IEEE Transactions on Power Apparatus and Systems, Vol. PAS-101, No. 6 June 1982

A TWO-STEP COMPENSATION METHOD FOR SOLVING SHORT CIRCUIT PROBLEMS

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ABSTRACT

This paper reports on the development of a unified approach for the solution of all types of short circuit problems. The basic approach is to consider a fault condition -- a fault type or combination of fault types and associated line outages -- as modifications to the parameters of the branches of the prefault network. An extension, referred to as the two-step compensation method, of the conventional compensation scheme was developed to account for the balanced and unbalanced nature of the modifications that networks undergo when faulted. The computationally efficient solution scheme was derived by using a decomposition of the modifications according to their balanced and unbalanced nature and exploiting the structural properties of the short circuit problem, notably sparsity. The solution approach is particularly useful for system-wide studies, in which specified fault conditions at a set of specified fault locations are analyzed sequentially, on large systems. A noteworthy feature of the proposed methodology is the natural manner in which mutuals are handled. Results of the application of this approach to investigate a variety of fault conditions on several systems, including a 2278 bus network, are presented.

INTRODUCTION

The short circuit analysis problem involves the solution of large scale network equations when faults on realistic sized systems are simulated. In addition to size, certain features of the problem complicate the short circuit calculations. For example, the unbalanced nature of many fault types of interest requires that three-phase systems be solved and the representation of mutual couplings in the network introduces many complexities. Many short circuit analysis methods are based on the evaluation of the bus impedance matrix $\underline{Z}$ and are variants of the classical Z-bus method [1]. The calculations for more complicated fault types become very cumbersome when the Z-bus approach is used. Solution schemes to overcome such difficulties for fault types such as line-end faults and line-out faults have been developed [2,3]. Recently, work focused on the development of more generalized solution methods using diakoptic techniques [4,5]. A novel aspect of the approach in [4] is the partitioning of the network into one balanced and several unbalanced subnetworks followed by the application of a sequence of linear transformations. The technique in [5] uses optimally ordered triangular factorization [6] on the torn networks to obtain efficiency and an iterative scheme for treating mutuals. The improvements in the computational aspects of the Z-bus method approach proposed in [7] are an important contribution to the short circuit analysis area. The scheme in [7] is a computationally efficient technique for evaluating only those elements of $\underline{Z}$ that are required in the actual computation of the fault calculations.

81 SM 456-3 A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE PES Summer Meeting, Portland, Oregon, July 26-31, 1981. Manuscript submitted February 2, 1981; made available for printing June 9, 1981.

This paper presents a unified approach for the solution of short circuit problems. Any balanced or unbalanced fault types or a combination of them and associated protective actions can be analyzed within the general framework of the proposed solution methodology. The basic approach is to consider a fault condition -- a fault and the ensuing protective action(s) -- as a modification of the parameters of the branches of the prefault network. To account for the balanced and unbalanced modifications to which a faulted power system is subjected, an extension referred to as the two-step compensation method of the conventional compensation method [8] was developed. In conceptual terms, this proposed approach evaluates the effects of all balanced modifications in the first step, and then calculates that of all unbalanced modifications in the second step. A major advantage in the use of the compensation approach is that the computations involved with evaluating all the terms of $\underline{Z}$ are avoided. In addition, this decomposition along the balanced/unbalanced nature of the modifications exploits the structural properties of the short circuit problem and consequently results in good computational efficiency.

The solution scheme for a single fault study is used as the basic building block for the system-wide short circuit study. For such a study, a set of faulted locations and a set of fault conditions postulated to occur at each location, are given. At each of the fault locations, the sequence of the specified fault conditions is studied. To avoid repeated calculations for the large sequence of single fault condition studies, a reordering of the rows of short circuit admittance matrix $\underline{Y}$ is introduced. In the reordered $\underline{Y}$, the rows corresponding to the nodes that are coincident with the branches undergoing modifications become the last rows of the matrix. An extension of the so-called sparse Z-bus formulation [7] was developed to compute only those elements of the inverse of Y which are necessary for the calculations of the system-wide study. The two-step compensation approach is then employed for each fault condition at each fault location. The new approach reduces considerably the overall computational and memory requirements.

The proposed approach was implemented in a production grade program. Results of its application to investigate a variety of fault conditions in large systems are very encouraging. The paper presents results on some test systems including a 2278 node network.

REVIEW OF THE COMPENSATION METHOD

The compensation method [8] is a computational scheme for simulating the effects of changes in the values of a network's passive elements. The approach is particularly effective when the number of element changes is small in comparison to the size of the network.

Consider a balanced network with (n+1) nodes, including the reference node, and b branches. The single-line representation of such a network is shown in Fig. 1. We can describe the network in its normal operating state by its nodal equation:

$$\underline{Y}\,\underline{e}^{o} = \underline{i}^{o} \tag{1}$$

where $\underline{Y}$ is the n x n short circuit nodal admittance matrix of the network, $\underline{e}^{o}$ is the n-vector of nodal voltages, and $\underline{i}^{o}$ is the vector of nodal current injections. Voltage vector $\underline{e}^{o}$ in (1) can be efficiently calculated using triangular factorization schemes which exploit the sparsity of $\underline{Y}$.

Next, consider the system in which changes are introduced in the values of some of the passive elements of the original network. In this case, the nodal equations becomes

$$\underline{Y}'\underline{e}' = \underline{i}^{o} \tag{2}$$

where $\underline{Y}'$ is the admittance matrix of the modified network and $\underline{e}'$ is the resulting nodal voltage vector which we wish to determine. Note that we assume that changes are made to the network elements only and that the current injection vector is unaltered.

When the compensation method is used, $\underline{e}'$ can be calculated without explicitly solving the network equation (2). Instead, the method makes use of the factors of $\underline{Y}$, which were obtained in solving (1). We first determine

$$S = \{\beta_1, \beta_2, \;\ldots\ldots,\; \beta_m\}$$

to be the set of m network branches that are modified. A branch modification occurs whenever any of its parameters is changed. The addition of a new branch or the removal of an existing branch are also considered to be network branch modifications. Denote the terminal nodes of each branch β_k in S by t and s and define T to be the set of all of these terminal nodes. Note that an element in T may correspond to the terminal node of more than one branch in S.

Consider the construction of an m-dimensional Thevenin equivalent of the original network, as seen from the terminal nodes of the branches in S, i.e., from the ports $\{(s_1 t_1), (s_2 t_2), \cdots\cdots, (s_m t_m)\}$. This equivalent will consist of an m-dimensional voltage vector $\underline{v}^{o}$ connected in series with an equivalent m x m impedance matrix $\underline{Z}^{o}$, as shown in Fig. 2. We denote it as the $\{\underline{Z}^{o}, \underline{v}^{o}\}$ Thevenin equvalent. The elements of $\underline{v}^{o}$ are the voltages across each of the branches of the original, unfaulted network, with the kth element

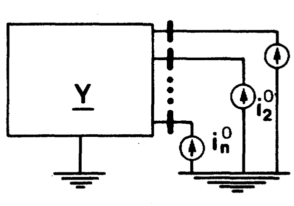

Fig. 1 Single phase representation of the pre-modification network.

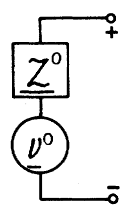

Fig. 2 Thevenin equivalent of the pre-modification network seen from the m ports formed by the terminal nodes of the m modified branches.

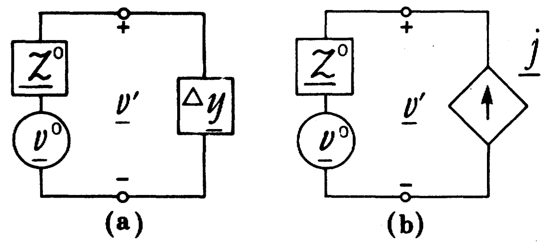

Fig. 3 The effects of the modifications represented by $\underline{\Delta y}$ (a) are identical to that of the dependent current vector $\underline{j}$ (b).

$$v^{o}_{\beta_k} = e^{o}_{s_k} - e^{o}_{t_k} \tag{3}$$

where $e^{o}_{s_k}$ and $e^{o}_{t_k}$ are the components of $\underline{e}^{o}$ corresponding to nodes s_k and t_k. The (ℓ,k)-th element of the m x m matrix $\underline{Z}^{o}$ is evaluated by removing all of the original current injections (i.e. setting $\underline{i}^{o}$ to $\underline{0}$), applying a 1 p.u. current across branch β_ℓ, and measuring the resulting voltage across branch β_k connecting nodes s_k and t_k. Define $\underline{1}^{(s_k)}$ and $\underline{1}^{(t_k)}$ as the s_k-th and t_k-th columns of the n x n identity matrix $\underline{U}_n$, and define $e^{(s_k t_k)}_{s_\ell}$ and $e^{(s_k t_k)}_{t_\ell}$ as the voltages that are set up at nodes s_ℓ and t_ℓ in response to the application of a 1 p.u. current across branch β_k. Then these nodal voltages can be calculated, as elements of $\underline{e}^{(s_k t_k)}$, by solving

$$\underline{Y}\,\underline{e}^{(s_k t_k)} = \underline{1}^{(s_k)} - \underline{1}^{(t_k)} \tag{4}$$

The value of $z^{o}_{\beta_\ell \beta_k}$ is given by

$$z^{o}_{\beta_\ell \beta_k} = z^{o}_{(s_\ell t_\ell)(s_k t_k)} = e^{(s_k t_k)}_{s_\ell} - e^{(s_k t_k)}_{t_\ell} \tag{5}$$

Thus, the entire matrix $\underline{Z}^{o}$ is readily determined by computing the m vectors $\underline{e}^{(s_k t_k)}$, $k = 1,2,\ldots,m$.

We replace the original network by its Thevenin equivalent to analyze the effect of the branch modifications. Let $\underline{y}$ and $\underline{y}'$ be the original and the modified network branch admittance matrices of the elements in S. Define the branch admittance modification matrix $\underline{\Delta y}$ by

$$\underline{\Delta y} = \underline{y}' - \underline{y} \tag{6}$$

We evaluate the effect of the modifications in the branches of S by considering the Thevenin equivalent with $\underline{\Delta y}$ connected across the branches in S, as shown in Fig. 3(a). If $\underline{j}$ is the vector of currents in the branches of S, then Kirchhoff's current and voltage laws obtain

$$\underline{j} = -\underline{\Delta y}\,\underline{v}' \tag{7}$$

$$\underline{v}' = \underline{v}^{o} + \underline{Z}^{o}\underline{j} \tag{8}$$

We combine equations (7) and (8) to obtain

$$\underline{j} = -\underline{\Delta y}\,(\underline{\bar{u}}_m + \underline{Z}^o \underline{\Delta y})^{-1}\underline{v}^o \tag{9}$$

Whenever $\underline{\Delta y}$ is invertible, then with $\underline{\Delta z} \overset{\Delta}{=} \underline{\Delta y}^{-1}$, we have

$$\underline{j} = -(\underline{Z}^o + \underline{\Delta z})^{-1}\underline{v}^o \tag{9a}$$

and both equations (9) and (9a) hold. Now $\underline{\Delta y}$ and the m-dimensional current source having the value $\underline{j}$ given by (9) are identical in their effects on the equivalent of the original network. Therefore, we may replace the branch modifications $\underline{\Delta y}$ by a current source $\underline{j}$, and the resultant voltage vector $\underline{v}'$ will be unaffected (see Fig. 3(b)). In the original network, it follows that the effect of the modifications in the branches of S may be represented by adding the compensating current vector $\underline{j}$ between the terminal nodes of the branches in S. This is shown in Fig. 4. Therefore, by superposition, the nodal equations of the modified system can be expressed as

$$\underline{Y}\,\underline{e}' = \underline{i}^o + \sum_{k=1}^{m} j_{(s_k t_k)}\underline{1}^{(s_k t_k)} = \underline{i}^o + \underline{\Delta i} \tag{10}$$

Here $j_{(s_k t_k)}$ is the kth element of $\underline{j}$ and $\underline{1}^{(s_k t_k)} = \underline{1}^{(s_k)} - \underline{1}^{(t_k)}$. From relations (1) and (10), and the assumed non-singularity of $\underline{Y}$.

$$\underline{e}' = \underline{e}^o + \sum_{k=1}^{m} j_{(s_k t_k)}\,\underline{e}^{(s_k t_k)} \tag{11}$$

Thus $\underline{e}'$ can be simply computed since the vectors $\underline{e}^{(s_k t_k)}$, $k = 1,2,\ldots,m$ are calculated earlier to evaluate $\underline{Z}^o$. The results in (11) indicate that we can determine the voltage vector $\underline{e}'$ of the modified network without refactoring $\underline{Y}'$, and simply using the factorization of $\underline{Y}$.

A reduction in the computational effort associated with the construction of $\underline{Z}^o$ may be possible. The evaluation of the elements of $\underline{Z}^o$ in eq. (5) requires solving the system in eq. (4) for $\underline{e}^{(s_k t_k)}$, $k = 1,2,\ldots,m$. Repeated solution of (4) may not be inefficient for small m. However, for large m we have developed a procedure that results in considerably lower computational effort. Let $\underline{e}^{(s_k)}$ be the solution of

$$\underline{Y}\,\underline{e}^{(s_k)} = \underline{1}^{(s_k)}$$

Due to linearity, for the system in (4) we have that

$$\underline{e}^{(s_k t_k)} = \underline{e}^{(s_k)} - \underline{e}^{(t_k)}$$

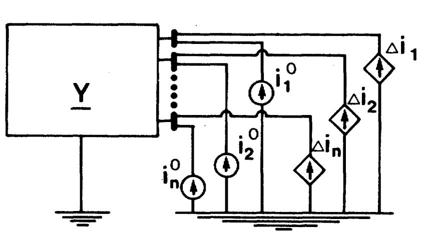

Fig. 4 The post-modification network.

We may consequently rewrite (5) as

$$z^o_{\beta_\ell \beta_k} = (e^{(s_k)}_{s_\ell} - e^{(t_k)}_{s_\ell}) - (e^{(s_k)}_{t_\ell} - e^{(t_k)}_{t_\ell}) \quad \ell, k = 1,2,\ldots,m \tag{12}$$

where $e^{(s_k)}_{s_\ell}$ is the component of $\underline{e}^{(s_k)}$ corresponding to node s_ℓ. Now consider the terms on the r.h.s. of (12). The term $e^{(s_k)}_{s_\ell}$, which is the voltage at node s_ℓ in response to a unit current injection at node s_k, is by definition the element $z_{s_\ell s_k}$ of the bus impedance matrix $\underline{Z} = \underline{Y}^{-1}$. Hence, (12) may be rewritten as:

$$z^o_{\beta_\ell \beta_k} = (z_{s_\ell s_k} - z_{s_\ell t_k}) - (z_{t_\ell s_k} - z_{t_\ell t_k}) \tag{13}$$

$$\ell, k = 1,2,\ldots,m$$

Note that each variable on the r.h.s. is an element of $\underline{Z}$. It is thus clear that the desired elements of $\underline{Z}^o$ may be easily computed once certain elements of $\underline{Z}$ are known. From (13), we see that the elements required are the self and transfer bus impedances in the rows and columns of $\underline{Z}$ corresponding to the nodes s_k and t_k of each branch β_k in S. We next develop expressions to efficiently compute these elements of $\underline{Z}$.

Consider the triangularization of the symmetric nodal admittance matrix $\underline{Y}$. Let the triangular factors of $\underline{Y}$ be $\underline{L}$ and $\underline{D}$:

$$\underline{Y} = \underline{L}\,\underline{D}\,\underline{L}^T$$

where the superscript T denotes matrix transposition. In the course of the triangularization, we use optimal ordering coupled with the restriction that the rows corresponding to the nodes in T, the set defined to consist of all the terminal nodes in S, are the last rows of the matrix. Now to construct $\underline{Z}^o$ we need only the elements of $\underline{Z}$ that correspond to the nodes in T. Consequently, only the last $|T|$ rows of the inverse of the reordered $\underline{Y}$ need be calculated. Since $\underline{Y}$ is symmetric, these elements may be computed recursively using the scheme in [7] :

$$z_{ij} = -\sum_{k>j} z_{ik}\ell_{kj} \qquad i < j$$

$$z_{jj} = d^{-1}_{jj} - \sum_{k>j} z_{jk}\ell_{kj}$$

where d_{jj} and ℓ_{ij}, $i < j$ are the nonzero elements of $\underline{D}$ and $\underline{L}$ respectively. Once the required z_{ij} have been calculated, the elements of $\underline{Z}^o$ are determined in a straightforward manner using (13). Note that using this approach, the vectors $\underline{e}^{(s_k t_k)}$ $k = 1,2,\ldots,m$, are not computed. $\underline{e}'$ is obtained by solving eq. (10) using $\underline{Y}$ in its factorized form and forward and back substitutions. With this approach, the m repeated solutions of eq. (4) are avoided. On the other hand, our method utilizes a nodal reordering scheme which may not be optimal, and consequently, the efficiency of the triangularization may suffer. The difference between the computational counts associated with the first approach and with our approach depends strongly on n, m, and on the structure of $\underline{Y}$. For systems of practical interest, extensive testing has shown that our approach for evaluating the elements in $\underline{Z}^o$ is computationally superior to the scheme using repeated solutions.

The extension of the compensation method to three-phase systems is rather simple. For the unmodified balanced three-phase system, the admittance matrix $\underline{Y}$ is a 3nx3n matrix. Each element in $\underline{Z}^o$ and $\underline{\Delta y}$ is a $3\times\overline{3}$ matrix, so that $\underline{Z}^o$ and $\underline{\Delta y}$ are of dimension 3m. The corresponding current $\underline{j}$ that represents the effect of the modification is a 3m vector. Due to computational considerations, the 0, +, and - sequence network representation of three-phase system is generally used in actual applications [9]. In this representation, the $3n\times 3n$ $\underline{Y}$ matrix becomes block diagonal, and the sequence networks of the pre-modification system are decoupled. Consequently, in the Thevenin equivalent, $\underline{Z}^o$ consists of three diagonal blocks corresponding to the three decoupled sequence networks and $\underline{v}^o$ has nonzero entries only in the positive sequence network components. Note that $\underline{\Delta y}$ is not a block diagonal matrix when unbalanced modifications are under considerations. Thus for three-phase networks, the system in (9) consists of 3m equations. Once $\underline{j}$ is determined using (9), $\underline{e}'$ can be calculated by (10). Now, the three-phase version of (10) may be decoupled into three n- dimensional systems because of the post-modification voltages for a three-phase system involves three systems of n equations, rather than a system of 3n equations. It is a simple exercise to show that the post-modification voltage for a three-phase system in the sequence coordinates are :

$$\underline{Y}_{(0)}\underline{e}'_{(0)} = \sum_{k=1}^{m} j_{\beta_k(0)}\,\underline{1}^{(s_k t_k)} \qquad \text{in the 0 sequence} \qquad (14a)$$

$$\underline{Y}_{(+)}\underline{e}'_{(+)} = \underline{i}^{o} + \sum_{k=1}^{m} j_{\beta_k(+)}\underline{1}^{(s_k t_k)} \qquad \text{in the + sequence} \qquad (14b)$$

$$\underline{Y}_{(-)}\underline{e}'_{(-)} = \sum_{k=1}^{m} j_{\beta_k(-)}\underline{1}^{(s_k t_k)} \qquad \text{in the - sequence} \qquad (14c)$$

where $\underline{Y}_{(0)}$, $\underline{Y}_{(+)}$ and $\underline{Y}_{(-)}$ are the diagonal blocks of $\underline{Y}$. Note that the balanced, pre-modification three-phase network, $\underline{i}^o_{(0)} = \underline{i}^o_{(-)} = \underline{0}$ and $\underline{Y}_{(+)} = \underline{Y}_{(-)}$. For the case of balanced modifications $\underline{\Delta y}$ is block diagonal so that $\underline{j}_{(0)} = \underline{j}_{(-)} = \underline{0}$. In that case only the positive sequence network in (14b) must be solved. This is identical to the single phase representation in eq. (10).

THE NATURE OF THE SINGLE FAULT STUDY

Short circuit studies are performed primarily for system protection and planning purposes. In these studies, the response of the power system to disturbances caused by faults and associated line outages are investigated. The study results are used to prescribe relay settings or to design specifications for future installations.

For any short circuit study, data describing the unfaulted network are given. These data specify the network topology, the line impedances and/or admittances, and the generation levels. Next, the various types of faults to which the network will be subjected, and the ensuing protective actions (line outages) are specified. The short circuit analysis must determine the postfault voltages and current flows.

We shall refer to each fault type together with its associated line outages as a fault condition and the analysis undertaken to investigate such a fault condition as a single fault study. In a single fault study, a fault and line outages may be regarded as modifications to the branches of the unfaulted network. For example, a system bus shorted to ground through a three-phase circuit can be considered as the addition of a new branch from the faulted node to ground, with the fictitious fault impedance in each phase. In a network with mutually coupled lines, whenever one or more members in a coupled group are modified, the parameters of all the lines in this group become modified. Because we can consider the various fault conditions as branch modifications, and because the number of branches to be modified for any single fault study is much smaller than the system size, the compensation method is a very appropriate algorithm to apply to the analysis of short circuits. For a balanced network subjected to a balanced fault only the positive sequence network is needed. The compensation scheme reviewed in the previous section may be applied to compute the postfault quantities. However, most of the faults of interest are unbalanced. In this case, the three-phase extension of the compensation scheme is used. We tabulated the solutions for some of the most common balanced and unbalanced fault types in Appendix A. In this table no associated outages are considered, i.e., each entry corresponds to a single modification, $m = 1$.

The use of the compensation method requires that $\underline{Y}$, the nodal admittance matrix of the prefault network, be known. This sparse matrix is easily constructed and may be efficiently stored [6]. One major advantage then of using the compensation scheme is that it can handle very large networks. In cases where only a small region of a given system is of interest, the reduced network obtained by eliminating all nodes outside the region of interest must be first determined. The admittance matrix of the reduced network then becomes $\underline{Y}$ for the region of interest. Clearly, the current injection vector of the given system is also reduced to derive $\underline{i}^o$. The network reduction is efficiently accomplished using Gaussian elimination. This flexibility associated with the admittance matrix for deriving reduced equivalents is an additional advantage of the admittance matrix based compensation method.

In the application of the conventional compensation scheme to short circuit studies, however, no explicit use of the properties of the fault conditions is made. We have developed an extension of the compensation method, which makes use of the structural properties of the matrices that represent the modifications of the various fault conditions. The proposed scheme, which we call the two-step compensation method for reasons that are made clear in the next section, solves any type of fault and yields savings in computational times and storage when compared to the conventional compensation method.

THE TWO-STEP COMPENSATION METHOD

This section outlines the generalized solution scheme we developed for solving any type of short circuit problems. The scheme is an extension of the conventional compensation method.

We classify the set of modifications of network branches into two categories:

(1) balanced modifications, in which identical changes occur in all three phases of a network branch; and
(2) unbalanced modifications, which constitute all other network branch modifications.

For example, three-phase faults and line outages are balanced modifications, while other faults such as single-line-to-ground are unbalanced modifications. If one or more branches in a group of mutually coupled branches undergo an unbalanced modification, then the modifications associated with all of the branches in this group will be unbalanced. Now, all the branches of S undergoing unbalanced modifications are first identified to construct the set $\mathcal{U}$. Then the set $\mathcal{B}$ consists of all other branches. Note that $\mathcal{B} \cap \mathcal{U} = \phi$. If the set $\mathcal{B}$ has p members and set $\mathcal{U}$ has q elements, then $p + q = m$. Let

$$\mathcal{B} = \{\beta_1^{\mathcal{B}}, \beta_2^{\mathcal{B}}, \ldots\ldots, \beta_p^{\mathcal{B}}\}$$

and

$$\mathcal{U} = \{\beta_1^{\mathcal{U}}, \beta_2^{\mathcal{U}}, \ldots\ldots, \beta_q^{\mathcal{U}}\}$$

Then

$$S = \{\beta_1^{\mathcal{B}}, \beta_2^{\mathcal{B}}, \ldots, \beta_p^{\mathcal{B}}, \beta_1^{\mathcal{U}}, \beta_2^{\mathcal{U}}, \ldots, \beta_q^{\mathcal{U}}\} \tag{15}$$

In the compensation framework the modifications in the branches in S are represented by the $3m \times 3m$ matrix $\underline{\Delta y}$. With the ordering used for S in (15), $\underline{\Delta y}$ has the form

$$\underline{\Delta y} = \left[\begin{array}{c|c} \underline{\Delta y}_{\mathcal{BB}} & \underline{0} \\ \hline \underline{0} & \underline{\Delta y}_{\mathcal{UU}} \end{array}\right]$$

The off diagonal blocks of $\underline{\Delta y}$ are zero since $\mathcal{B} \cap \mathcal{U} = \phi$. The submatrix $\underline{\Delta y}_{\mathcal{BB}}$ ($\underline{\Delta y}_{\mathcal{UU}}$) is $3p \times 3p$ ($3q \times 3q$) and represents the balanced (unbalanced) modification in the p (q) branches in $\mathcal{B}$ ($\mathcal{U}$). We may express $\underline{\Delta y}$ as

$$\underline{\Delta y} = \underline{\Delta y}^{\mathcal{B}} + \underline{\Delta y}^{\mathcal{U}} \tag{16a}$$

where

$$\underline{\Delta y}^{\mathcal{B}} = \left[\begin{array}{c|c} \underline{\Delta y}_{\mathcal{BB}} & \underline{0} \\ \hline \underline{0} & \underline{0} \end{array}\right] \quad \text{and} \quad \underline{\Delta y}^{\mathcal{U}} = \left[\begin{array}{c|c} \underline{0} & \underline{0} \\ \hline \underline{0} & \underline{\Delta y}_{\mathcal{UU}} \end{array}\right] \tag{16b}$$

Note that $\underline{\Delta y}^{\mathcal{B}}$ ($\underline{\Delta y}^{\mathcal{U}}$) represents the balanced (unbalanced) modifications in the m branches in S.

We make use of this decomposition of $\underline{\Delta y}$ to derive a computationally efficient scheme for evaluating the compensating current $\underline{j}$ [see Fig. 3(b)] that has the same effect as the modifications represented by $\underline{\Delta y}$ [see Fig. 5(a)]. We proceed as follows leaving the mathematical details to Appendix B. As in the compensation method we replace the pre-modification network in Fig. 5(a) by its Thevenin equivalent "seen" from the 3m ports formed by the terminal nodes of the branches in S. The components $\underline{Z}^0$ and $\underline{v}^0$ have the form

$$\underline{Z}^0 = \left[\begin{array}{c|c} \underline{Z}^0_{\mathcal{BB}} & \underline{Z}^0_{\mathcal{BU}} \\ \hline \underline{Z}^0_{\mathcal{UB}} & \underline{Z}^0_{\mathcal{UU}} \end{array}\right] = \left[\begin{array}{cccccc} \underline{Z}^0_{\mathcal{BB}(0)} & \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{BU}(0)} & \underline{0} & \underline{0} \\ \underline{0} & \underline{Z}^0_{\mathcal{BB}(+)} & \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{BU}(+)} & \\ \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{BB}(-)} & \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{BU}(-)} \\ \underline{Z}^0_{\mathcal{UB}(0)} & \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{UU}(0)} & \underline{0} & \underline{0} \\ \underline{0} & \underline{Z}^0_{\mathcal{UB}(+)} & \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{UU}(+)} & \underline{0} \\ \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{UB}(-)} & \underline{0} & \underline{0} & \underline{Z}^0_{\mathcal{UU}(-)} \end{array}\right]$$

and

$$\underline{v}^0 = \left[\begin{array}{c} \underline{v}^0_{\mathcal{B}} \\ \hline \underline{v}^0_{\mathcal{U}} \end{array}\right] = \left[\begin{array}{c} \underline{v}^0_{\mathcal{B}(0)} \\ \underline{v}^0_{\mathcal{B}(+)} \\ \underline{v}^0_{\mathcal{B}(-)} \\ \underline{v}^0_{\mathcal{U}(0)} \\ \underline{v}^0_{\mathcal{U}(+)} \\ \underline{v}^0_{\mathcal{U}(-)} \end{array}\right]$$

and are obtained as described in the section on the compensation method. Thus the effects of $\underline{\Delta y}$ are computed using the system in Fig. 5(b). Now the decomposition in (16) allows us to replace the admittance $\underline{\Delta y}$ by the admittances $\underline{\Delta y}^{\mathcal{B}}$ and $\underline{\Delta y}^{\mathcal{U}}$ connected in parallel, as shown in Fig. 5(c). As a first step, we introduce the effects of the balanced modifications represented by $\underline{\Delta y}^{\mathcal{B}}$ into the pre-modification network. The resultant network is represented by its Thevenin equivalent with components $\underline{Z}^{\mathcal{B}}$ and $\underline{v}^{\mathcal{B}}$ "seen" from the same 3m ports. The open circuit voltage vector is simply the voltage across the admittance $\underline{\Delta y}^{\mathcal{U}}$. The short circuit impedance matrix $\underline{Z}^{\mathcal{B}}$ is the equivalent impedance of $\underline{Z}^0$ in parallel with $[\underline{\Delta y}^{\mathcal{B}}]^{-1}$ Thus the $\{\underline{Z}^{\mathcal{B}}, \underline{v}^{\mathcal{B}}\}$ Thevenin equivalent represents the pre-modification network prior to introducing the modifications represented by $\underline{\Delta y}^{\mathcal{U}}$.

As the second step, we apply the extension of the compensation method to three-phase systems to this pre-modification network [see Fig. 5(d)]. Let $\underline{j}^{\mathcal{U}}$ denote the compensation current whose effect is identical to that of the modification represented by $\underline{\Delta y}^{\mathcal{U}}$ [see Fig. 5(e)]. It is shown in Appendix B that

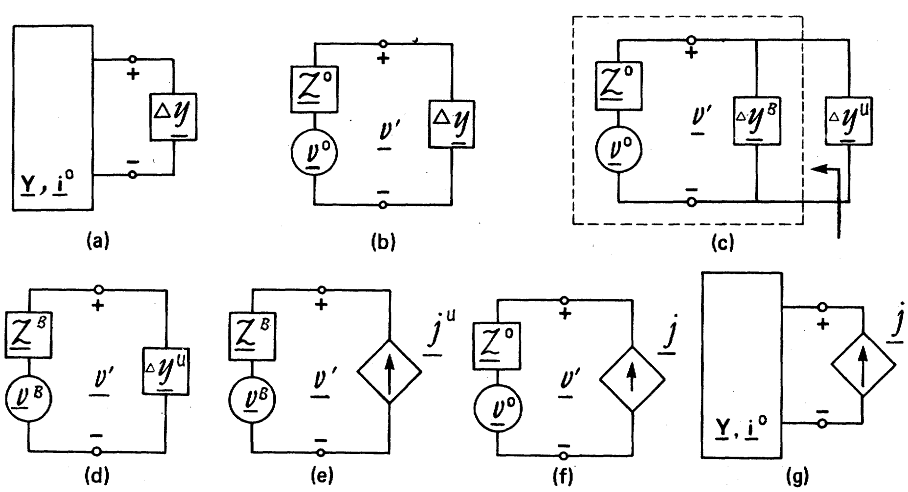

Fig. 5 The networks used in the derivation of the two-step compensation method.

$$i^{U} = \begin{bmatrix} i^{U}_{B} \\ \hline i^{U}_{U} \end{bmatrix} = \begin{bmatrix} 0 \\ \hline -\Delta y_{UU}(\bar{U}_{3q} + Z^{B}_{UU}\Delta y_{UU})^{-1} \, v^{B}_{U} \end{bmatrix} \quad (17)$$

with Z^{B}_{UU} and v^{B}_{U} given by

$$Z^{B}_{UU} = Z^{o}_{UU} - Z^{o}_{UB}\Delta y_{BB}(\bar{U}_{3p} + Z^{o}_{BB}\;\Delta y_{BB})^{-1} Z^{o}_{BU} \quad (18)$$

$$v^{B}_{U} = v^{o}_{U} - Z^{o}_{UB}\;\Delta y_{BB}(\bar{U}_{3p} + Z^{o}_{BB}\;\Delta y_{BB})^{-1}\; v^{o}_{B} \quad (19)$$

Some algebraic manipulation shows that the networks in Figs. 5(e) and 5(f) are equivalent. As shown in Appendix B, the current vector i in Fig. 5(f) is given by

$$i = i^{B} + i^{U}$$

where

$$i^{B} = \begin{bmatrix} i^{B}_{B} \\ \hline i^{B}_{U} \end{bmatrix} = \begin{bmatrix} -\Delta y_{BB}(\bar{U}_{3p} + Z^{o}{}_{BB}\;\Delta y_{BB})^{-1}(v^{o}_{B} + Z^{o}_{BU} i^{U}_{U}) \\ \hline 0 \end{bmatrix}$$

Thus (20)

$$i \;= [\; i^{B}_{B} \;\vdots\; i^{U}_{U} \;]^{T} \quad (21)$$

is the compensation vector that represents the combined effects of the balanced and unbalanced modifications. Finally, replacing the Thevenin equivalent by the pre-modification network we obtain the desired result as shown in Fig. 5(g).

The proposed approach is a generalized compensation scheme for solving efficiently any branch modifications. The conventional compensation becomes a special case when $\Delta y^{U} = 0$. Then, $i^{U}_{U} = 0$ in (17). For $\Delta y^{B} \neq 0$, $i^{B}_{B} \neq 0$ and its only nonzero sequence quantity is

$$i^{B}_{B(+)} = -\Delta y_{BB(+)}(\bar{U}_{p} + Z^{o}_{BB(+)}\Delta y_{BB(+)})^{-1} v^{o}_{B(+)}$$

This result is identical to eq. (9) of the conventional compensation method.

In the two-step compensation method, systems with lower dimensions than those in the conventional approach are solved. Furthermore, the decomposition of the balanced and unbalanced modifications allows all computations, with the exception of the calculation of i^{U} in eq.(17), to be performed directly in the sequence components. Clearly, these characteristics reduce the computational and storage requirements when compared to the conventional compensation method.

In terms of both computer memory requirements and execution time, the factorization based schemes are superior to the traditional Z-bus method [7, 10]. Both the conventional and the two-step compensation methods are factorization based schemes. The major difference between the conventional and the two-step approaches lies in the computation of the compensation current vector i; in the conventional scheme, eq. (9) is used, while for the two-step scheme eqs. (18), (19), (17) and (20) are used. For the reasons given above, the memory requirements of the two-step approach are slightly less than those of the conventional scheme. As an indication of the computer time requirements, the number of multiplication operations in the computation of i may be examined. The upper bounds of the numbers of multiplications for the conventional and the two-step schemes are compared in Appendix C. In terms of the number of branches p undergoing balanced modifications, it can be shown that whenever $p \neq 0$, the number of multiplications for the conventional scheme is greater than that for the two-step approach. For the case $p = 0$, the conventional and the two-step schemes are identical.

APPLICATION OF THE TWO-STEP COMPENSATION METHOD TO SHORT CIRCUIT STUDY

The two-step compensation method provides a unified approach for solving any type of short circuit problems. Since any fault condition is equivalent to a set of balanced and/or unbalanced branch modifications, any fault type or combination of fault types and associated protective actions can be analyzed using this solution scheme. Consider, for example, a line-end fault. This is a shunt fault which occurs at or near one end of a faulted branch, with the branch at that end open. At the other terminal node, called the connecting end, no action occurs. General short circuit solution techniques are very cumbersome in handling this fault type; specialized solution approaches have been developed for this purpose [2,3]. The line-end fault consists of two distinct simultaneously occurring modifications of the faulted branch: one is the complete removal of the branch, and the other is its replacement with a new branch which represents the fault. The former is a balanced modification. The latter may be either balanced or unbalanced. If the fault is a three-phase short circuit, then the modification is balanced. If it is a single-line-to-ground fault, then the branch addition is a single phase addition, and the modification is unbalanced. Note that in this case, unbalanced modifications are also introduced to any branches that are mutually coupled to the added branch.

The two-step compensation method is unique among short circuit solution schemes in that it uses directly the Y array. A noteworthy feature of this approach is the natural way in which it handles mutual couplings. The presence of mutuals requires no additional work once the Y and Δy arrays are constructed.

In addition to making the solution approach computationally efficient, the decomposition of the branch modifications according to their balanced/unbalanced nature is useful in another way. In the study of certain fault types, the computation of the compensating current i is complicated by the conventional compensation scheme. In such cases the computation of i^{U} followed by i^{B} as in the two-step approach can significantly simplify the calculations. A typical example is in solving the single-line-to-ground line-end fault.

The decomposition of the modified branches into two disjoint subsets as is done in the derivation of the two-step compensation method is a feature that is useful in its own right. The derivation of eqs. (17)-(21) holds for any two subsets B and U of S with the property that $B \cup U = S$ and $B \cap U = \phi$ since the structure of Δy in eq. (16) is preserved. Consider one application of eq. (18) where B is chosen to be the set of outaged lines and their mutual couplings, a total of p branches, and U is a set of q branches. The expression for Z^{B}_{UU} in eq. (18) gives the equivalent impedance of the network after the line outages in the set B take place as seen from the q ports formed by the terminal nodes of the q branches in the set U. A typical application is in the study of a line-out fault at a specified bus, where it is desired to construct the equivalent impedance of the faulted network as seen from this bus. This equivalent impedance determines the fault duty at the given bus under the line-out contingency.

THE SYSTEM-WIDE SHORT CIRCUIT STUDY

Typically, protection engineers are interested in performing a system-wide short circuit study. For such a study, a set of fault locations and a set of fault conditions are specified. One or more fault conditions are to be investigated at each of the specified fault locations. Each of the fault conditions at a fault location is studied sequentially and independently. After all of the fault conditions at one location have been analyzed, the procedure is repeated at the next fault location. The process continues until all of the specified fault conditions at all of the specified locations have been analyzed. Hence, a system-wide study is a sequence of independent single fault studies. In this section, we discuss incorporation of the system-wide study into the framework of the two-step compensation scheme.

For each single fault study ℓ in the system-wide study, a set S_ℓ of m_ℓ modified branches is defined. Let T_ℓ denote the set of nodes coincident with the branches in S_ℓ. We define $T \triangleq \bigcup_\ell T_\ell$. In the application of the compensation approach, the $\{\underline{Z}_\ell^0, \underline{v}_\ell^0\}$ Thevenin equivalent of the prefault network as seen from the m_ℓ ports defined by the terminal nodes of the m_ℓ branches in S_ℓ is obtained.

For the effective application of the two-step compensation method to the system-wide short circuit study, some preliminary work on $\underline{Y}$ is necessary. $\underline{Y}$ is reordered in such a way that the rows which correspond to the nodes in T become the last $|T|$ rows of the matrix. While for a single fault study ℓ, $|T_\ell|^2$ elements of $\underline{Z}$ must be evaluated, for a system-wide study not all the elements of the $|T| \times |T|$ submatrix $\underline{Z}_{|T|}$ of $\underline{Z}$, formed by the last $|T|$ rows and $|T|$ columns of $\underline{Z}$, are required. For example if two of the single fault studies ℓ and k in the system-wide study have the property that $T_\ell \neq T_k$, $s \in T_\ell$, $s \notin T_k$, $t \in T_k$ and $t \notin T_\ell$, then the element z_{st} of $\underline{Z}$ is not required and need not be evaluated. The values of the elements of interest in $\underline{Z}_{|T|}$ are obtained using the recursive formula in [7].

The triangularization of $\underline{Y}$ proceeds in two steps. First the first $n - |T|$ rows are optimally reordered and triangularized. Next, the elements in $\underline{Z}_{|T|}$ which must be evaluated are identified from the information on the specified fault conditions. The corresponding elements in the last $|T|$ rows of $\underline{Y}$ are treated as nonzero variables even though their values may be 0 [11]. The last $|T|$ rows are then optimally ordered and triangularized.

NUMERICAL RESULTS

We have developed SHOCAP (SHOrt Circuit Analysis Package) which implements the two-step compensation scheme discussed in this paper. SHOCAP is a production grade software package that can be used for studying short circuits on systems with up to 3000 buses, 4000 lines and 1000 mutual couplings. System-wide studies, which analyze various fault conditions at up to 250 sequential fault locations, can be carried out with SHOCAP. Fault types studies by SHOCAP include three phase and single-line-to-ground bus faults, line-end faults and line-out faults. SHOCAP can be used to study any faults simultaneously with arbitrary number of specified line outages.

We have tested the SHOCAP on a large number of systems with sizes ranging from 7 buses up to several thousand buses. For the purpose of this paper we restrict our attention to five test systems. Their characteristics are given in Table I. For each of the five systems, a system-wide study with a single fault location and three balanced and unbalanced fault types is performed. The fault location is an arbitrary bus in the system. At each location the fault conditions studied are three-phase and single-phase

. bus fault
. line-end fault on one line coincident with the fault location bus
. line-out fault on one line coincident with the fault location bus.

TABLE I: CHARACTERISTICS OF THE TEST SYSTEMS

System	Number of Buses	Number of Lines	Number of Mutuals
A	7	16	11
B	23	32	9
C	31	72	10
D	94	124	27
E	2278	3628	787

For systems B,C,D and E additional system-wide studies with 10 faulted locations and the same fault types are performed. Computational times for the triangularization phase and the fault calculation phase in SHOCAP are plotted in Fig. 6 as a function of system size. All computation times are for an IBM 3033. Notice that the decrease in efficiency of the triangularization of $\underline{Y}$ when the fault locations are increased from 1 to 10 has an insignificant effect for each of the four systems.

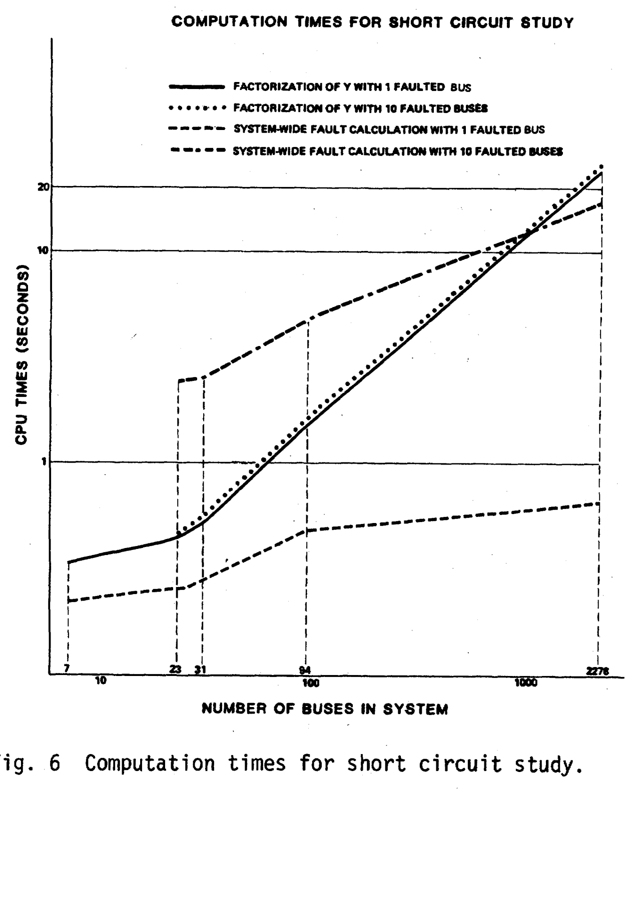

Fig. 6 Computation times for short circuit study.

For system E an additional system-wide study with 100 faulted locations and the same type of faults was made. Table II presents the computational times for system E for the three system-wide studies.

TABLE II: COMPUTATIONAL TIMES FOR SYSTEM E

SHOCAP PHASE	CPU sec. required for system-wide study with		
	1 fault location	10 fault locations	100 fault locations
Factorization of $\underline{Y}$	24.5	24.6	30.5
Fault Calculations per fault location	0.662	1.776	2.32

CONCLUSION

A unified approach for solving all types of short circuit problems has been presented. The approach is based on the two-step compensation method. We develope this computationally efficient methodology by extending the conventional compensation scheme for unbalanced and balanced modifications. This approach was derived by making detailed use of the structural properties of the short circuit problem and is particularly effective in dealing with large systems. Any fault condition, i.e., any fault type or combination of fault types and associated protective actions can be solved using this new methodology. The proposed scheme works especially well on system-wide studies in which specified fault conditions at a set of specified fault locations are analyzed sequentially. A noteworthy feature of the solution methodology is the natural way in which mutuals are handled. The SHOCAP software was developed to implement the new solution scheme. Extensive tests indicate that it is a powerful tool for short circuit studies particularly for large scale systems.

ACKNOWLEDGEMENT

The authors acknowledge the many useful discussions they had with the engineers in the System Protection Department at PGandE. In particular, we thank Messrs. E.H. Gove and W.R. Searway who were generous with their advice during the development of SHOCAP.

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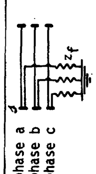
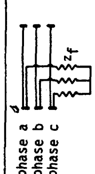
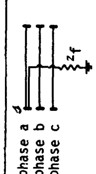
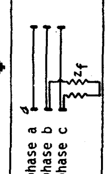
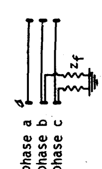
APPENDIX A

COMMONLY STUDIED FAULT TYPES IN THE COMPENSATION FRAMEWORK

Fault Type	Fault Configuration	Branch Modification Matrix $\underline{\Delta y}$	Compensation Current $\underline{i}$ in Sequence Components
Three-phase to ground		$\frac{1}{z_f}$ Note: balanced fault; use eq. (14b) only	$\frac{-e^0_\delta(+)}{z_{\delta\delta}(+)+z_f}$
Three-phase short circuit		$\frac{1}{z_f}$ Note: balanced fault; use eq. (14b) only	$\frac{-e^0_\delta(+)}{z_{\delta\delta}(+)+z_f}$
Single-line to-ground		$\frac{1}{3z_f}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$	$\frac{-e^0_\delta(+)}{z_{\delta\delta}(0)+z_{\delta\delta}(+)+z_{\delta\delta}(-)+3z_f}\begin{bmatrix}1\\1\\1\end{bmatrix}$
Line-to-line		$\frac{1}{2z_f}\begin{bmatrix}0&0&0\\0&1&-1\\0&-1&1\end{bmatrix}$	$\frac{-e^0_\delta(+)}{z_{\delta\delta}(+)+z_{\delta\delta}(-)+2z_f}\begin{bmatrix}0\\1\\-1\end{bmatrix}$
Double-line-to-ground		$\frac{1}{3z_f}\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix}$	$\frac{e^0_\delta(+)}{D}\begin{bmatrix}z_{\delta\delta}(-)+z_f\\-z_{\delta\delta}(0)+z_{\delta\delta}(-)+2z_f\\z_{\delta\delta}(0)+z_f\end{bmatrix}$ $D=[z_{\delta\delta}(0)z_{\delta\delta}(+)+z_{\delta\delta}(+)z_{\delta\delta}(-)+z_{\delta\delta}(0)z_{\delta\delta}(-)]+2z_f[z_{\delta\delta}(+)+z_{\delta\delta}(-)+z_{\delta\delta}(0)]+3z_f^2$

APPENDIX B: THE TWO-STEP COMPENSATION METHOD

The two step compensation method is based on the decomposition of the modifications of the m branches in eq. (16). The decomposition is based on the balanced/unbalanced nature of the modifications. In the first step, only the effect of the balanced modifications represented by Δy^B is considered. In the second step, the additional effect of the unbalanced modifications represented by Δy^u is evaluated. We refer to Figs.5(a)-5(g) to derive eqs. (17)-(21).

Consider the system in Fig. 5(d) obtained in the course of applying the three-phase compensation method with modifications represented by Δy^u. It follows from eq. (9) that

$$i^u = -\,\Delta y^u(\bar{U}_{3m} + Z^B \Delta y^u)^{-1} v^B \tag{B1}$$

Using the identity

$$\left[\begin{array}{c|c} \bar{U}_\alpha & A \\ \hline 0 & B \end{array}\right]^{-1} = \left[\begin{array}{c|c} \bar{U}_\alpha & -A\,B^{-1} \\ \hline 0 & B^{-1} \end{array}\right],$$

where B is a $\gamma \times \gamma$ nonsingular matrix and A is any $\alpha \times \gamma$ array. We may show that

$$i^u = \left[\begin{array}{c} i^u_B \\ \hline i^u_u \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline -\Delta y_{uu}(\bar{U}_{3q} + Z^B_{uu}\Delta y_{uu})^{-1} v^B_u \end{array}\right] \tag{B2}$$

assuming $(\bar{U}_{3q} + Z^B_{uu}\Delta y_{uu})$ is nonsingular. Now, the components Z^B and v^B of the Thevenin equivalent "seen" from the 3m ports formed by the terminal nodes of the branches in S prior to introducing the unbalanced modifications are evaluated by referring to Fig. 5(c). Thus,

$$Z^B = [(Z^o)^{-1} \| \Delta y^B]^{-1} = [(Z^o)^{-1} + \Delta y^B]^{-1}$$

$$= Z^o - Z^o \Delta y^B\,(\bar{U}_{3m} + Z^o \Delta y^B)^{-1} Z^o \tag{B3}$$

$$v^B = (\bar{U}_{3m} + Z^o \Delta y^B)^{-1} v^o \tag{B4}$$

where we make use of the matrix inversion formula [12]. Since to evaluate i^u in eq. (B2) only Z^B_{uu} and v^B_u are required. We may use (B3), (B4) and the identity

$$\left[\begin{array}{c|c} C & 0 \\ \hline D & \bar{U}_\gamma \end{array}\right]^{-1} = \left[\begin{array}{c|c} C^{-1} & 0 \\ \hline -D\,C^{-1} & \bar{U}_\gamma \end{array}\right]$$

for any nonsingular $\alpha \times \alpha$ matrix C and any $\gamma \times \alpha$ matrix D, that

$$Z^B_{uu} = Z^o_{uu} - Z^o_{uB}\,\Delta y_{BB}(\bar{U}_{3p} + Z^o_{BB}\,\Delta y_{BB})^{-1} Z^o_{Bu} \tag{B5}$$

$$v^B_u = v^o_u - Z^o_{uB}\,\Delta y_{BB}(\bar{U}_{3p} + Z^o_{BB}\,\Delta y_{BB})^{-1} v^o_B \tag{B6}$$

All matrices in (B5) and (B6) have block diagonal structure. Thus the sequence arrays $Z^B_{uu(0)}$, $Z^B_{uu(+)}$, and $Z^B_{uu(-)}$ are obtained directly from the decoupled system in (B5). Furthermore, the only nonzero component of v^B_u is $v^B_{u(+)}$. Then in the decoupled system in (B6), $v^B_{u(0)} = v^B_{u(-)} = 0$ and only the positive sequence part need be evaluated.

Once i^u is known, we next consider the voltage v' across the 3m modified branches. From Fig. 5(e) or equivalently, eq. (8) of the compensation method we have

$$v' = v^B + Z^B i^u \tag{B7}$$

Upon substituting (B3) and (B4) into (B7) we have

$$v' = (\bar{U}_{3m} + Z^o\Delta y^B)^{-1} v^o + Z^o i^u - Z^o \Delta y^B (\bar{U}_{3m} + Z^o \Delta y^B)^{-1} Z^o\, i^u$$

$$= v^o + Z^o i^u - Z^o[\Delta y\,(\bar{U}_{3m} + Z^o\Delta y^B)^{-1}(v^o + Z^o i^u)]$$

$$= v^o + Z^o i^u + Z^o i^B \tag{B8}$$

where

$$i^B \triangleq -\Delta y^B(\bar{U}_{3m} + Z^o\Delta y^B)^{-1}[v^o + Z^o i^u] \tag{B9}$$

Now,

$$i^B = \left[\begin{array}{c} i^B_B \\ \hline i^B_u \end{array}\right] = \left[\begin{array}{c} -\Delta y_{BB}(\bar{U}_{3p} + Z^o_{BB}\Delta y_{BB})^{-1}(v^o_B + Z^o_{Bu} i^u_u) \\ \hline 0 \end{array}\right] \tag{B10}$$

The sequence currents $i^B_{B(0)}$, $i^B_{B(+)}$, and $i^B_{B(-)}$ are obtained individually in evaluating (B10) since the system is decoupled. Moreover, only the nonzero $v^o_{B(+)}$ enters in the evaluation of i^B_B. With i^u and i^B known and

$$i = i^u + i^B = [\, i^B_B \;\vdots\; i^u_u \,]^T \tag{B11}$$

It follows from (B8) that

$$v' = v^o + Z^o i \tag{B12}$$

which corresponds to the network in Fig. 5(f). Thus i is the compensation current that represents the effects of both the balanced and unbalanced modifications.

APPENDIX C

COMPARISON OF THE UPPER BOUNDS OF THE NUMBER OF MULTIPLICATIONS TO COMPUTE THE COMPENSATION CURRENT VECTOR i

Conventional Compensation Method		Two-step Compensation Method	
Operation in eq. (9)	Multiplications*	Operation	Multiplications*
$Z^o\Delta y$	(a)=$6m^3$	eq. (18)	(d)=$2(p^3+\frac{1}{3}p^3+qp^2-\frac{1}{3}p+qp^2+q^2p)$
$(\bar{U}_{3m}+Z^o\Delta y)^{-1}v^o$	(b)=$\frac{1}{3}(3m)^3+(3m)^2-\frac{3m}{3}$	eq. (19)	(e)=$2p^2+qp$
		eq. (17)	(f)=$6q^3+\frac{1}{3}(3q)^3+(3q)^2-\frac{3q}{3}+9q^2$
$\Delta y(\bar{U}_{3m}+Z^o\Delta y)^{-1}v^o$	(c)=$9m^2$	eq. (20)	(g)=$2(pq+2p^2)$
Upper Bound	(a)+(b)+(c)	Upper Bound	(d)+(e)+(f)+(g)

* based on the assumption that $Z^o_{(+)} = Z^o_{(-)}$ and $v^o_{(0)} = v^o_{(-)} = 0$

p = number of branches undergoing balanced modifications, q = number of branches undergoing unbalanced modifications and m = p+q.

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Discussion

H. E. Brown (Cary, NC): The authors are to be commended for continuing the effort to improve short circuit analysis methods.

The limitation of the excessive memory requirement of the Z-matrix building algorithm (1) is overcome by combining sparse matrix techniques and compensation methods for short circuit studies involving line alterations (additions and removals).

Previously, the limitation imposed by the storage requirements of the Z-matrix was overcome by an axis discarding technique (2) in which axes, not of immediate interest, were discarded and only axes in the study area, and those required in making line alterations were retained. Both the positive and zero sequence matrices were computed and stored in memory. With the computer available in 1966 (32,000 K words of memory) studies of systems of 1,500 buses and 4,500 lines, three phase and single phase faults, became routine. Mutual couplings have little effect in the zero sequence network when they are remote from the area of interest. The program selected, from the system wide mutual coupling data, only the study area for pocessing. The building algorithm (1) was modified to include two additional routines required for mutually coupled radial lines and mutually coupled loop closing lines using the method of Reitan (3, 4). The program package included all the features described in the paper, and in addition gave preliminary settings for certain types of relays.

The discussor would like to know the speed advantage of the new method and any other advantages, that are not evident, over the older Z-matrix method.

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Manuscript received August 21, 1981.

G. Gross and **H. W. Hong:** We thank Prof. Brown for his discussion and acknowledge our appreciation of his interest in our work.

The major motivation for the work reported in this paper was to derive a generalized scheme for solving all types of fault conditions. One important consideration was the speed of the solution scheme. Since the system-wide analysis of large-scale systems was a primary objective, it became necessary to develop an efficient computation technique. The major advantages of the two-step compensation method which we developed for short circuit analysis are:

- high computational efficiency due to the use of triangular factorization;
- the additional enhancement in computational efficiency due to the incorporation of the sparse Z method;
- generalized solution methodology for analyzing all fault types with associated line outages; and
- consideration of mutual couplings within the general framework of the solution scheme.

The computational advantages of the triangular factorization-based conventional compensation method over the Z bus method are detailed in the last paragraphs of the section entitled "The Two-Step Compensation Method" of the paper. The application to the analysis of short circuit problems of the two-step compensation method, which generalizes the conventional compensation method, results in further reducing the overall computational requirements. The comparison of the multiplication counts between the conventional compensation and the two-step compensation methods is presented in Appendix C. For any applications of practical interest, the two-step scheme requires less computational effort and storage. The SHOCAP implementing the two-step compensation method was developed to replace the Z bus based method in use at PGandE since the 1960s. The axis discarding technique developed by Prof. Brown and his colleagues had been incorporated into the old short circuit program in the 1970s. One major limitation of the old program was the performance of system-wide studies. For example, for a system-wide study with 150 faulted buses, the axis discarding scheme involves the formulation of a 350 dimensional Z. This imposes rather severe storage requirements even when matrix symmetry is taken into account. Moreover, a rather substantial computational effort is involved in the construction of the reduced Z. With the application of the two-step compensation method based SHOCAP program for the system-wide study of 150 faulted buses, no more than 1000 elements of Z are computed. The computations involved in the evaluation of the postfault voltages are substantially reduced. Two additional advantages of the two-step compensation scheme are the natural way in which the mutuals and line outages are handled. As explained in the paper, the incorporation of mutuals involves no approximation and the consideration of line outages induces no additional computational burden.

Manuscript received October 13, 1981.