dots,T$. Let the water flow from facility i to facility j in the t-th subperiod be x_{ij}^t. The travel time from facility i to facility j is a constant τ_{ij}. At a reservoir $k\in R$, s_k^t denotes the storage level at the end of the subperiod t. We define $\underline{x}^t$ to be the vector of flows in subperiod t; its components are the flows x_{ij}^t. Let $\underline{s}^t=[s_1^t,\dots,s_r^t]$. We define $\underline{x}\triangleq[\underline{x}^1,\underline{x}^2,\dots,\underline{x}^T]$ and $\underline{s}\triangleq[\underline{s}^1,\underline{s}^2,\dots,\underline{s}^T]$.

At each source node $v_i\in S$, a non-negative number b_i^t indicates the natural inflow for subperiod t, $t=1,\dots,T$. At each sink node $v_j\in T$, a non-positive number b_j^t indicates the demand for water in subperiod t, $t=1,\dots,T$. At each intermediate node v_i, $b_i^t=0$ for all t. We define the n-dimensional vector $\underline{b}^t=[b_1^t,\dots,b_n^t]$. We use the notation $\underline{b}=[\underline{b}^1,\underline{b}^2,\dots,\underline{b}^T]$.

Formulation of the Constraints

We next describe the formulation of some of the basic operational constraints in terms of the hydro system variables $\underline{s}^t$ and $\underline{x}^t$. The operational constraints given here include some of the most important ones for hydroelectric systems. However, not all types of constraints are treated here. Any of the constraints not explicitly treated here can be formulated along analogous lines.

Water Flow Balance Equation

At each source node v_i, the sum of outflows equals the natural inflow in each subperiod t:

$$\sum_{\substack{j\\(v_i,v_j)\in A}} x_{ij}^t = b_i^t \quad \text{for each } v_i\in S. \tag{1}$$

At each sink node v_j, the sum of inflows equals the demand in each subperiod t:

$$-\sum_{\substack{i\\(v_i,v_j)\in A}} x_{ij}^{t-\tau_{ij}} = b_j^t \quad \text{for each } v_j\in T. \tag{2}$$

Here the travel time of inflows is taken into account.

At each intermediate node v_i, conservation of flows must be observed by keeping the sum of inflows equal to the sum of outflows. The travel time of inflows is again considered. Thus

$$\sum_{\substack{j\\(v_i,v_j)\in A}} x_{ij}^t - \sum_{\substack{j\\(v_j,v_i)\in A}} x_{ji}^{t-\tau_{ji}} = 0 \quad \text{for each } v_i\in Q. \tag{3}$$

In relations (2) and (3), losses due to seepage or evaporation are neglected. Such losses, when significant, may be represented as a demand and modeled as an additional sink node of type II.

If an intermediate node v_i represents a reservoir, the storage level of the reservoir in subperiod t-1 needs to be taken into account:

$$s_i^t = s_i^{t-1} - \sum_{\substack{j\\(v_i,v_j)\in A}} x_{ij}^t + \sum_{\substack{j\\(v_j,v_i)\in A}} x_{ji}^{t-\tau_{ji}} \quad \text{for each } v_i\in R. \tag{4}$$

In Eq. (4) for $t=1$, s_i^0 is the initial storage level of reservoir v_i.

Fixed Limits on Flows and Storage Levels

For each arc $(v_i,v_j)\in A$, an upper bound and a lower bound on the flow are imposed in each subperiod. These bounds may represent the maximum capacity of the corresponding pipe or canal, certain operational policies, contractual obligations or legal requirements. We have for each subperiod t,

$$\ell_{ij}^t \le x_{ij}^t \le u_{ij}^t \text{ , for each } (v_i,v_j)\in A. \tag{5}$$

Note that $0\le \ell_{ij}^t$ and that u_{ij}^t may be infinity when no upper bound on the flow is imposed. The limits ℓ and u may be different for each subperiod.

For each reservoir $v_i\in R$, there exist minimum and maximum bounds corresponding to the physical storage level limits. Thus, in each subperiod the storage s_i^t of reservoir v_i must satisfy the constraint:

$$w_i^t \le s_i^t \le z_i^t \quad \text{for each } v_i\in R. \tag{6}$$

Variable Flow Bounds

Recall from the discussion above that the maximum flow from the forebay to the afterbay at a power house is in general a nonlinear function of the head. To simplify notation, we assume that the tailwater elevation is constant. (However, the treatment of the case where the head depends on the afterbay water surface elevation is a straightforward extension of the development given here.) Consider a node $v_i \epsilon P$ corresponding to a power plant. Let v_{j_i} denote the node corresponding to the forebay of this power plant. Now the instantaneous maximum flow through the power house $x_{j_i i}$ is a function of the storage level of the forebay s_{j_i} at that instant. A typical curve showing this nonlinear dependence is given in Fig. 5. Due to the time frame for the h.s.s.p. -- the subperiod is the smallest time unit considered -- it is customary to represent this dependence by replacing the storage level by its average value during the subperiod [12].

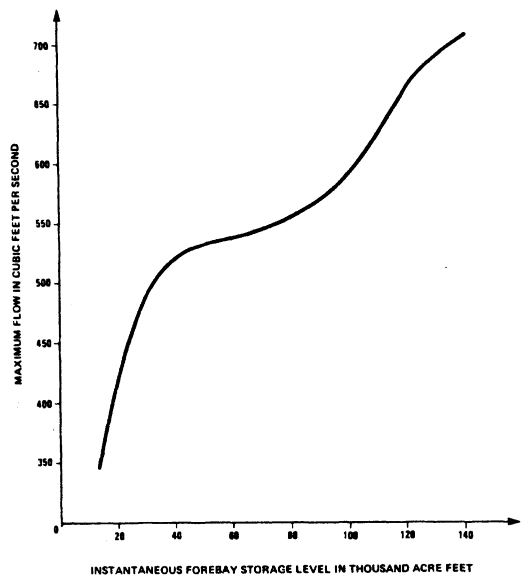

Fig. 5. Maximum flow through a power plant as a function of the instantaneous forebay storage level.

The curve given in Fig. 5 is redrawn in Fig. 6 where the units along the axes are changed. Figure 6 is obtained from Fig. 5 by making the required conversion from cubic feet per second to acre feet per week assuming a one-week subperiod. Thus, the maximum flow through power house i in subperiod t is represented as a nonlinear function of the average storage level of the forebay during the subperiod. The curve giving this relationship is identical to that for the instantaneous quantities.

It is possible to bound this nonlinear function by piece-wise linear segments. It is particularly convenient from the point of view of the solution methodology to choose these segments so that the feasible set of flows is bounded by a convex polytope. The piece-wise linear bound shown in Fig. 6 is convex.

In the piece-wise linear representation, the flow through the power house must satisfy constraints of the form:

$$x^t_{j_i i} \leq \alpha_{i\ell}(s^{t-1}_{j_i} + s^t_{j_i}) + \beta_{i\ell}, \text{ for } \ell = 1,\dots,L_i, \text{ each } v_i \epsilon P \tag{7}$$

where L_i denotes the number of linear segments used to bound the flow through the power house v_i. In (7), the average storage level in the subperiod t is evaluated

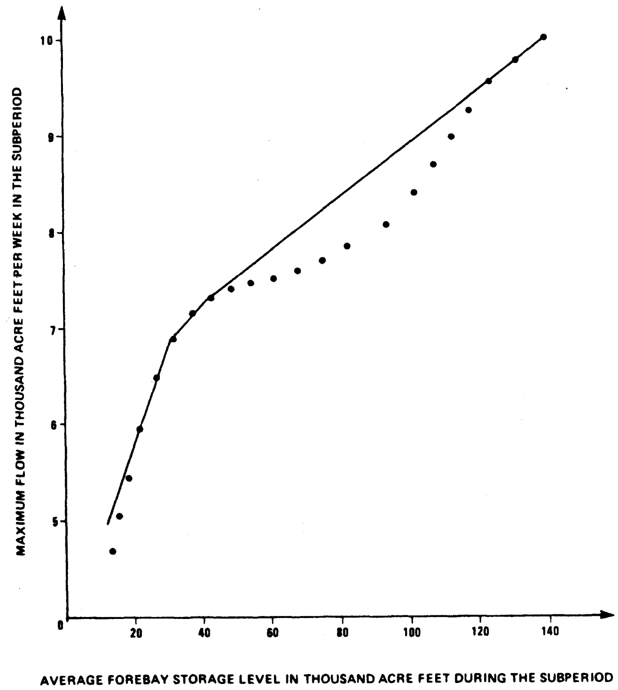

Fig. 6. Maximum flow and its piece-wise linear bounds as a function of average forebay storage level.

from the storage level at the beginning and the end of the subperiod. The constants $\alpha_{i\ell}$ and $\beta_{i\ell}$ are the slope and the offset, respectively, of each line segment. Note that when $L_i = 1$ and $\alpha_{i1} = 0$, this corresponds to the case of a fixed upper limit imposed on the flow.

Storage Index Decree

A storage index decree constraint is imposed on the combined amount of storage of a specified subset of reservoirs. For a subset $S' \subset R$ of reservoirs at which a storage decree is imposed, a minimum d^t on the storage level is specified for each subperiod t. Thus a storage index decree gives rise to the constraint:

$$\sum_{v_i \epsilon S'} s^t_i \geq d^t \tag{8}$$

Forced Spill Condition

Spillage, which is the uncontrollable flow of water, takes place to ensure that the reservoir capacity is not exceeded. At a reservoir $v_i \epsilon R$, let $x^t_{iq_i}$ denote the flow on the spillway. The case of no spillage corresponds to the condition:

$$\text{if } s^t_i < z^t_i \ , \ x^t_{iq_i} = \ell^t_{iq_i} . \tag{9}$$

The spillage condition, on the other hand, is given by

$$\text{if } s^t_i = z^t_i \ , \ x^t_{iq_i} \geq \ell^t_{iq_i} . \tag{10}$$

Note that (9) and (10) may be combined into a single equality constraint. Thus spillage at a reservoir v_i gives rise to the constraint:

$$(z^t_i - s^t_i)(x^t_{iq_i} - \ell^t_{iq_i}) = 0 \tag{11}$$

We denote by R' the subset of reservoirs where the relation (11) must be satisfied.

Objective Function

The energy generated by a hydro unit depends on the head and the flow through the power house. An example of such a relation is shown in Fig. 4. Let node v_i correspond to a power house and let its forebay be associated with node v_{j_i}. The energy E_i^t generated in subperiod t is a nonlinear function of the instantaneous flow and head. Since the subperiod is the smallest time unit considered, it is customary to represent the energy generated as a function of the average head in subperiod t and the water flow $x^t_{j_i i}$. Here, we again assume that the tailwater elevation remains constant. The average head is expressed in terms of the average storage level $1/2(s^{t-1}_{j_i} + s^t_{j_i})$ of forebay v_{j_i},[12]. Hence E_i^t is expressed as a function:

$$E_i^t = \tilde{f}_i(x^t_{j_i i}, s_i^{t-1}, s_i^t) \tag{12}$$

In a power system the value of the energy generated by a hydro plant may vary from plant to plant for reasons such as transmission losses and unit capacity factors. The energy E_i^t generated is then assigned to the value $V_i^t[E_i^t]$, which is the relative worth of energy generated at power plant v_i in subperiod t. The value $V_i^t[E_i^t]$ is expressed as a function of $x^t_{j_i i}$, s_i^{t-1} and s_i^t:

$$V_i^t[E_i^t] = V_i^t[\tilde{f}_i(x^t_{j_i i}, s_i^{t-1}, s_i^t)] \triangleq f_i^t(x^t_{j_i i}, s_i^{t-1}, s_i^t) \tag{13}$$

The objective of the hydro scheduling problem is to maximize the total value of energy generated in all the power houses in the study period. The objective function of the hydro scheduling problem is thus given by

$$f(\underline{x},\underline{s}) \triangleq \sum_{t=1}^{T} \sum_{i\in P} f_i^t(x^t_{j_i i}, s_i^{t-1}, s_i^t)$$

Hydro Scheduling Problem Formulation

We can formulate the hydro scheduling problem in terms of the flow vector $\underline{x}$ and storage level $\underline{s}$:

$$\text{(H): Maximize } f(\underline{x},\underline{s}) = \sum_{t=1}^{T} \sum_{i\in P} f_i^t(x^t_{j_i i}, s_i^{t-1}, s_i^t) \tag{14}$$

subject to

$$A\underline{x} + B\underline{s} = \underline{b} \tag{15}$$

$$\underline{\ell} \le \underline{x} \le \underline{u} \tag{16}$$

$$\underline{w} \le \underline{s} \le \underline{z} \tag{17}$$

$$D\underline{x} + F\underline{s} \le \underline{r} \tag{18}$$

$$(z_i^t - s_i^t)(x^t_{iq_i} - \ell^t_{iq_i}) = 0 \quad \text{for each } v_i \in R', \; t = 1,\ldots,T \tag{19}$$

In the problem formulation (H) we have expressed the water flow balance relations (1)-(4) as Eq. (15). The upper and lower bounds on flows of Eq. (5) are given in vector form by Eq. (16). Similarly, the upper and lower bounds on the storage levels stated in Eq. (6) are restated in vector form in Eq. (17). The variable flow bounds given in Eq. (7) and the storage index decree given in Eq. (8) can be combined as shown in Eq. (18). The construction of the matrices A, B, D, and F is straightforward. This is given in Appendix A.

Characteristics of Problem (H)

We have formulated the h.s.s.p. as a nonlinear programming problem. The objective function $f(\underline{x},\underline{s})$ is nonlinear and continuously differentiable since without any loss of generality, it may be assumed that the functions $f_i^t(\underline{x},\underline{s})$ are continuously differentiable. However, f may not be concave.

The constraints of the h.s.s.p. have a number of noteworthy characteristics. All the constraints with the exception of those in (19) representing the forced spill conditions are linear. The constraint matrices A,B,D,F are sparse. The matrices A and B are essentially incidence matrices [1]. Each column of A or B contains exactly two non-zero entries 1 and -1. In each row of D and F which corresponds to a variable flow bound constraint, there are at most three non-zero entries. For a storage index decree, the corresponding row in D has only zeros.

The number of rows in A or B is much larger than that in D or F. This is true in general because the number of power houses in the hydro system is much smaller than the number of total hydro facilities.

The feasible set $\Omega^{(H)}$ of problem (H) is defined and its characterization is given in Appendix B. A feature of note is that the constraints (19) representing the forced spill conditions may cause the set $\Omega^{(H)}$ to be nonconvex.

SOLUTION METHODOLOGY

For systems of practical interest, the mathematical program (H) for hydro scheduling is a large-scale optimization problem. Consider the task of determining the weekly schedules for a one-year study period for a hydro system. For a system with ten reservoirs and five power houses, the formulation (H) may have as many as 2500 decision variables, 2000 linear equalities, 5000 fixed bound linear inequalities, an additional 500 linear inequalities and 300 nonlinear equalities representing the forced spill conditions. In addition, the objective function is nonlinear. Thus, the determination of the weekly hydro schedules entails the maximization of a nonlinear function subject to a large number of linear and nonlinear constraints.

The major difficulty in solving problem (H) is the presence of the nonlinear constraints (19). The discussion in Appendix B points out that these constraints may cause the feasible set $\Omega^{(H)}$ to be nonconvex. Standard nonlinear programming techniques are ineffective in treating such constraints because they cause the feasible set to be non-smooth. The standard algorithms could easily jam at some point which is not a local optimum and consequently will be unable to solve problem (H).

A possible way to handle the constraints of Eq. (19) is outlined in Appendix B. This approach converts the continuous programming problem (H) into a mixed integer program by introducing a binary variable for each forced spill condition. However, for a large river system, the resulting nonlinear mixed integer program becomes very large. The currently available optimization techniques for mixed programs would require excessive computational resources. Such techniques would be unable to solve the problem for a large river system within reasonable time on today's computers.

Given the difficulty or intractability of solving problem (H) with available techniques, we developed an alternate approach. We propose a method based on the relaxation of the constraints (19) in (H). The basic idea is to convert problem (H) into a nonlinear program with only linear constraints by appending the equalities in (19) to the objective function as a penalty term via the exact penalty method [6]. The problem in the converted form is considerably easier to solve. In addition, it can be shown that a solution of the reformulated problem is also a solution of problem (H) under certain conditions.

Reformulation of Problem (H)

Consider the problem

(R): Maximize $h(\lambda;\underline{x},\underline{s}) = f(\underline{x},\underline{s}) - \lambda p(\underline{x},\underline{s})$ (20)

subject to

$$A\underline{x} + B\underline{s} = \underline{b} \tag{15}$$

$$\underline{\ell} \le \underline{x} \le \underline{u} \tag{16}$$

$$\underline{w} \le \underline{s} \le \underline{z} \tag{17}$$

$$D\underline{x} + F\underline{s} \le \underline{r} \tag{18}$$

where

$$p(\underline{x},\underline{s}) = \sum_{t=1}^{T} \sum_{i\in R'} (z_i^t - s_i^t)(x_{iq_i}^t - \ell_{iq_i}^t) \tag{21}$$

Here the original objective function of (H) is modified by the penalty term p in (21) representing violations of the forced spill conditions. All the other constaints remain identical. The basic idea in selecting a value for the penalty coefficient λ is to ensure that a (local) solution of (R) is a local optimum of (H).

A basic result in nonlinear programming state that under certain conditions and for sufficiently large λ, a local optimum of (R) is also a local optimum of (H). In fact, our numerical work has shown that we can always find a value λ to ensure that the solution of (R) is locally optimal for (H) when problem (H) is feasible, or is nearly feasible for (H) when problem (H) is infeasible.

Now, problem (R) is considerably easier to solve than problem (H) since all the constraints are linear. The function h has similar characteristics to the objective function f: it is continuously differentiable and may not be concave. Due to the nonconcavity of the objective function, a solution of (R) corresponds to a local maximum of problem (R), and an optimization algorithm may not be able to determine the global maximum of (R). Moreover, formulation (R) has certain structural characteristics that can be exploited to develop a computationally efficient solution scheme. Recall that the coefficient matrices of the constraints (15)-(18) are all highly sparse: only a few non-zero entries per column whereas the number of rows may be in the thousands. The proposed solution scheme takes advantage of this sparsity.

An additional important feature of (R) is the fact that it may be viewed as a network flow problem, with nonlinear objective functions and additional linear side constraints. We make use of this characteristic to determine a starting point for the solution algorithm.

Determination of a Starting Point

The computation time for solving a nonlinear program such as (R) is highly dependent on the starting point $(\underline{x}^o,\underline{s}^o)$. A considerable reduction in the overall solution time may be obtained if a "good" starting point, i.e., one that is nearly feasible for problem (R), is available. We present next a scheme for determining such a starting point. The basic idea is to find a point $(\underline{x}^o,\underline{s}^o)$ that satisfies most of the constraints of problem (R) and that maximizes an approximation of the objective function (20). To do this, the proposed scheme casts problem (R) into a network flow framework.

We introduce a scheduling network $G^* = (N^*,A^*)$ which spans over the time horizon of T subperiods and is based on the topological hydro network G. For each subperiod t, we define a node set

$$N^t \triangleq \{v_1^t,\ldots,v_n^t\} \tag{22}$$

to be an exact copy of the node set N with the index t, $t=1,\ldots,T$. Next, we introduce the super sink node v^*, where $v^* \notin N^t$, $t=1,\ldots,T$. We define an arc set

$$A^t \triangleq \{(v_i^t, v_j^{t+\tau_{ij}}) \mid (v_i,v_j)\in A,\ t=1,\ldots,T;$$

$$\text{use the convention that } v_j^{t+\tau_{ij}} = v^* \text{ if } t+\tau_{ij} > T\} \tag{23}$$

The set A^t is derived from the arc set A by introducing the necessary modifications to take into account the travel times between two connected hydro facilities. Note that whenever the travel times are negligible, A^t and A are identical. From each fictitious sink node v_n^t, we introduce a fictitious arc (v_n^t,v^*), $t=1,\ldots,T$. Let

$$A_n \triangleq \{(v_n^t,v^*) \mid t=1,\ldots,T\} \tag{24}$$

The water stored from one subperiod to the next in each reservoir node $v_i \in R$ is represented by the flow along the arc (v_i^t,v_i^{t+1}) for $t=1,\ldots,T-1$, and along the arc (v_i^T,v^*) for the final subperiod. Let

$$A_s \triangleq \{(v_i^t,v_i^{t+1}) \mid v_i\in R, t=1,\ldots,T-1\} \cup \{(v_i^T,v^*) \mid v_i \in R\} \tag{25}$$

The scheduling network $G^* = (N^*,A^*)$ is defined by

$$N^* = \bigcup_{t=1}^{T} N^t \cup \{v^*\} \tag{26}$$

$$A^* = \bigcup_{t=1}^{T} A^t \cup A_n \cup A_s \tag{27}$$

We use the scheduling network G^* as the framework within which we represent all the water flows in the hydro system. By definition, at each node of the network G^*, the conservation of flow holds. This means that Eq. (15) is satisfied. The water flow x_{ij}^t from facility i to facility j is represented by the flow along the arc $(v_i^t,v_j^{t+\tau_{ij}})$ of A^*. Note that his flow is constrained by the lower and upper bounds ℓ_{ij}^t and u_{ij}^t, respectively, and thus the constraints in (16) are satisfied. The storage level s_i^t at the end of subperiod t in reservior v_i is represented by the flow on the arc (v_i^t,v_i^{t+1}) of A^*. Note that for $t=T$, $v_i^{T+1} = v^*$. This flow has lower and upper bounds of w_i^t and z_i^t, respectively. In this way, the constraints in (17) are satisfied. It follows from this discussion that a feasible flow on the scheduling network G^* corresponds to a vector $(\underline{x},\underline{s})$ that satisfies constraints (15)-(17). Furthermore, a feasible solution of (R) corresponds to a feasible flow on G^* that satisfies the linear side constraints (18). Hence, we may view problem (R) as a network flow problem on the scheduling network G^* with additional linear side constraints (18) and nonlinear objective function (20).

Clearly, a feasible flow on G^* would constitute an acceptable starting point for (R) since all but the constraints in (18) are satisfied. Note that this starting point is "nearly feasible" since the number of constraints in (18) is considerably smaller than those in Eqs. (15)-(17), so that most of the constraints of (R) are satisfied. However, a still better starting point is possible by optimizing an approximation to the objective function of (R). Consider the objective function with the penalty term neglected, i.e., λ is assumed to be zero, $h(0;\underline{x},\underline{s})$. A first order Taylor series expansion about $(\underline{x},\underline{s}) = (\underline{\ell},\underline{w})$ yields

$$h(0;\underline{x},\underline{s}) - h(0;\underline{\ell},\underline{w}) \cong \left.\frac{\partial h}{\partial \underline{x}}\right|_{(\underline{\ell},\underline{w})} \underline{x} + \left.\frac{\partial h}{\partial \underline{s}}\right|_{(\underline{\ell},\underline{w})} \underline{s} \quad (28)$$

Recalling the definition of f in eq. (14), we may rewrite (28) as

$$\cong g(\underline{x},\underline{s}) \triangleq \sum_{t=1}^{T}\left\{\sum_{v_i \in P} c_i^t x_{j_i}^t + \sum_{v_i \in R} d_i^t s_i^t\right\} \quad (29)$$

where c_i^t and d_i^t are the derivatives of the objective function $h(0;\underline{x},\underline{s})$ with respect to $\underline{x}$ and $\underline{s}$, respectively, at $(\underline{\ell},\underline{w})$. We use the solution of the linear network flow problem

$$\text{(N): Maximize } g(\underline{x},\underline{s}) = \sum_{t=1}^{T}\left\{\sum_{v_i \in P} c_i^t x_{j_i}^t + \sum_{v_i \in R} d_i^t s_i^t\right\} \quad (29)$$

subject to

$$A\underline{x} + B\underline{s} = \underline{b} \quad (15)$$

$$\underline{\ell} \leq \underline{x} \leq \underline{u} \quad (16)$$

$$\underline{w} \leq \underline{s} \leq \underline{z} \quad (17)$$

as the starting point for (R). This problem may be solved rapidly using a primal network simplex algorithm [9]. Thus, a starting point for (R) can be computed with little effort.

The Solution Algorithm

The proposed solution scheme has three major procedures:

INIT: determine the starting point $(\underline{x}^o,\underline{s}^o)$

SOLVE: solve problem (R)

LAMBDA: check for feasibility of solution for problem (H) and updating scheme for parameter λ

The information flow in these three procedures is shown in Fig. 7.

In the actual computer implementation we utilize two state-of-the-art optimization routines for the INIT and SOLVE procedures. We use the NETFLO [9] program for the INIT procedure to solve the linear network flow problem (N). The primal network simplex method of this package produces the optimal solution very rapidly by using efficient data structures for the network repre-

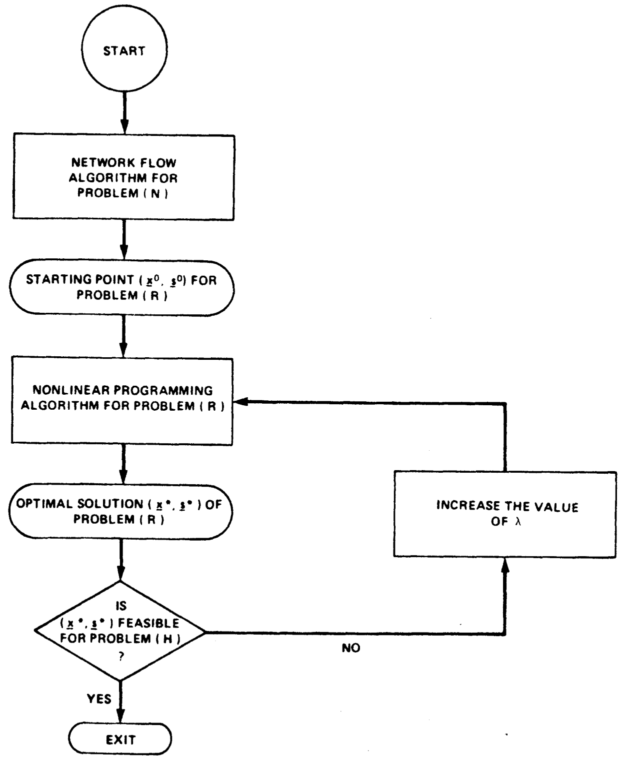

Fig. 7. The information flow among procedures of the proposed solution algorithm.

sentation. The solution is used as the starting point for solving the nonlinear program (R).

The MINOS [5] optimization package is used for solving problem (R). The quasi-Newton scheme of MINOS is very efficient for handling the linear constraints and nonlinear cost function of (R). The MINOS algorithm takes full advantage of the sparsity of the constraints.

The phase of the algorithm concerned with the choice of λ is based on a number of considerations. Clearly, the proper value of λ depends on the objective function $f(\underline{x},\underline{s})$ of problem (H). In addition, the value of $p(\underline{x},\underline{s})$ is dependent on the scaling of the decision variables $\underline{x}$ and $\underline{s}$. Thus, an important consideration is that the decision variables $(\underline{x},\underline{s})$ and the objective function $f(\underline{x},\underline{s})$ must be scaled appropriately. This is also required to ensure good accuracy and numerical stability in the optimization algorithm. The scaling of the decision variables is based on the equality constraint (15). In order to avoid numerical overflows, we define new variables $(\underline{x}',\underline{s}')$ and new vector $\underline{b}'$ to be a scaling factor multiples of $(\underline{x},\underline{s})$ and $\underline{b}$, respectively. The scaling factor depends on system characteristics, such as the magnitude of inflows. An additional scaling is introduced in the objective function f. This second scaling factor multiplies the objective function $f(\underline{x}',\underline{s}')$. Again, this scaling factor's value is dependent on system characteristics, such as the magnitude of total energy generation.

The scaling operations may be viewed essentially to be equivalent to the selection of the units for the decision variables and the objective function. Based on extensive experimentation, we determined that for the river systems of interest appropriate values of the scaling factor for the decision variables is to have the elements of $\underline{b}'$ expressed in 10 acre feet for weekly scheduling and in 100 acre feet for monthly scheduling. For the objective function, the scaling factor is chosen so that one unit of the value of the energy generation corresponds to the value of ten megawatt hours of energy.

The scaling operations have the effect of bounding the range of values for λ leading to a locally optimum solution of (H). Under the usual set of assumptions made, we know that for sufficiently large λ a local optimum $(\underline{x}^*,\underline{s}^*)$ of (R) is also a local optimum of (H) if $(\underline{x}^*,\underline{s}^*)$ is feasible for (H). Thus to find a local optimum of (H), the iterative process depicted in Fig. 7 is used. The process starts with an initial value of λ. Our experiments indicated that $\lambda = 0.01$ or $\lambda = 0.1$ is a good initial value. This value is increased by multiplying it with a constant factor in each iteration until a value of λ is reached for which the solution $(\underline{x}^*,\underline{s}^*)$ of (R) is feasible for (H). The multiplicative factor for λ is chosen to be 10. With this choice of parameters, a proper value of λ to obtain a local optimum of (H) was found in at most two iterations for virtually all cases.

COMPUTATIONAL RESULTS

We have implemented the proposed solution technique in the HYdro System Scheduler (HYSS) software package. HYSS is a user friendly interactive tool with fast response time. Typical applications of HYSS are:

-- hydro energy generation and capacity forecasts

-- operational decision making for determining the amounts of drafts through the power houses, setting end-of-the-year storage levels of the reservoirs, planning maintenance schedules and scenario simulation for various inflow forecasts

-- planning studies such as cost benefit analysis of planned power plants including various planning alternatives, and fuel acquisition planning.

We illustrate the application of HYSS by presenting numerical results of monthly and weekly scheduling on three river systems. They have drainage areas ranging from three hundred square miles to five thousand square miles. Table I presents the characteristics of each river system. In addition, the characteristics of the corresponding network G are given.

The sizes of problem (H) for the monthly scheduling of the three river systems are given in Table II. The study period is one year.

Table III presents the corresponding information for the weekly hydro scheduling of the test river systems. The study period for each system is given in the second solumn of the table.

Computation times for solving these monthly and weekly scheduling problems are given in Table IV. All CPU times are on the NAS 9050 computer. The solution times in Table IV are for the final values of λ, i.e., a single iteration for the solution of the proper value of λ is needed. The values are in the interval [0.1, 1.0]. For comparison purposes, the computation times without going through the INIT procedure and using only the SOLVE procedure with $(\underline{x}^o,\underline{s}^o) = (\underline{\ell},\underline{w})$, are given. The processing times for input data generation and output report writing are excluded.

These results point out the usefulness of the INIT procedure. We can speed up the total solution time by a factor of two to three when the initial solution point is given by the INIT procedure.

Table I: Characteristics of Test Systems

River System	Hydro Facilities		Characteristic of Network G				
	Reservoirs	Power Houses	Source Nodes	Sink Nodes	Intermediate Nodes	Total Nodes	Arcs
a	10	5	11	3	30	44	60
b	9	8	11	1	18	30	40
c	13	12	14	5	44	63	86

Table II: The sizes of problem (H) for the monthly scheduling of the test river systems

River System	Number of Decision Variables			Number of Constraints			Number of Fixed Bounds		Number of Forced Spill Conditions (19)
	$\underline{x}$	$\underline{s}$	Sum	Eq. (15)	Eq. (18)	Sum	Eq. (16)	Eq. (17)	
a	680	120	800	528	24	552	377	240	48
b	480	108	588	360	12	372	265	216	24
c	997	156	1153	756	60	816	496	312	51

Table III: The sizes of problem (H) for the weekly scheduling of the test river system

River System	T	Number of Decision Variables			Number of Constraints			Number of Fixed Bounds		Number of Forced Spill Conditions (19)
		$\underline{x}$	$\underline{s}$	Sum	Eq. (15)	Eq. (18)	Sum	Eq. (16)	Eq. (17)	
a	50	2854	500	3354	2200	100	2300	1587	1000	186
b	33	1320	297	1617	990	33	1023	738	594	66
c	36	2998	468	3464	2268	180	2448	1492	936	116

Table IV: Computation times for monthly and weekly scheduling

Problem Type	River System	CPU Times (Seconds)			CPU Time for SOLVE only (without INIT solution)
		INIT	SOLVE	TOTAL	
Monthly	a	0.9	3.5	4.4	8.9
	b	0.6	1.5	2.1	4.8
	c	2.1	6.9	8.0	14.1
Weekly	a	6.8	53.6	60.4	127.9
	b	2.8	6.8	9.6	32.9
	c	13.8	46.5	60.3	120.7

CONCLUSION

We have presented a very general framework for the formulation and solution of the h.s.s.p. The framework allows for the incorporation of virtually all types of constraints that are imposed in the planning and the actual operation of hydro systems. A noteworthy feature of the problem formulation given in the paper is the explicit representation of the forced spill conditions. The development and major characteristics of the computationally efficient solution methodology are described. Typical numerical results for actual river systems illustrate the capabilities of the proposed solution scheme. In general, for the river systems of interest, the monthly scheduling problems for a one-year study period can be solved within ten seconds. The weekly scheduling problem for a one-year study period can be solved in less than three minutes.

ACKNOWLEDGMENTS

We thank Ed Horciza, Gary Freeman, David Ward and Ted Taylor of PG&E's Hydro Generation Department for their continuous encouragement and support of the work reported here. Harriet Sollod and Satenik Ambarian are responsible for the major part of the software development. We thank Dr. Paul Gribik and Gene Sand for their numerous comments and suggestions which were instrumental in clarifying many of the concepts presented in this paper.

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APPENDIX A: CONSTRUCTION OF MATRICES A,B,D AND F

We use the scheduling network $G^* = (N^*,A^*)$ defined in eqs. (26) and (27) to describe the entries of matrices A and B in eq. (15). Let $A = [\underline{a}_1,\ldots,\underline{a}_n] = [a_{ij}]$ and $B = [\underline{b}_1,\ldots,\underline{b}_m] = [b_{ij}]$. Each row of A and B corresponds to a node of N^*. The rows are ordered according to the numbering of the nodes in N^*. Each column of A corresponds to an arc of $\bigcup_{t=1}^{T} A^t UA_n$, a subset of A^*. Suppose a column $\underline{a}_j$ corresponds to the arc $(v_\ell^t, v_r^{t+\tau_{\ell r}})$ with the convention that $v_r^{t+\tau_{\ell r}}$ is v^* if $t+\tau_{\ell r} > T$. Then the components of $\underline{a}_j$ are given by

$$a_{ij} = \begin{cases} -1 \text{ , if row i corresponds to node } v_\ell^t \\ 1 \text{ , if row i corresponds to node } v_r^{t+\tau_{\ell r}} \\ 0 \text{ , otherwise} \end{cases}$$

The matrix B is similarly constructed. Each colum of B corresponds to an arc in A_s. Suppose a column $\underline{b}_j$ corresponds to the arc (v_ℓ^t, v_ℓ^{t+1}). Then, the components of $\underline{b}_j$ are given by

$$b_{ij} = \begin{cases} -1 \text{ , if row i corresponds to node } v_\ell^t \\ 1 \text{ , if row i corresponds to node } v_\ell^{t+1} \\ 0 \text{ , otherwise} \end{cases}$$

The matrices D and F in eq. (18) represent the constraints (7) and (8). Let $D = [\underline{d}_1,\ldots,\underline{d}_n] = [d_{ij}]$ and $F = [\underline{f}_1,\ldots,\underline{f}_m] = [f_{ij}]$. Each row of D and F corresponds to an inequality in eq. (7) or (8). Each column of D corresponds to an arc in $\bigcup_{t=1}^{T} A^t UA_n$. Suppose a column $\underline{d}_j$ corresponds to the arc $(v_\ell^t, v_r^{t+\tau_{\ell r}})$. Then, the only nonzero components of $\underline{d}_j$ is $d_{ij} = 1$ where row i corresponds to an inequality in eq. (7) for forebay v_ℓ and power house v_r. Note that for row i corresponding to eq. (8), $d_{ij} = 0$. Each column of F corresponds to an arc in A_s. Suppose a column $\underline{f}_j$ corresponds to an arc (v_ℓ^t, v_ℓ^{t+1}). Then, these are at most two nonzero components in $\underline{f}_j$. If row i corresponds to an inequality in (7) limiting the forebay v_ℓ associated with power house node v_p, then $f_{ij} = -d_{pq}$ where q denotes the piece-wise linear segment that bounds the flow through the power house. If row i corresponds to an inequality in (8) and $v_\ell \in S'$, then $f_{ij} = 1$.

APPENDIX B: CHARACTERIZATION OF THE FEASIBLE SET OF PROBLEM (H)

The feasible set $\Omega^{(H)}$ of problem (H) is defined by

$$\Omega^{(H)} \triangleq \{(\underline{x},\underline{s}) | (\underline{x},\underline{s}) \text{ satisfies eq. (14)-(19)}\}$$

Let Ω be a convex polytope defined as

$$\Omega \triangleq \{(\underline{x},\underline{s}) | (\underline{x},\underline{s}) \text{ satisfies eqs. (14)-(18)}\}$$

Note that $\Omega^{(H)}$ is a subset of Ω. For each $v_i \in R'$ and for $t = 1,\ldots,T$, let

$$\Psi_i^t \triangleq \Omega \cap \{(\underline{x},\underline{s}) | s_i^t = z_i^t\}$$

$$\Phi_i^t \triangleq \Omega \cap \{(\underline{x},\underline{s}) | x_{iq_i}^t = \ell_{iq_i}^t\}$$

Assuming the components of $\underline{u}$ and $\underline{z}$ are finite, the set Ω is a bounded polytope. Moreover, since z_i^t is the upper bound of s_i^t and $\ell_{iq_i}^t$ is the lower bound of $x_{iq_i}^t$, the sets Ψ_i^t and Φ_i^t are faces of the polytope Ω if $\Psi_i^t \neq \phi$ and $\Phi_i^t \neq \phi$, respectively. Thus, they are bounded polytopes themselves. For $v_i \in R'$ and t such that $\Psi_i^t \neq \phi$ and $\Phi_i^t \neq \phi$, a feasible point $(\underline{x},\underline{s})$ for problem (H) must be on a face Ψ_i^t or Φ_i^t of the polytope Ω. A possible situation in two dimensions is sketched in Fig. 8.

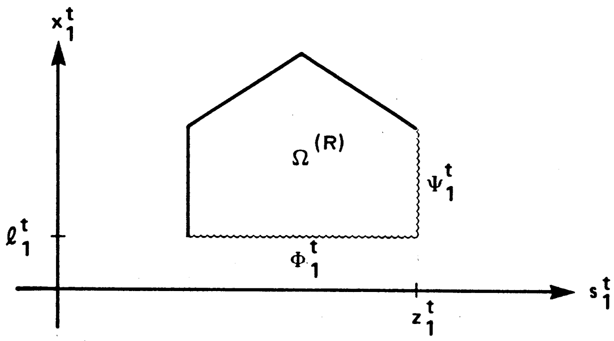

Fig. 8. The polytope faces Ψ_i^t and Φ_i^t for the case of Ω in two dimensions.

The feasible set $\Omega^{(H)}$ of problem (H) is expressed as

$$\Omega^{(H)} = \bigcap_{\substack{v_i \in R' \\ t=1,\ldots,T}} (\Psi_i^t \cup \Phi_i^t)$$

It follows that the set $\Omega^{(H)}$ is not empty if and only if for each $v_i \in R'$ and t, $\Psi_i^t \cup \Phi_i^t \neq \phi$. Moreover, if for some i and some t, $\Psi_i^t \neq \phi$ and $\Phi_i^t \neq \phi$, then the set $\Omega^{(H)}$ is not convex.

A standard technique [2] to deal with the nonconvexity of the feasible set due to conditions of the type given by Eq. (19) is to introduce a binary variable $\mu_i^t \in \{0,1\}$ to represent the binding constraint in each condition (19). In this approach, we set $\mu_i^t = 1$ iff $(\underline{x},\underline{s}) \in \Psi_i^t$, and $\mu_i^t = 0$ iff $(\underline{x},\underline{s}) \in \Phi_i^t$; we replace each condition (19) with the two linear inequalities:

$$s_i^t \geq \mu_i^t (z_i^t - w_i^t) + w_i^t$$

$$x_{iq_i}^t \leq \mu_i^t (u_{iq_i}^t - \ell_{iq_i}^t) + \ell_{iq_i}^t$$

In this way, the problem is changed into a mixed integer programming problem.

Yoshiro Ikura received B.S. in Physics and M. Eng. in Computer Science from University of Tokyo, Japan, in 1974 and 1976, respectively. He was awarded the degree of Ph.D. in Operations Research from Cornell University in 1981.

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Discussion

David Ford (Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, CA): Yoshiro Ikura and George Gross have described an interesting application of mathematical programming to a practical problem, and they have recognized the need for flexibility to model a broad range of physical, environmental, and legal constraints on system scheduling. The constraints they include in their formulation represent the important characteristics of and limitations on operation of most reservoir systems. The objective function they have chosen is appropriate for the hydroelectric planning studies for which the model is designed. Expansion of the objective function to incorporate the value of capacity for satisfying peak demands might make the model more useful for planning.

Previous Research

The authors have overlooked two successfully implemented solutions for similar hydro-system scheduling problems. Sigvaldason [4] formulated the scheduling problem as a pure network-flow programming problem with a linear approximation of the relationship of energy, head, and flow. In this formulation, the range of feasible storage volume and discharge values is subdivided, and appropriate unit benefits or costs are associated with values in each range. The convex objective function thus defined also allows specification of inter-reservoir operating rules for a multiple reservoir system. Specification of such rules is not addressed by the authors. For example, consider two reservoirs on parallel streams, either of which can release water to generate energy. The model proposed by the authors is indifferent regarding this decision, even if one of the reservoirs will be emptied in the process. This is generally unacceptable in practice. The second similar solution is that proposed by Martin [2]. Martin uses an iterative procedure analogous to the successive linear programming procedure of Palacios-Gomez [3]. The linear programming problem is solved at each iteration with the network-with-gains algorithm developed by Jensen [1]. This network algorithm allows accounting for evaporative losses; these losses are ignored by the authors but may, in fact, be significant in arid climates.

Forced Spill Condition

The need for emphasis on forced spills is not clear, and the author's proposed model of the spill seems unnecessarily complex. Practically if the storage, s^t_i, computed from Eq. (4) with maximum possible controlled release exceeds the reservoir capacity, z^t_i, *all* excess water (z^t_i-s^t_i) will be released to avoid jeopardizing the dam. This operation policy can be modeled more simply than proposed by the authors in Eq. (11). The total reservoir release can be defined as the sum of controlled and uncontrolled release, and a large penalty can be assigned to uncontrolled release. This should force the uncontrolled release to be zero unless the upper bound of Eq. (6) would be violated.

Determination of a Starting Point

The conclusion about the value of a "good" starting point is important. With a water-resources system model, a near-optimal solution often can be determined simply by applying a heuristic operating policy. This cost-saving step commonly is ignored, and the resulting search for an initial feasible solution requires significant effort.

Value of Model

Ikura and Gross present results of application of their proposed model for analysis of monthly and weekly operation of three river systems. Interpretation of the value of the model from a water resources engineering standpoint is difficult without additional detail. What is the physical layout of each system? What is the capacity of each hydropower plant? What is the release pattern determined by the model? Does it fluctuate rapidly in the weekly analyses? (Such fluctuation is a common problem with application of mathematical programming models of reservoir systems.) The writer would be interested in a comparison of energy generation with scheduling determined by the model and energy generation with scheduling determined with the existing operating procedure. What savings of cost or increase of profit is possible?

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Manuscript received January 4, 1984.

H. Habibollahzadeh and **J. Bubenko** (Royal Institute of Technology, Stockholm, Sweden): This paper considers Hydro System Scheduling with Forced Spill Condition. The special feature of this problem is its network structure. The authors use the NETFLO [9] that exploits this structure to obtain only an initial solution. The points that were noticed when reading the paper were as follows:

1. The constraint representing the forced spill condition is obtained from combination of Eq. (9) that corresponds to no spill condition, and Eq. (10) that corresponds to forced spill condition, in a multiplication manner. This results in a nonlinear constraint. Such a problem can be handled with the addition of one arc in the network representation such as Fig. 1. This figure considers reservoir i during hour t. The x^t_1 and x^{t+1}_1 are the contents of reservoir 1 at the beginning of hours t and t+1 respectively. Y^t_1 is the sum of all inflows to the reservoir, such as natural inflow and discharges from the upstream resevoir. There are two types of discharges considered in this figure: u^t_1, the discharge through the power plant that contributes to the power production, and s^t_1, the flow through the spillway that has no contribution to the power production. There are upper bounds on all arcs of this figure except the spillage, so as the x^{t+1}_i and u^t_i reach their maximum, the rest of the water is discharged through the arc representing the spillage, to keep the flow balance. Since the spillage has no contribution to the power production (it has zero coefficients in the objective function), it will be minimized in the optimization procedure. Refs. [14] and [16] have successfully used such a technique.
2. The penalty method used to handle the nonlinear constraints is a rather slow method. Application of such a method to a large scale hydro system such as Swedish State Power Board with 60 TWh yearly hydro energy production and up to 30,000 decision variables would require high CPU time.
3. The MINOS algorithm used takes only advantage of the sparsity of the constraint but application of Network Flow Algorithms such as in Refs. [14]-[16] will also exploit the network structure of the problem.

Considering points 1, 2 and 3, we would like to ask the authors if they have compared their method with full application of network flow algorithm, such as Refs. [14]-[16] and what is their judgment?

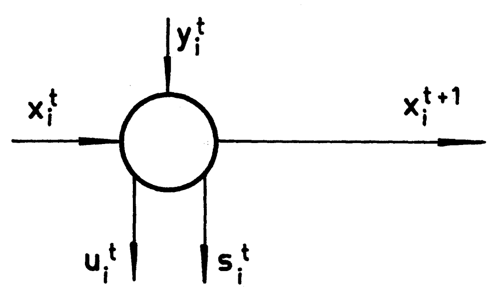

Fig. 1. Plant i during hour t.

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Manuscript received February 3, 1984.

Melville Hanscom (Institut de Recherche d'Hydro-Quebec, Varennes, Quebec, Canada): The authors should be commended for their superb application of linear network programming and linearly-constrained nonlinear programming to computationally efficient large-scale hydro system scheduling. A distinctive feature of their nonlinear programming approach is that it introduces the use of a superlinearly convergent algorithm. Furthermore, in contrast with [1], [2] and [3], the authors consider not only the linear equalities modeling the water flow conservation constraints but also linear inequalities and nonlinear equalities modeling the maximum flow through a plant and the forced-spill conditions respectively.

A couple of questions are in order, however:

1. According to the authors, one contribution of their work is that it explicitly considers forced-spill conditions. For such conditions, what are the advantages of the approach used in this paper compared with that described in [4]?
2. a) In the INIT procedure, did scaling result in substantial improvements in convergence speed?
   b) In the SOLVE procedure, have the authors considered the possibility of making that procedure more storage-efficient by using a truncated-Newton scheme [5] instead of a quasi-Newton scheme? The superlinearly convergent truncated-Newton scheme should be more storage-efficient because, when solving by conjugate-gradients the well-known reduced Newton equation

$$Z^T \nabla^2 f \, Zp = -Z^T \nabla f$$

where Z is the projection matrix of the reduced gradient method, $\nabla^2 f$ and ∇f are the Hessian and gradient of the objective function f and p is the Newton direction, it requires storage of the vector $Z^T \nabla^2 f \, Zp$ only, whereas the quasi-Newton scheme requires storage of a matrix approximating the inverse of the reduced Hessian $Z^T \nabla^2 f Z$. For details, see [4] and [5] for example.

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Manuscript received February 3, 1984.

Jerson Kelman, Leontina M. V. G. Pinto, and **Mario V. F. Pereira** (CEPEL, Rio de Janeiro, Brazil): We would like to congratulate the authors on a very interesting paper. Hydro system scheduling is a difficult large-scale and nonlinear problem, and the authors have made relevant contributions for its solution. There are some questions we would like to raise:

1) What is the percentage difference betweent the objective function evaluated at the starting point and the final solution? Is this difference due mainly to the non-linearity of the objective function or to the constraints (18) and (19)? What percentage of constraints type (18) were tight in the final solution?
2) Eq. (13) represents the worth of energy generated at a particular plant as a function of the plant state alone, that is, without considering the energy generated at other plants of the system. Can the authors comment on the criteria used to determine the relative worth of energy in different plants and different subperiods? Can the algorithm accommodate nonseparable objective functions?
3) In predominantly thermal systems, the total hydroenergetic production can be easily "absorbed," that is, can be used to save fuel in expensive thermal plants. However, in predominantly hydro systems, the total amount of hydro production can often exceed the total load. Since any amount of energy produced beyond this value is simply wasted, it is necessary to introduce load balance equations at each subperiod as well as a "target" state at the end of the study period, which will minimize future energy shortages. Do the authors believe that their algorithm could be used to solve this problem?
4) The natural inflows into each stream during each subperiod are assumed to be known. The algorithm is supposed to be applied to the scheduling of several weeks (there is one example of 50 weeks). We wonder how it is possible to have such a forecasting capability. Has any comparison been made between the solution obtained from the forecasted and the observed inflows?
5) Why is spillage only acceptable when the reservoir is full? Is it possible that some spillage from a reservoir would result in a net gain for the system capability of meeting the load, with due consideration to the downstream cascade of hydroplants? Why not penalize the spillage in the objective function regardless of the reservoir state, rather than use the nonlinear constraint (19)?
6) The afterbay water surface elevation is a function of the flow between the forebay and afterbay nodes and also of the eventual backwater effect due to downstream reservoir water surface elevation. Do the authors foresee any difficulty to take into account this last effect?

Manuscript received February 13, 1984.

Thomas M. Lekane TRACTIONEL S. A., Brussels, Belgium): The authors have to be commended for their very interesting and well written paper. This work is a good example of the capability of the state of the art optimization techniques for solving large-scale problems from the power industry.

We would appreciate it if the authors would clarify some points:

1. Could they describe how the benefit function is elaborated on? Is it computed by running a production cost model before solving the hydro problem or is it recomputed at every iteration of the solution algorithm? What kind of approach is used to approximate the non-differentiable objective function which results usually from a production cost evaluation?
2. The constraints (19) associated with the forced spill conditions imply that the feasible set $\Omega(H)$ of problem H is non-convex. Could the authors clarify why such a formulation of the forced spill conditions is required and why the adjunction of an arc representing the spill flow in the network of the system is not sufficient?
3. We developed several years ago a model for optimally scheduling the weekly operation of a multireservoir system hour by hour [1]. The problem was formulated as a large-scale nonlinear network program with a quadratic objective function. A reduced gradient type algorithm based on Murthagh and Saunders' work but taking advantage of the network structure of the constraint matrix was implemented. Since, the code has been extended to be able to consider a nonlinear objective function. It would be very interesting to know the improvement in the computation time that can be achieved by such a procedure over the authors' algorithm when the problem does not require the introduction of the constraints (7) (8).

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Manuscript received February 13, 1984.

J. Stedinger (U.S. Geol. Survey, Reston, VA): The authors are to be congratulated on their lucid discussion of hydropower optimization issues and their solution to the forced-spill problem. I have two questions and then a suggestion. First, could the authors please clarify how their NAS 9050 computer compares with other machines. Second, for sufficiently large λ, a local optimum of (R) is a feasible local optimum for (H). What is the penalty or reason for not initially choosing λ large, say 10? This should eliminate any need to iteratively solve (R).

My suggestion is that inspection of the structure of the reservoir system and the anticipated inflows may allow elimination of some forced-spill constraints, thereby simplifying solution of (R). Dr. Ikura indicated in personal communications that these constraints correspond to small reservoirs in the stream system's headwaters which cannot be controlled during the winter and early spring. I would expect that optimal policies have these small upstream reservoirs drawn down as far as possible in the late fall before their gates are closed for the winter. That being the case, from the given inflows one could explicitly calculate when the reservoirs would fill corresponding to $s^t_i = Z_i$, thereby allowing $X^t_{ij} \geq 0$. Before that time and while the gates were closed, X^t_{ij} must equal zero or some specified lower bound ℓ^t_{ij} reflecting the value of seepage and unregulated flows. Alternatively, if such a headwater reservoir has no controllable gates, then its entire sequence of releases X^t_{ij} can be predetermined external to the optimization models because the releases will depend only on the reservoir's capacity, the initial storage volume, and the subsequent series of natural uncontrolled inflows. These considerations should allow some forced-spill contraints to be eliminated allowing a more nearly optimal and feasible initial solution to (R) to be identified by the authors' efficient INIT network flow algorithm which did not include the forced-spill constraints.

Manuscript received February 16, 1984.

Claude R. Gagnon and **Richard A. Duncan** (Boeing Computer Services Company, Tukwila, WA): The paper presents a hydro scheduling problem formulation and a method of solution to this nonlinear programming problem. The maximum turbine discharge constraint is approximated by a piece-wise linear function. A nonlinear constraint for "forced spill" is treated as a penalty factor. The resulting problem has a nonlinear objective function with linear constraints and is solved using MINOS. A network flow algorithm is used to provide a good starting point for MINOS.

The need for the constraint (11) for "forced spill" evades us. Clearly, water will be spilled whenever the reservoir storage capacity is exceeded—this is covered by the content constraints (6). That water should not be wasted (unnecessarily spilled) is covered by the objective function. We see no need for any other constraint. In any case, the meaning of constraint (11) is not clear; its satisfaction by no means guarantees satisfaction of (9) and (10). Is there a typographical error in these?

The use of a network flow algorithm to provide a good starting point for MINOS is claimed to provide faster solution times. How much was gained by this?

Manuscript received February 21, 1984.

J. C. Dodu and **K. Ea** (Electricite de France, Direction des Etudes et Recherches, Clamart, France): In the paper by Messrs. Ikura and Gross, the description of the hydro system and the mathematical formulation of its scheduling were particularly clear and well presented. However, concerning the formulation, we feel that more precise details should be given on the following points:

1) The proposed method is a deterministic method. Moreover, according to the authors, it can be used, equally well, for daily as for annual scheduling. However, at annual level, it is essential to take into account the uncertainties, particularly those which affect the water inflows in the reservoirs. Is it not contradictory, in a mid-term scheduling, to ignore an important factor, such as the uncertainties, and, at the same time, to describe accurately the operation of the system (e.g., taking into account the variation in the maximum flow through a powerhouse as a function of the head, or taking into account the forced spill conditions).
2) In the optimal scheduling problem of a hydro system, it is classic to introduce end-point conditions: for example, to impose that the final storage level of each reservoir is equal to a given value; or to give an economic value to the water stock at the end of the study period. Would it not be necessary to complete the formulation with a condition of this type so as to compare more objectively the possible trajectories of the storage level of each reservoir?
3) The originality of this paper lies in the consideration of the forced spill conditions [11]. The presence of these conditions seriously complicates the problem since they are nonlinear and the domain that they define is nonconvex, whereas the other constraints formulated by the authors are linear. In our opinior, the two following situations may be considered: either the constraints [11] are often violated at the optimum of a problem analogous to problem (H) but without the forced spill conditions (i.e., the problem defined by the relations [14] to [18], with linear constraints and a nonlinear objective function): this proves that it is economically justified to spill when the reservoir storage level is not at its maximum. In this case, would it not be necessary to examine the advantages, from an economic viewpoint, of constructing installations allowing for this possibility? Or the conditions [11] are rarely violated: is it not possible, therefore, to neglect them, at least in the annual deterministic scheduling which does not require such a detailed description of the system as the daily scheduling?
4) An important point concerning the objective function to be maximized needs to be developed, since this will clarify, for the reader, the way in which both the problem of hydro generation facilities scheduling and the problem of thermal generation facilities and transmission system scheduling are separated: how do we obtain the functions $V^t_i[E^t_i]$, which give the value of the energy, E^t_i, produced by the plant v_i in the time interval t.

The resolution method proposed by the authors uses two methods mentioned in the references: a penalty method due to Han and Mangasarian, and an algorithm due to Murtagh and Saunders, allowing a mathematical program, with a nonlinear objective function and linear constraints, to be solved. It would be advantageous, for the reader, to know the broad lines of these methods and to know why the authors chose these, and not other methods, to solve their problem. We would also like the authors to reply to the two following questions:

1. The solution obtained by solving the flow problem (N) is not a feasible solution for problem (R). How, under these conditions, can this be used as a starting point to solve problem (R)?
2. The structure of the constraint matrix of (R) is particular, since the matrix [A,B] is a node-arc incidence matrix of a graph G^*. Is this property made use of in the MINOS program?

A structure similar to that of (R) can be seen in the weekly scheduling model of the reservoirs used at Electricite de France [14]. The objective function of this model is linear and, therefore, the problem to be resolved is a linear program. In this case, a partitioning method can be used, that is, the dividing of the basis matrix, M, of the linear program into four submatrices, one of which is nonsingular. This submatrix is also the node-arc incidence matrix of a forest (i.e., a set of trees). A system such that $Mx = a$ can then be solved without having to know all the elements of M^{-1}. All that needs to be done is to store in the computer memory an n_1 - vector indicating which of the arcs of G^* make up the forest, and a matrix of dimension $n_2 \times n_2$, taken from M^{-1}, n_1 denoting the number of rows of matrix A and n_2 denoting the number of rows of matrix D. Could a technique of this kind be applied in solving (R) with the help of the Murtagh and Saunders algorithm?

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Manuscript received February 21, 1984.

Walter O. Wunderlich, **James E. Giles**, and **Heinz-Dieter Waffel** TVA, Norris, TN): The authors present an interesting application of mathematical programming methods to the hydro system scheduling problem. Their use of standard mathematical programming packages (NETFLO and MINOS) to aid in the problem solution is a practice that should become widespread. Our concerns do not center, then, on the solution software but rather on terminology and problem formulation.

In the description of the hydro system the authors use some unusual terminology. For example, they state "a hydro unit usually has two associated reservoirs." Actually it may have one, two, or no associated reservoirs. The water elevation on the upstream side is usually called the headwater elevation, and on the downstream side, the tailwater elevation. In case of an impulse turbine, the lower level is the axis of the exiting water jet which is above tailwater level at all times so that it is of no import whether the tailwater is constant or not. The authors also say that "a river system with hydro generation facilities is a multipurpose development." All one can say is that it is a power development. Fig. 6 shows an unusually shaped maximum turbine discharge versus storage level curve. It is practically the same as Fig. 5 whose ordinate transforms into the ordinate of Fig. 6 after multiplication with the unit conversion factor 86,400x7/(43,560x1,000). The convex approximation used by the authors may correct the curve in the right direction.

The paper represents a deterministic scheduling model that is proposed for mid-term and short-term scheduling; typically, for one year of one week, respectively. Each of the study periods can be subdivided into subperiods, presumably weeks in the first case and days in the second. The inflow and minimum water demands are assumed to be known for each subperiod. It is not quite clear why the forced spill condition is so important. It seems that a minimum spill is made at those dams which take the flow out of the natural river channel and transfer it to a powerhouse or chain of powerhouses in another basin. It is not obvious why this "minimum spill" condition cannot be violated for augmenting hydropower benefits downstream.

The forced spill condition could alternately be implemented as a two-step linear preemptive priority constraint as part of a lexicographic linear goal programming formulation with $s^t_i \leqq {}^t_i$ as the first priority constraint and $x^t_{iq} = \ell^t_{iq}$ as the second priority constraint. We refer in this matter to an application of this approach for large-scale multiweek reservoir system scheduling using MPSX (Waffel, 1980). It appears that solutions to the reservoir system scheduling problem as formulated by the authors could exhibit erratic discharges from one subperiod to the next. For scheduling applications it may be necessary to restrict the variability of reservoir releases and elevations by additional constraints such as rate of change constraints on discharges and reservoir elevations or guidance over time. A typical reservoir schedule calculated by the authors' method would clarify this point.

The authors assign a value to each individual hydro unit and subperiod hydropower generation and add these values over all plants and subperiods of the study period to obtain the objective function. Since the hydropower value is described as being only a function of release and storage change at individual plants, it seems that the value of the total hydro generation to the power system is prespecified rather than computed directly as a function of load demands and the status of other non-hydro generating plants. This seems to be a simplifying assumption for a mixed hydrothermal system. We may refer in this matter to our own effort of establishing such a value-finding capability for a hydrothermal system that is based on the evaluation of the total power system (Giles and Wunderlich, 1983).

The objective function of Eq. (14) contains only the sum of values over all study periods. This function is incomplete unless it is used over very long study periods which are all simultaneously included in the optimization. Such an approach amounts to scheduling over the study period with foresight. This means that the model in its near-term schedule makes full use of information on inflows that may occue later in the study period, and vice versa. For example, if low flow occurs later in the period, the model will store water now and if high flow occurs later, it will dump water now. Unless such foresight is justified by reliable long-term forecasts, the schedules calculated may be invalid if the assumed flows do not materialize. All the model can do is to show what the best operation would be over the study period if such inputs occurred. In a planning situation, this can provide useful information on the use of the available system capability. For scheduling purposes, the study period length should be reduced to a length for which reliable forecasts are available. This could be from a few weeks to as short as one week for a weekly time step model or from perhaps a fortnight to not more than a few days for a daily time step model. This then requires the use of long-range guides at the end of these truncated study periods. These guides can be in the form of reservoir level targets or more elaborate value functions. In the latter case, the optimal policy over the shortened study period is found by maximizing the sum of the near-future value plus the expected long-range value. For more detail on this foresight problem we refer to another recent publication (Wunderlich and Giles, 1982) that deals with near-future and long-range scheduling. The long-range guides can be precomputed, whereupon a shortened study period can be scheduled. A weekly subperiod model that implements this approach for the TVA hydrothermal system by using only one week as the near future period has been recently completed and is described in Ref. 3.

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Manuscript received February 27, 1984.

Yoshiro Ikura and **George Gross:** We appreciate very much the interest of the discussers in our work. We thank them for their thoughtful and pertinent comments which add considerably to the value of the paper. Also, we welcome the opportunity to provide further clarification on a number of aspects of our work.

Before responding to the specific questions raised, we wish to shed some light on the background and objectives of the work reported here. This will, we believe, provide the reader with a better perspective on our work. To start with, our aim was to develop a very general solution methodology for the h.s.s.p.. The formulation of the h.s.s.p. adopted and the model developed are so general that virtually any hydro system considerations, constraints and policies of practical interest can be accommodated within the framework of this formulation and this model. Our proposed optimization technique has three important characteristics:

- is nonlinear and is capable of optimizing very general h.s.s.p.'s
- makes use of well-tested optimization techniques and software
- highly efficient since it exploits the structure of the h.s.s.p.

From the outset, our aim was to use well-validated optimization techniques and associated software which was reliable and well maintained. We arrived at the construction of the optimization algorithm through several stages. The early attempts of using linear programming—conventional network flow techniques for the h.s.s.p.—were not successful since the nonlinearities due to the head, variable flow constraints and especially, the forced spill condition could not be taken into account. After various additional attempts, we concluded that the MINOS package for optimizing a general nonlinear cost function with linear constraints was a good basis for the construction of a solution methodology. Furthermore, we made good use of the network structure of the h.s.s.p. and implemented the NETFLO package as part of the solution methodology. These two computationally efficient and well tested software packages have proved to be highly reliable. Within the framework of our h.s.s.p. formulation and solution methodology, we believe, the scheduling of water release for any river system can be accomplished efficiently.

We should also mention that our initial aim was to develop a deterministic solution methodology. The reason for this was that it seemed feasible to come up with a sufficiently fast optimizing scheduler to be used for multiple scenario analysis. While a stochastic scheduler which would be able to directly account for the uncertainty in hydrologic forecasts would be a higly desirable tool, the present state-of-the-art in stochastic optimization combined with the large scale nature of the h.s.s.p. would make such a solution scheme numerically intractable.

Next, we proceed with responding to the specific issues raised.

The Forced Spill Condition

Virtually every discussion raised some questions on the forced spill condition (f.s.c.), particularly, concerning its necessity, treatment and implications on the solution methodology. Some of the confusion stems from the fact that in the paper we failed to explicitly distinguish between forced and controllable spills. A controllable spill requires a controllable gate and a spillway. By definition, system operators have direct control over the timing and location of controllable spills. As several discussers correctly pointed out, for controllable spills to occur, the reservoirs need not be full. While unavailable for hydro generation at the powerhouse immediately downstream from the reservoir, controllable spills have direct economic benefit since they can be used for generation in the other downstream plants. Typically, an operator releases water from storage as a controllable spill in a subperiod in order to provide sufficient reservoir storage capacity for the anticipated peaking inflows in a future subperiod.

Forced spills, on the other hand, occur at each reservoir whose storage level reaches its upper limit. Any inflow into a reservoir that would increase the storage level beyond its maximum capacity results in a forced spill. Such a spill is due to the physical reservoir capacity limits and its occurrence is outside the control of system operators. The quantity of water in the forced spill is not available for generation at the powerhouse immediately downstream from the reservoir at which the forced spill occurs; however, it may be used for generation at powerhouses further downstream unless the associated reservoir storage levels are also at maximum capacity. For the efficient operation of a river system, operators schedule reservoir releases and storage levels so as to minimize the loss of generation due to forced spillage.

It follows that system operators can use controllable spills in an attempt to avoid forced spills in future subperiods. It is both necessary and thus important, to explicitly represent the f.s.c.'s in the scheduling of the river system.

In terms of the modeling of the f.s.c., relations (9) and (10) are imposed in addition to the constraint limits of the inequalities in (5) and (6). We use the quality constraint in Eq. (11) to represent the f.s.c. Under the assumption that the inequalities (5) and (6) hold, it can be shown that *the equality in (1) is equivalent to the relations (9) and (10).* This clarification is intended to do away with the misunderstanding of Messrs. Gagnon and Duncan concerning Eq. (11).

A number of discussers questioned the inclusion of Eq. (11) in the constraint set of the h.s.s.p. We must stress that the constraints of type (11) express the intertemporal effects in the scheduling network, i.e., the coupling across subperiods due to the timing of releases from storage and the possible occurrence of forced spills. It is precisely this coupling that makes critical the determination of water releases from storage before the peaking inflow season in order to avoid spills. As explained in the section 'Solution Methodology' equalities of the type of Eq. (11) introduce combinatorial complexity in the problem solution. We agree with the suggestion of Messrs. Habibollahzadeh, Bubenko, Ford, and Lekane that a *controllable release* may be treated through the introduction of an additional arc representing the path of the release bypassing the powerhouse immediately downstream. However, we believe that such "linear" treatment coupled with the introduction of a large penalty term for the forced spill flow is inadequate for handling the discrete phenomenon due to the f.s.c. Such an approach cannot lead to the enforcement of relations (9) and (10) or equivalently (11).

Consider the following situation. The multi-subperiod schedule of a river system with no controllable spillage facilities is to be determined. The optimal solution using our h.s.s.p. formulation is characterized by forced spill of x at some reservoir in a subperiod with peak inflows. Moreover, all downstream reservoirs have storage levels such that this forced spill must bypass all downstream powerhouses and consequently cannot be used for generation. We denote this to be solution A. Next, consider the formulation proposed in the various discussions: a penalty term is appended to the objective functions and additional arcs are introduced to represent spillage from the reservoirs. The optimal solution for their formulation is characterized by a controllable release x during a subperiod preceding that with peak inflows with the spill utilized at some of the downstream powerhouses. We denote this to be solution B.

At the solution B, there is a penalty cx in the objective function, where c is come large number. Now, the optimal solution A for our h.s.s.p. formulation is a feasible solution for this formulation. Solution A also introduces a penalty term cx to the objective function. Note that solution B results in more energy generation than solution A. On the other hand, solution B is *not* feasible for our h.s.s.p. formulation because the river system has no controllable spillage facilities. Thus, the formulation proposed by the discussers using additional arcs and a penalty term is inadequate for ensuring the satisfaction of relation (11), and hence may yield solutions that are physically not meaningful. We agree with Dr. Ford that the presence of Eq. (11) adds considerable complexity. However, we disagree that this complexity is "unnecessary", as he claims. The f.s.c.'s must be explicitly modeled to ensure a realistic representation of the physical system. Otherwise, if the approach proposed by Dr. Ford and the other discussers is followed, situations such as the one just described may occur.

Prof. Stedinger's suggestion to make use of the structure of the hydro system to reduce the number of f.s.c. constraints, is a reasonable scheme. Such a scheme is valid at the reservoirs located at the uppermost part (headwaters) of the stream and requires knowledge of the initial conditions. The latter constraint implies that for a multiple year scheduling study the Stedinger scheme would be useful only for the first year. On the other hand, for studies of duration shorter than one year, there may be benefits associated with the use of the Stedinger scheme. Note that the reduction in the number of decision variables entails additional data input and preprocessing calculations.

We are unable to comment on the use of the Waffel's lexicographic linear goal programming approach discussed by Messrs. Wunderlich, Giles and Waffel due to the unavailability of the reference cited. As to the Escudero work cited by Dr. Hanscom, a major difference is due to the fact that the Escudero formulation is in terms of a nondifferentiable objective function. From our reading of the Escudero paper, we did not see the explicit representation of the f.s.c. We did not investigate the issue of using a nondifferentiable function for the representation of the f.s.c.

With respect to the remarks on the f.s.c. in point 3 of the discussion by Messrs. Dodu and Ea, we have the following comments. The first situation considers the case where the optimum to the problem (H) without Eq. (11) frequently violates Eq. (11). We cannot conclude from this that the economics of installing facilities for controllable spillage needs to be investigated. The reason is that the problem solved by the discussers does not represent the physical system since the f.s.c.'s are ignored. Certainly, one can use our HYSS methodology to study the economics of such installations; however, the f.s.c.'s must be explicitly represented. On the other hand for the second situation in which conditions (11) are rarely violated, it may be permissible to neglect them under certain conditions. For general applications, the hydro scheduling tool should have the capability to solve a wide variety of situations including those where the constraints (11) are explicitly represented since the solution of the h.s.s.p. formulation with (11) and without (11) may or may not have the same optimal solution.

Uncertainty of Stream Flows

The discussion of Dodu and Ea, Kelman, Pinto and Pereira, and Wunderlich, Giles and Waffel showed considerable concern over the fact that the proposed methodology ignores the uncertainty inherent in the natural inflows. As we stated in the beginning of this closure, our goal from the outset was to develop a fast, deterministic scheduling tool, capable of analyzing a wide range of scenarios. We decided to stay away from stochastic optimization in view of the limitations of such techniques for dealing with large scale problems.

We agree with the discussers that it would be desirable to have a stochastic optimization-based scheduler which could take into account the uncertainty in the stream flows. While, in the deterministic framework we adopted, we cannot capture these stochastic effects directly, we developed the capability to make sensitivity studies very quickly to evaluate the impacts of changes in the forecasted inflows.

Corresponding to each assumed forecast, the optimal water release and storage strategy are determined. Through the analysis of these studies the sensitivity of the schedules with respect to changes in the forecasted natural inflows can be evaluated. One of the strengths of the HYSS methodology is its ability to represent in detail the hydro system characteristics. It is precisely this detailed representation that permits the evaluation of the sensitivity information in a meaningful framework. It follows that we find, in response to the question posed by Messrs. Dodu and Ea, absolutely no contradiction in taking into account nonlinear phenomena to develop a detailed deterministic model and ignoring the uncertainty in the natural inflows.

The proposed HYSS is a tool for "scheduling over the study period with foresight," in the words of Messrs. Wunderlich, Giles and Waffel, since the forecasted inflows are assumed to be known with 100% certainty. As the discussers have remarked, the longer the study horizon, the larger is the uncertainty. However, in actual use HYSS can deal effectively with the uncertainty. The reason for this is that in the actual application of HYSS, the forecast lead time is sequentially decreasing. For example, for a one-year schedule performed on a monthly basis, the schedule of the first month only is used. After the first month the updated forecast of inflows and the new initial conditions are used to schedule months 2-12. This process is repeated for each subsequent month. Note that for each subsequent subperiod, the study period is shorter and with that the forecast uncertainty decreases. This corrective strategy allows at each stage to account for deviations of actual vs. forecasted inflows.

In summary, we believe that HYSS's effectiveness as a scheduling tool for both shorter and longer (over one year) periods is not impaired by the fact that it is deterministic. The capability to perform sensitivity analysis plus the manner in which the tool is used are very effective in dealing with the inherent uncertainty on the natural inflows.

Objective Function

The majority of the questions concerning the objective function focused on its evaluation—in particular, how to determine the value $V_i^t[E_i^t]$ of the hydro energy E_i^t generated by plant v_i in subperiod t. In a hydro-thermal system the worth of subperiod energy generated by a hydro unit is a function of total system load, economics of the thermal units and the status of the thermal and the other hydro units. Thus the value of the energy generated by a particular hydro plant in subperiod t depends, in general, on the other hydro units in the same subperiod as well as in the other subperiods through factors such as unit planned outages. Hence, many factors come into play in the hydro-thermal coordination problem.

For the scheduling of the hydro system alone, we introduce a simplifying assumption to allow the evaluation of the objective function of Eq. (13) in a straightforward manner. We assume that the worth of energy generated by powerhouse v_j during subperiod t is *independent* of the status of the other hydro units in any subperiod of the scheduling period. This assumption fails to hold in general. However, in a system such as PG and E, the energy generated by any one powerhouse is a small fraction of the total energy generated by the entire supply system. Under these conditions such a simplifying assumption on unit independence is not unreasonable. This assumption implies that the subperiod energy generated by each powerhouse is expressed as a function of the release and the storage change during the subperiod.

The value associated with the subperiod energy generated by each powerhouse is obtained from a production costing-marginal costing study as follows. Given a set of specified hydro energy allocations for each subperiod in a study period, the production costing determines the exact loading order of all supply system units. The expected system marginal energy costs may then be evaluated at any load level. For evaluating Eq. (13) for the h.s.s.p. the subperiod marginal energy cost at the loading point of the hydro unit is used. However, there is an approximation involved since the subperiod energy generated by a hydro unit as determined from solving the h.s.s.p. may not equal the value specified for the allocated energy of that unit used to evaluate the marginal costs. In actuality, for a given subperiod, the loading of a hydro unit depends on the allocated energy of all the other hydro units, which, in turn, depend on the marginal costs. For a more precise evaluation one would have to iterate between the production costing and hydro scheduling phases until convergence.

Hydro thermal coordination requires a more detailed representation of the interaction between the hydro and thermal units. However, for hydro system scheduling, we believe that no iterations are necessary. If indeed a more detailed representation were required, then the production costing-marginal costing calculations would have to be carried out at each iteration of the optimization algorithm. In addition to the tremendous increase in computational resources, the incorporation of production costing into the objective function would give rise to many complications. Chief among these, as pointed out by Mr. Lekane, is that the objective function would become nondifferentiable. It is questionable whether the integration of a detailed probabilistic production costing into the HYSS deterministic optimization for solving the h.s.s.p. would at all be numerically tractable.

The discussion of Kelman, Pinto, and Pereira touched on the question of nonseparable objective function. Our nonlinear optimization scheme using the MINOS algorithm can handle any continuously differentiable nonlinear objective function including nonseparable functions.

Dr. Ford asked us to address the issue of specifying inter-reservoir operating rules for a multiple reservoir system within the framework proposed in the paper. We have used two approaches. In the first approach, to treat reservoirs distinctly in a multi-reservoir system, their storage level bounds are modified to reflect a specific preference. If the modified bounds lead to infeasibility, further readjustments are necessary. This is a relatively straightforward approach but requires a good "feel" for the system in order to effect the necessary modifications of the bounds.

The second approach for specifying a preference for the use of water from a certain reservoir in a multi-reservoir configuration is to introduce a small gain factor for the release from that reservoir. The aim in adding a term due to this gain factor to the objective function is to implement a release strategy which indicates preference for the reservoir associated with the gain factor. This approach is more complex than the first since a number of other factors may come into play. Adjustment of the gain factor may be necessary. Finally, a combination of these two approaches may also be used to specify inter-reservoir operating rules for a multi-reservoir system.

Modeling Isses

The various discussers commented and raised questions on a number of additional issues, all related to the modeling aspect of this work. We shall deal with these issues in this section.

We did not explicitly discuss the modeling of evaporative losses in the paper. This led Dr. Ford to believe that these losses are ignored. This is not the case since, in fact, we have run many cases with evaporation explicitly represented. Within our framework, the basic idea is to model the loss due to evaporation as an additional water demand. If the value of this loss is fixed, then an additional sink node and an arc form the reservoir where the evaporation occurs to this sink are introduced. The demand at the sink is set equal to the evaporative loss. If, on the other hand, the evaporative loss is expressed as a fraction of the reservoir storage, the loss is represented by introducing an additional arc from the reservoir node to a sink of type II. The flow on this arc is related to the storage level at the end of the subperiod. The latter is represented as an arc in the scheduling network from the present to the next subperiod. The flow along these arcs is constrained by a linear relationship to represent the fraction of water lost due to evaporation. For example, a loss of 1% of the reservoir contents in one subperiod results in constraining the flows along the two arcs to have a ratio of 1:99.

There were two questions on the modeling of additional effects within the objective function. Conceptually, there is no problem incorporating any additional effect within the objective function as long as the effect can be expressed as a function of the water releases and storage level decision variables. For example, Dr. Ford inquired whether the value of capacity to satisfy peak demand is one such effect. The answer is yes since this capacity is a function of the heads and the flows through the powerhouses. Dr. Kelman, Ms. Pinto, and Mr. Pereira asked whether the backwater effect due to the downstream reservoir water surface evaluation can be incorporated within the afterbay surface elevation. Again, if it is possible to obtain a good representation of this effect in terms of water releases and storage levels, we see no difficulty in accounting for it.

The issue of terminal effects was discussed by both Messrs. Dodu and Ea and messrs. Wunderlich, Giles, and Waffel. We included the end point conditions of the reservoirs as part of Eq. (17). The fixed bounds on the reservoir storage corresponding to the target level set by the system operators is specified at each reservoir in the final subperiod. Alternatively, we can account for the economic value of the final water stock by appending a term to the objective function. There is no conceptual difficulty in incorporating the value of the end conditions in the objective function. In practice, however, this evaluation may be very difficult.

In response to question 3 of the Kelman, Pinto and Pereira discussion, we see no difficulty in introducing additional constraint for predomoninantly hydro systems. The load balance equation can be accommodated as a side constraint.

Messrs. Wunderlich, Giles, and Waffel questioned the use of the term "multi-purpose development" for a river system. While the utility's principal objective is energy generation, the operation of the river system as a whole has a number of additional objectives: flood control, irrigation, environmental conservation, leisure and fishing. Chapter 4 of Ref. [12] describes the incorporation of multiple objectives in planning water resource systems. For our work we selected the energy generation as the objective and incorporated the other objectives within the constraints of the h.s.s.p.

Solution Methodology

Several of the discussers commented and raised questions concerning the solution scheme we developed for the h.s.s.p. As we stated in the beginning of this Closure, from the outset our aim was to cast the formulation of the h.s.s.p. into such a framework that existing optimization techniques could be efficiently adopted. We have already elaborated in an earlier paragraph on the reasons for the choice of the particular solution techniques. This explanation should provide the answer to Messrs. Dodu's and Ea's question as to the reason for the selection of these particular techniques over other methods.

The basis of our optimization scheme is the MINOS algorithm [5]. Messrs. Habibollahzadeh and Bubenko correctly pointed out that MINOS does not explicitly take advantage of the network flow structure of the h.s.s.p. formulation. The basic network flow constraints are expressed in terms of the coeffient matrices A and B in Eq. (15). In respnse to Messrs. Dodu and Ea, MINOS exploits fully the sparse structure of these matrices. Were it not for the presence of the side constraints in Eq. (18), a method that takes advantage of the network flow structure would undoubtedly be more efficient.

Mr. Lekane in his discussion explained how he used such an approach when the side constraints of Eqs. (7) and (8) or, equivalently of Eq. (18), are ignored. We expect that solution times are reduced with the incorporation of efficient network data structures for solving a nonlinear h.s.s.p. with pure network flow constraints. However, constraints such as those of Eqs. (7) and (8) and others not explicitly discussed in the paper, e.g., flows with gains and losses, fixed ratio of flows on two arcs, and variable lower bounds, play an important role in the h.s.s.p. formulation. As such, their representation as side constraints in the form in Eq. (18) cannot be ignored. It is unlikely that for the h.s.s.p. with the side constraints in (18), the incorporation of network data structures would, in fact, result in substantial improvement in solution times. While there is considerable amount of research ongoing in the development of solution schemes for the inclusion of side constraints in network flow problems (see, e.g., [14]) and the exploitation of the network flow structure within general nonlinear programming problems (e.g., [15]), we are not aware of the existence of a general purpose software in the public domain for the solution of nonlinear programming problems with network flow structure.

Considering the limitations of the state-of-the-art for the solution of problems such as the h.s.s.p., we developed an alternative strategy. We use the efficient network simplex method NETFLO [9] to solve the simplified problem (N) in which the side constraints of Eq. (18) are ignored. The solution serves as an effective starting point for solving the nonlinear problem (R) with MINOS. This strategy allows the direct incorporation of these two well-tested software packages bypassing the need to develop and implement special purpose optimization schemes. One important aspect of this solution strategy is the linkage of the NETFLO and MINOS programs in such a way that the optimal basis for problem (N), represented as a spanning tree in the NETFLO routine, when expressed in algebraic terms, becomes the starting basis in the MINOS algorithm.

Messrs. Dodu and Ea inquired about the direct incorporation of certain network data structures within our MINOS-based solution methodology. This is possible; however, the incorporation of such data structures is a rather major undertaking. In view of our remarks above that such techniques may not significantly reduce computational times, the appropriateness of expending efforts on the implementation of these data structures is questionable.

In his discussion, Dr. Hanscom proposed the use of the truncated Newton method [16] for the purpose of reducing storage requirements. In implementing our solution methodology, the availability of sufficient storage was not a problem. For the MINOS package, there is the capability of specifying an upper limit on the size of the reduced Hessian matrix so that the solution technique switches to conjugate gradient whenever the number of super basic variables exceeds this limit. For the problems for which HYSS is used, a limit of 50 is sufficient to preclude switching over to a conjugate gradient technique. By far, the most intense requirements on storage are due to the representation of the linear constraints in compressed form and the factorized form of the basis. These requirements exceed greatly those associated with the reduced Hessian array. However, for cases where the h.s.s.p. is highly nonlinear so that the number of variables appearing in the nonlinear component of the objective function is very large, the use of the truncated Newton may be more appropriate.

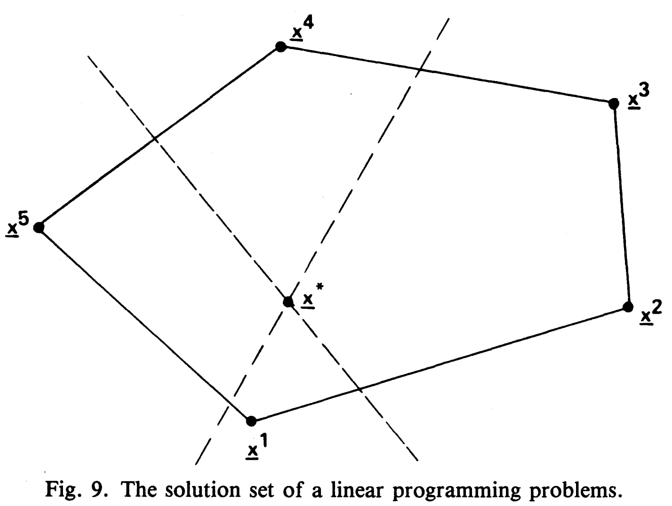

Fig. 9. The solution set of a linear programming problems.

Messrs. Ford, Wunderlich, Giles and Waffel raise the question whether the optimal solutions determined by HYSS exhibit erratic behavior across subperiods. Such behavior is possible because of the "extremal" nature of the solution technique. To illustrate the meaning of this statement, consider the solution of a linear programming problem by the simplex method. Since such a problem may have multiple optima, i.e., the objective function attains the same value at multiple points, the solution set corresponds to the convex hull of these points, i.e., a polytope. Fig. 9 gives an example of a problem whose solution is defined by five extremal points x^i, $i = 1, 2, \ldots, 5$. It may happen that the desired solution point is none of these extremal points but some interior point x^*. The simplex method, however, will find one of the x^i as the optimum. Such an optimum is characterized by certain extremal values in its components: e.g., in the h.s.s.p for a multi-subperiod optimization, the flow through a powerhouse may be at full capacity in certain subperiods and at minimum capacity at subsequent subperiods. One way to eliminate such fluctuating behavior in the optimal solution is through the introduction of additional constraints in the problem formulation. The addition of two constraints for the example in Fig. 9 would result in x^* as the external optimum of the modified feasible region. The simplex method may be used to determine x^*. Typically, the additional constraints required for this purpose are obtained by changing the upper and lower limits of variables.

H.s.s.p.'s, similar to the linear programming problem discussed above, may have multiple optimal solutions. These solutions may be characterized by the extremal values of their components. Consequently, it is possible to have fluctuating values across subperiods or between two cases when small perturbation sensitivity studies are made. In general, system operators wish to see solutions with a continuous pattern of discharge values. The application of the "cutting plane" approach outlined above with the HYSS can be successful in eliminating much of the erratic behavior in the optimal solutions due to extremal values. This corrective strategy is a post-optmization task that is carried out after careful analysis of the solution results.

Prof. Stedinger raised a question on the selection of an appropriate initial value for λ. The reason for the initial value assigned is to preclude the possibility of overflows or underflows in the optimization calculations. A large initial value of λ may result in numerical problems of this type. The value of λ must be judiciously chosen taking into account the ratio of the magnitude of the objective function $f(x,s)$ to that of the penalty term $p(x,s)$. The scaling described in the paper together with the initial value between 1.0 and 0.01 for λ serve to avoid possible over- or underflows. In response to the other question of Prof. Stedinger, all numerical results were computed on the (NAS/National Advanced Semiconductor) 9050 computer. This is a 9 MIPS (million instructions per second) machine that is roughly equivalent to the IBM 3081.

The Starting Point

The INIT step of the solution scheme used for determining the starting point (x°, s°) raised a number of questions. The starting point for solving problem (R) is computed using the NETFLO scheme to optimize the relaxed problem (N). As Messrs. Dodu and Ea remarked, this point is not, in general, feasible for problem (R). However, it is "almost" feasible for problem (R) when the number of side constraints in Eq. (18) is considerably smaller than the total number of constraints in Eqs. (15)-(17). This condition is nearly always met in cases of practical interest. Moreover, this starting point is a substantial improvement over the conventionally used starting point (l, w), the default option in the MINOS algorithm.

A good starting pont for solving problem (R) results in reducing the overall computational requirements. Messrs. Gagnon and Duncan requested some quantification of the gain attributable to the the use of the INIT step. The savings realized through the use of the INIT step is a function of the number of side constraints in Eq. (18). For cases where the side constraints of Eq. (18) constitute less than 5% of the set of linear constraints of Eqs. (15)-(18) of problem (R), we found that the ratio of the computational time with the INIT step to that without the INIT step is roughly 1:2. Thus, in cases of practical interest, the use of the INIT step results in halving the total computational time.

The discussion by Kelman, Pinto and Pereira, raised the question of how close is the value of the objective function at the starting point to that at the optimum. Numerical results indicate that, in general, at the starting point the objective function attained abut 95% of its value at the optimum. This is strong experimental evidence to support the claim that the solution obtained from the INIT step is "almost" optimal for problem (R). Typically, 20%-50% of the side constraints of Eq. (18) are tight at the optimal solution for problem (R).

Dr. Hanscom inquired about the effect of scaling in the INIT step. We have found that the procedure of the INIT step is virtually insensitive to scaling. By normalizing the h.s.s.p. formulation using a unit of flow in the reasonable range of 10-100 acre feet, any possibility for overflows is precluded.

One additional comment on the INIT step stems from our private communication with Dr. Hanscom. He proposed the sequential use of the INIT procedure to further reduce the total computational time to solve problem (R). Since the computations in the INIT step are very quick, the repeated sequence of linearizing and using the INIT procedure promises to be a viable approach for computing quickly the optimum of problem (R). Experimental testing is necessary to determine if the repeated use of the INIT procedure always results in computational savings and to quantify these savings.

Value of the HYSS Methodology

Dr. Ford asked us to quantify the benefits of the proposed model for the PGandE river systems. He also requested a more detailed description of the PGand E hydro network. The size of the PGandE hydro systems varies from a watershed covering a couple of hundred square mile area with several megawatt capacity to that covering a several thousand square mile area with over fifteen hundred megawatt capacity. Each system has its unique characteristics: some are almost a chain of run-of-river facilities, while others have reservoirs with large storage capacity (up to one million acre feet) and complex network configurations.

In use for over two years, the HYSS package has proved to be a very powerful tool for studying a number of important issues associated with the effective management of the large PGandE river network. In qualitative terms, some of the major benefits of the use of HYSS are: improvement in the accuracy and quality of schedules, capability to make more detailed analysis of alternatives, increased confidence in the operational planning studies, and a better handle on environmental aspects of hydro operations. For example, the forecasts by HYSS are valuable for specifying the power purchase contracts and planning the fuel inventory. Scenario analysis capability of HYSS is extensively used in cost-benefit analysis of hydro unit constructions. Based on the results from HYSS, it was decided to bring the Kerchkoff 2 hydro plant on-line six months ahead of schedule, resulting in large savings in production costs.

For quantitative evaluation of the savings, we performed a number of backcasting studies to compare the HYSS results using the historical natural inflows to actual measurements. These studies indicate that as much as a 10% increase in the value of the hydro energy could be realized under the condition of a 100% certain knowledge of the inflows. In general, our studies with HYSS indicate that its use may increase the overall hydro energy generation by 2-3%. In addition to the obvious economic benefits, this represents a significant contribution in improving the fuel security of the company due to the resulting reduction in foreign oil dependency.

We hope that we were able to address effectively the large number of issues brought up by the discussers. We again wish to express our appreciation for their interest in our work and our thanks for their efforts.

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