EVALUATION OF TRANSMISSION CONGESTION IMPACTS ON ELECTRICTY MARKETS

BY

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ABSTRACT

Congestion in the transmission network has become a critical problem for the electricity markets in the restructured power industry. Congestion has a wide range of impacts from the way the system is operated to the behavior of each player in the market. The presence of congestion may prevent the use of the lowest-priced resources to meet the demand and may, in addition, facilitate the attempt of a particular seller to exercise market power. Many observers of the industry see congestion as a key barrier for the establishment of vibrant competitive markets.

This thesis studies the impacts of congestion on the individual market players and the market as a whole, in general, and the quantification of these impacts when a seller attempts to exercise market power by varying its offer prices, in particular. We also investigate the role of price-responsive demand in the mitigation of the possible exercise of market power. We make use of a two-layer analytical framework that integrates the physical characteristics of power systems with the appropriate economic aspects of electricity markets. We effectively formulate the characterization of congestion and present a comprehensive set of measures to evaluate the impacts of congestion on both the individual players and the entire market. We report simulation results on five test systems of various sizes to illustrate the nature of the impacts on each player and the market as a whole. We conclude from the simulation that the impacts of congestion on the individual players and on the entire market are bounded for price-responsive demand. Each measure used in the studies is asymptotic. In this way, the attempt of a particular seller to exercise market power fails to produce the seller's desired increase in profits.

Some of the players are *free riders* while others are negatively impacted by the attempt of market power exercise. In terms of the overall market impacts of congestion, these are similar in each test system. However, the redispatch effects vary among the systems, but have the common feature that they are asymptotic.

ACKNOWLEDGMENTS

I am profoundly indebted to my supervisor, Professor George Gross, for the guidance and support he provided throughout the course of this work. His patience, despite my many, many questions, is greatly appreciated.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

In this chapter, we set the stage for the work presented in this thesis. We start out by discussing the motivation for the research pursued for the thesis. We outline the scope of the efforts undertaken and the contributions made. We also include a brief literature survey of publications relevant to the problems investigated and provide a brief outline of the remainder of this report.

1.1 Motivation

The liberalization of the electricity industry around the world has as its principal goals the introduction of competition into electricity markets and their effective exploitation for the benefit of society. The introduction of competitive markets aims to reduce prices, improve the quality of services, and on a longer-term basis, make markets more efficient. The salient characteristics of electricity – the lack of major storage capabilities, the *just-in-time-manufacturing* nature of electricity, and the central role played by the transmission and distribution networks – make electricity markets different from any other markets and pose major challenges in their operations. One problem that is becoming increasingly critical is that of transmission congestion. In the competitive environment, investments in the transmission network have not kept pace with the increasing demand for transmission services. This situation has led to bottlenecks in many networks. In addition, the unbundling of transmission from the other electricity services – generation, marketing, and ancillary services – combined with the shift to more decentralized decision making have resulted in making the coordinated operation of the system more complex.

One consequence of transmission congestion is that not all the desired transactions may be accommodated in actual operations; therefore, not all of the energy in such cases may be provided by the suppliers who offer it at the lowest prices. The explicit consideration of the transmission network effects may result in the purchase of some energy from alternative sources at possibly higher prices. However, if the *willingness to pay* of the buyers is exceeded by the price at which this higher-priced energy is offered, then some of the demand may remain unsatisfied in the particular market. This situation is especially critical for an area with demand that exceeds local generation. Reliance on imports of electricity from other areas therefore becomes the only means to meet the demand. In the absence of demand-price responsiveness, the suppliers in the importing area may exercise *local* market power, particularly when transmission congestion constraints limit the capability to import into that area.

Congestion is not a new phenomenon since power systems, by their very nature, have always operated under constrained conditions. In the traditional vertically integrated utility structure of the pre-open-access-transmission regime, we referred to such conditions as *constrained operations*. The power system operator selected the generator output levels so as to ensure system security, i.e., so that no physical and operational limits were violated. The single integrated decision-making entity could make any necessary trade-offs between security and economics. In the restructured environment, the operation of the system is unbundled, meaning that generation, transmission, distribution, and system control are separate entities and may have different ownership and control. The new situation, with the many independent decision makers, makes the operation of a secure network very challenging, particularly when the market economics are considered. Congestion management is concerned with the formulation and implementation of efficient procedures to coordinate the actions of all market participants, while ensuring the maintenance of the system reliability. This is among the responsibilities of the independent grid operator (IGO), the entity set up to control system transmission operations without ownership of the facilities under its control. The procedures, to be effective, must be robust, fair, and transparent to all the market players. The assessment of congestion management schemes involves the evaluation of the impacts of congestion. For this purpose, meaningful metrics to measure these impacts, along both the energy and the dollars dimension, are needed.

The congestion in the grid must be studied for the reference case as well as the set of postulated contingencies that may befall the system. Congestion impacts the market and market players in many ways. The presence of congestion on the grid may prevent the use of the lowest-priced resources to meet the load and therefore may result in a generation/demand schedule that will incur higher costs than that for the transmission-unconstrained market. Consequently, a loss in the market efficiency results due to the change in the generation/demand schedule. The measures of the resulting change in terms of both the energy and the dollars incurred are very important impacts that need to be evaluated from the point of view of each individual player and the market as a whole. In this way we obtain the quantification of how each market player fares under congestion and the reassignment of the individual player profits/savings due to congestion.

Congestion facilitates gaming by some sellers who wish to increase their profits. The measure of the extent to which a particular seller is able to exercise market power is an important aspect of the congestion impacts. The role of the network is critically important in the measurement of market power exercise effects. This is particularly true since any action adopted by a particular player impacts directly how the player and the other players fare. Such assessments are a key focus of this work.

There are other impacts of congestion. For example, pollution may also increase when the system is congested due to the use of older units or less efficient plants or both, that replace the part of the generation that cannot be supplied by the lower-priced resources due to network considerations. Such impacts, however, are beyond the scope of this thesis and are not considered further. While we take into account the explicit consideration of reliability, we do not evaluate the impacts of congestion on reliability. Such an investigation is also beyond the scope of this report.

In addition of the impacts of congestion in the short term, congestion also has long-term effects on the market and on the individual players. The study of such effects is beyond the scope of this work and constitutes an important topic for future research.

1.2 Literature Survey of Related Previous Work

Effective congestion management schemes are critically important for the smooth functioning of competitive electricity markets. Several studies have focused on assessing certain impacts of congestion associated with specific transmission congestion management approaches. The analysis in [1] evaluates the transmission congestion costs for the two basic paradigms: the so-called pool and bilateral. The nodal-pricing-based approach is used in the framework of the pool paradigm. For the so-called bilateral model, an approach based on cost allocation is constructed. The authors in [1] explore different cost allocation schemes, from a pro rata basis to a constraint allocation mechanism, considering the impacts of the counter-flows. The method for congestion relief used in the bilateral model considers a least-cost formulation with incremental and decremental offers. Therefore, the two approaches result in the same dispatch; however, the costs to the buyers in the transmission-constrained market are different. This work only evaluates how the buyers are unequally impacted under the two distinct paradigms.

Different congestion management schemes are discussed in [2] for three different transmission management models: the optimal power flow model deployed in the United Kingdom, part of the United States, and in Australia and New Zealand; the price and congestion control model used in the Nordpool market; and the U.S. bilateral transactions model. For each model, the analytic aspects of the congestion management are discussed, describing the background of each model and its strengths and weakness. However, there is no quantitative evaluation of the impacts of congestion in each market when the various models are in practice.

The NordPool market is also discussed in [3], where the focus is on the Norwegian system. The authors in [3] define the socioeconomic costs of having a congested path, i.e, the costs incurred by the society because congestion. These socioeconomic costs serve as the basis for establishing new tools to use in congestion management that give the system operator and the network owners incentives to operate the transmission system in the most efficient way and to invest in the transmission network. The main concern of the study is the creation of adequate long-term signals to incentive further developments of the transmission system.

Two congestion management schemes used in the NoordPool market are also studied in [4]. A unified framework for the study of the congestion management schemes in different jurisdictions is provided, using a consistent set of metrics. The study provides a side-by-side comparison of the various congestion management schemes analyzed through the use of these metrics providing good insights on the impacts of congestion in the various jurisdictions.

In the area of market power exercise, the discussion in [5] provides a comprehensive analysis of the understanding of market power in electricity market, and how market power is different from competitive behavior. Further insights are provided in [6], where the impacts of the transmission system on imperfect competition in the restructured markets are assessed. The authors show that transmission limits have a significant impact on the outcomes of the market when a player is gaming, indicating that reducing transmission congestion would mitigate market power. The identification of a situation where market power is possible is particularly important in the area of market monitoring. The study in [7] provides a good overview of the key elements needed for an effective design and implementation of market monitoring systems.

A key factor in the control of market power exercise is the existence of competition in electricity markets. However, the introduction of competition has been developed basically in the supply-side entities. The active participation of the demand-side entities in the electricity markets remains minimal. The studies in [8] and [9] analyze the important role that demand responsiveness can play in competitive electricity markets. They show how the increase of the elasticity of the demand alleviates congestion and improves the efficiency of the electricity markets.

Unfortunately, none of the references cited above evaluate the impacts of congestion on the individual players when a seller attempts to exercise market power.

1.3 Scope and Contribution of This Thesis

In this thesis, we study the impacts of congestion on the individual market players and the market as a whole, in general, and the quantification of these impacts when a seller located in an importing zone attempts to exercise market power, in particular. One of our interests is to explicitly assess the critical role of the network in electricity markets. We investigate the role of price-responsive demand in the mitigation of possible market power exercise. We make use of a two-layer framework that integrates the physical characteristics of power systems with the economic aspects of electricity markets. We discuss the characterization of congestion using this framework and define a comprehensive set of measures to evaluate the impacts of congestion on the individual players and the entire market. We assess the impacts of congestion in five test systems varying in size from 2 to 57 buses. Our work addresses the market for a specific hour in the day-ahead framework.

Congestion impacts some players negatively and others positively. The key contribution of this thesis is the finding that these impacts on the players and the market are limited when demand is price responsive. In fact, we observe that the increase or decrease of the profits or benefits achieved by the various players under congestion is asymptotically bounded. Such asymptotic behavior is also present in the measure of the inefficiency that arises in the market due to congestion.

In addition, the simulation results reported here allow us to conclude some significant findings about the impacts of congestion on the market players and the markets themselves. We observe that any attempt of a particular seller to exercise market power is not too worthwhile since the seller's profits can increase in only a small range, thereby providing little incentive to the seller to change its offer prices. Other important findings concern the existence of *free riders* and negatively impacted players in the various markets. Another finding is that congestion does not impact uniformly each market in terms of the *redispatch* of the units, so no general conclusions can be drawn on the redispatch effects. But, both the amount of redispatch power and redispatch costs are characterized by their asymptotic nature in every market of each test system. The thesis documents these impacts in detail in the tests systems simulated

In Chapter 2 we review the mathematical framework used to study congestion. We use a framework consisting of a physical network layer and an electricity market layer within which we can identify and characterize congestion. In Chapter 3 we define and discuss the congestion metrics used to measure the impacts of congestion and illustrate them through some simple examples. In Chapter 4 we provide a more in-depth characterization of congestion to gain some insights about the information provided by the congestion measures defined in Chapter 3. Simulation results for five test systems are reported in Chapter 5. Chapter 6 summarizes the key results of our studies and points out directions for future work.

This thesis has also three appendices to provide a self-contained documentation of the work. Appendix A provides a summary of the acronyms and the notation used in this thesis. In Appendix B we characterize the optimal solution of the transmission-constrained problem. The analytical development provides the basis for the relationship used in the chapters of the thesis. The data of the five test systems used in the simulation studies are provided in Appendix C.

2. MATHEMATICAL FRAMEWORK

This chapter describes the framework used for the study of congestion. The framework consists of two interconnected layers as shown in Figure 2.1 [10]. One layer deals with the model of the transmission network and the other deals with the model of the electricity market.

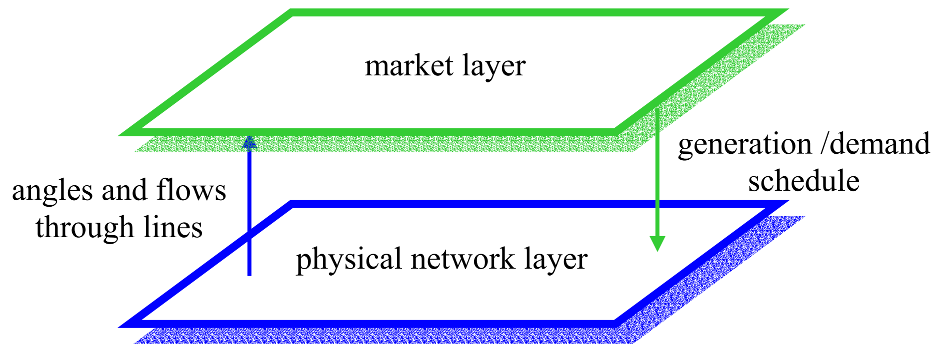

Figure 2.1: The two-layer framework structure.

The physical network layer presents the modeling of the key physical elements of the of power system. In this layer, the relationship between the generation/demand at each node and the line flows is established. The congestion conditions are defined in this layer. The market layer presents the modeling of the electricity market in terms of the bids and offers of sellers and buyers, respectively. This layer contains the representation of the determination of the market equilibrium which provides the resulting generation/demand schedule.

The interaction between the two layers is through the information flows, as indicated in Figure 2.1. The market layer provides the *preferred* transmission schedules including the nodal injections and withdrawals. This information flows to the network layer so as to verify the feasibility of the desired transaction schedule. The feasible schedule, which the physical network can accommodate, is then determined. This schedule explicitly indicates the congested lines in the network and provides information to the market layer to determine the energy prices at each node in the system and the impacts of congestion. The structure of the framework provides the flexibility to analyze issues related to the provision of transmission services in competitive electricity markets.

2.1 The Physical Network Layer Model

Consider a power system consisting of a set of (N + 1) buses $\mathcal{N} = \{0,1,2,...,N\}$ and a set of L lines $\mathcal{L} = \{\ell_1, \ell_2,...,\ell_L\}$, with $N \leq L$, where each bus is connected to at least one other bus. Each line is denoted by the ordered pair $\ell = (n,m)$ where n is the *from node,* and m is the *to node* with $n, m \in \mathcal{N}$. Let 0 denote the slack bus. We consider a lossless model of the network, in which only the reactance of the line is considered. Let $\underline{\boldsymbol{B}}_d \in \Re^{L \times L}$ represents the *diagonal branch susceptance matrix*,

$$\underline{\boldsymbol{B}}_{\boldsymbol{d}} = \text{diag}\left[b_{\ell_1}, b_{\ell_2},...,b_{\ell_L}\right], \tag{2.1}$$

where b_ℓ is the susceptance of the line ℓ. For each line ℓ,

$$b_\ell = \left(x_\ell\right)^{-1}, \tag{2.2}$$

where, x_ℓ is the reactance of line ℓ. We assume that each line has $x_\ell > 0$.

Definition 2.1: For a network with $(N+1)$ buses and L lines, we define the *augmented branch-to-node incidence matrix* to be the $L \times (N+1)$ matrix

$$\hat{\underline{A}} = \begin{bmatrix} a_{\ell_1 0} & a_{\ell_1 1} & \cdots & a_{\ell_1 N} \\ a_{\ell_2 0} & a_{\ell_2 1} & \cdots & a_{\ell_2 N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{\ell_L 0} & a_{\ell_L 1} & \cdots & a_{\ell_L N} \end{bmatrix} \in \Re^{L\times(N+1)}, \qquad (2.3)$$

where, for $\ell = (n,m)$

$$a_{\ell h} = \begin{cases} 1 & \text{if } h = n \\ -1 & \text{if } h = m \\ 0 & \text{otherwise} \end{cases}. \qquad (2.4)$$

■

Note that the definition of $\hat{\underline{A}}$ implies that

$$\hat{\underline{A}}\underline{1}^{N+1} = \underline{0}, \qquad (2.5)$$

where $\underline{1}^{N+1} \in \Re^{N+1}$ is the $N+1$ vector with each component being 1.

Definition 2.2: For a network with $(N+1)$ buses, we define the *augmented node-to-node susceptance matrix* to be the $(N+1)\times(N+1)$ matrix given by

$$\hat{\underline{B}} = \hat{\underline{A}}^T \underline{B}_d \hat{\underline{A}} = \begin{bmatrix} b_{00} & b_{01} & \cdots & b_{0N} \\ b_{10} & b_{11} & \cdots & b_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N0} & b_{N1} & \cdots & b_{NN} \end{bmatrix} \in \Re^{(N+1)\times(N+1)}. \qquad (2.6)$$

Note that $\hat{\underline{B}}$ is symmetric under the assumption that $x_\ell > 0$, $\forall \ell \in \mathcal{L}$.

■

We construct the *branch-to-node incidence matrix* $\underline{A}$ by removing the column of $\hat{\underline{A}}$ corresponding to the slack node, i.e.,

$$\underline{\boldsymbol{A}} = \begin{bmatrix} a_{\ell_1 1} & a_{\ell_1 2} & \cdots & a_{\ell_1 N} \\ a_{\ell_2 1} & a_{\ell_2 2} & \cdots & a_{\ell_2 N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{\ell_L 1} & a_{\ell_L 2} & \cdots & a_{\ell_L N} \end{bmatrix} \in \Re^{L \times N} \tag{2.7}$$

and the *node-to-node susceptance matrix*

$$\underline{\boldsymbol{B}} = \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N} \\ b_{21} & b_{22} & \cdots & b_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NN} \end{bmatrix}. \tag{2.8}$$

$\underline{\boldsymbol{B}} \in \Re^{N \times N}$ and is symmetric.

Proposition 2.1: $\underline{\boldsymbol{B}}$ is nonsingular.

Proof: We prove this claim by making use of two facts:

(a) The diagonal matrix $\underline{\boldsymbol{B}}_d$ is positive definite since each line has a nonzero susceptance [11].

(b) The N columns of $\underline{\boldsymbol{A}}$ form a linearly independent set [12] since each bus of the system is connected to at least one other bus; also

$$rank(\underline{\boldsymbol{A}}) = rank(\underline{\boldsymbol{A}}^T) = N \,.$$

The facts *(a)* and *(b)* imply that

$$(\underline{\boldsymbol{B}}_d)^{1/2} \underline{\boldsymbol{A}}\underline{\boldsymbol{x}} = \underline{\boldsymbol{0}} \Leftrightarrow \underline{\boldsymbol{x}} = \underline{\boldsymbol{0}} \,.$$

Then we have that

$$\underline{\boldsymbol{x}}^T \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}}\underline{\boldsymbol{x}} = \underline{\boldsymbol{x}}^T \underline{\boldsymbol{A}}^T (\underline{\boldsymbol{B}}_d)^{1/2} (\underline{\boldsymbol{B}}_d)^{1/2} \underline{\boldsymbol{A}}\underline{\boldsymbol{x}} = \left\| (\underline{\boldsymbol{B}}_d)^{1/2} \underline{\boldsymbol{A}}\underline{\boldsymbol{x}} \right\|^2 = 0 \Leftrightarrow \underline{\boldsymbol{x}} = 0 \,.$$

We conclude then that $\underline{\boldsymbol{B}} = \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}}$ is a positive definite matrix, and therefore, $\underline{\boldsymbol{B}}$ is nonsingular. ∎

We next rewrite $\hat{\underline{\boldsymbol{B}}}$ as

$$\hat{\underline{\boldsymbol{B}}} = \begin{bmatrix} \boldsymbol{b}_{00} & \underline{\boldsymbol{b}}_0^T \\ \underline{\boldsymbol{b}}_0 & \underline{\boldsymbol{B}} \end{bmatrix}, \tag{2.9}$$

where

$$\underline{\boldsymbol{b}}_0 = \left[b_{01}, b_{02}, \ldots, b_{0N} \right]^T .$$

From (2.5) we have

$$\hat{\underline{\boldsymbol{B}}} \underline{\boldsymbol{1}}^{N+1} = \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}} \underline{\boldsymbol{1}}^{N+1} = \underline{\boldsymbol{0}} .$$

Since $\underline{\boldsymbol{B}}$ is nonsingular (2.9) implies that

$$\underline{\boldsymbol{b}}_0 = -\underline{\boldsymbol{B}} \underline{\boldsymbol{1}}^N . \tag{2.10}$$

This result will be used later in this work.

We now set up of the power flow equations for the network. For bus *n*, $n \in \mathcal{N}$ let P_n^S denotes the power injected at bus *n*, P_n^B the power withdrawn at bus *n,* and θ_n the voltage phase angle at bus *n*. We write the power flow relations under the following assumptions [13]:

- Real power losses are neglected.
- Phase angle differences are small.
- Bus voltage magnitudes are approximately 1.0 per unit.

Under these assumptions the power flow equations have the form

$$\underline{\boldsymbol{P}}^S - \underline{\boldsymbol{P}}^B = \underline{\boldsymbol{B}} \underline{\boldsymbol{\theta}}, \tag{2.11}$$

$$P_0^S - P_0^B = \underline{\boldsymbol{b}}_0^T \underline{\boldsymbol{\theta}}, \tag{2.12}$$

where

$$\underline{\boldsymbol{P}}^S \triangleq [P_1^S, P_2^S, ..., P_N^S]^T \in \Re^N ,$$

$$\underline{\boldsymbol{P}}^B \triangleq [P_1^B, P_2^B, ..., P_N^B]^T \in \Re^N ,$$

and

$$\underline{\boldsymbol{\theta}} \triangleq \left[\theta_1, \theta_2, ..., \theta_N\right]^T \in \Re^N .$$

Let f_ℓ denote the real power flow on line ℓ and

$$\underline{\boldsymbol{f}} \triangleq [f_{\ell_1}, f_{\ell_2}, ..., f_{\ell_L}]^T \in \Re^L .$$

Then, we have that

$$\underline{\boldsymbol{f}} = \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}} \underline{\boldsymbol{\theta}} . \tag{2.13}$$

The constraints on the real power flows on the lines of the network are represented through the following inequality:

$$\underline{\boldsymbol{f}} \leq \underline{\boldsymbol{f}}^{max} , \tag{2.14}$$

where

$$\underline{\boldsymbol{f}}^{max} \triangleq [f_{\ell_1}^{max}, f_{\ell_2}^{max}, ..., f_{\ell_L}^{max}]^T \in \Re^L ,$$

and

f_ℓ^{max} is the maximum active power flow allowed through line $\ell \in \mathcal{L}$.

Definition 2.3: We call the line ℓ *congested* if the actual flow through the line equals the line flow limit, i.e.,

$$f_\ell = f_\ell^{max} .$$

We call the transmission system *congested* if there are one or more congested lines in the network.

■

We use this characterization of congestion throughout this work.

2.2 The Market Layer

A *market mechanism* is a setup to allow buyers and sellers to exchange a commodity [14]. In this work, we consider markets in which the commodity is the electric energy measured in megawatthours. While there exist different market mechanisms that can be implemented, in this study we consider only the uniform-price double auction [15].

We define one hour as the smallest indecomposable unit of time. We can define different markets depending on the time at which market decisions are taken. In Figure 2.2 we show the time frame for markets. We focus in the day-ahead market, which corresponds to a collection of 24 separate energy markets, one for each hour of the next day. In particular, we study the market decisions corresponding to a specified hour *h* of the day-ahead market. For the sake of simplicity, throughout this thesis we omit the mention of hour *h* in the presentation of each result.

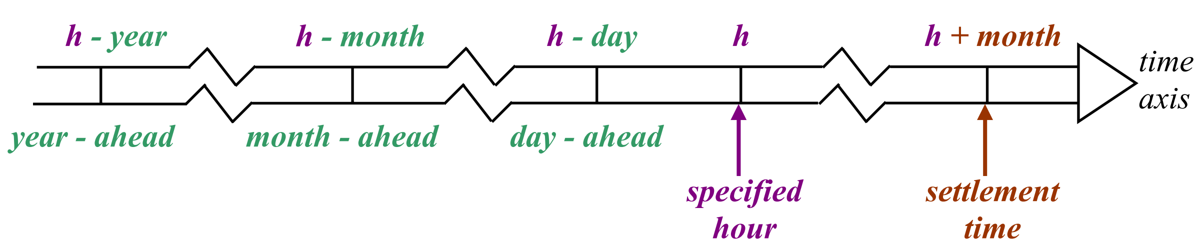

Figure 2.2: Time frame for markets.

We consider the electricity market consisting of a set of M^S sellers $\mathbb{S} = \left\{ S_1, S_2, ..., S_{M^S} \right\}$ and a set of M^B buyers $\mathbb{B} = \left\{ B_1, B_2, ..., B_{M^B} \right\}$.

Definition 2.4: The *offer function* $\sigma^{S_i}(\cdot) : \mathfrak{R}_{+0} \to \mathfrak{R}_{+0}$ of seller $S_i \in \mathbb{S}$ maps each MWh/h into a price.[1] The offer function indicates the price at which the seller is willing to sell a specified quantity of megawatthour per hour for the hour h. We denote by $\sigma_n^S(\cdot)$ the aggregated offer function of sellers at bus *n*.

■

Definition 2.5: The *bid function* $\nu^{B_j}(\cdot) : \mathfrak{R}_{+0} \to \mathfrak{R}_{+0}$ of buyer $B_j \in \mathbb{B}$ maps each MWh/h into a price. The bid function indicates the price at which the buyer is willing to buy a specified quantity of megawatthour per hour for the hour h. We denote by $\nu_n^B(\cdot)$ the *aggregated bid function* of buyers at bus *n.*

■

To be able to participate in the market, the buyers and sellers submit sealed offers and bids, respectively, to the centralized electricity market operator (CEMO). The CEMO is the independent entity responsible for the operation of the market. The CEMO determines the accepted offers and bids, the market clearing price, and the market clearing quantity for each hour *h*. In doing so, the CEMO also determines the quantity sold by each seller and the quantity bought by each buyer. Figure 2.3 shows the structure of the market and the direction of the energy and money flows in the market.

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[1] $\mathfrak{R}_{+0} = \mathfrak{R} \cup \left\{ 0 \right\}$.

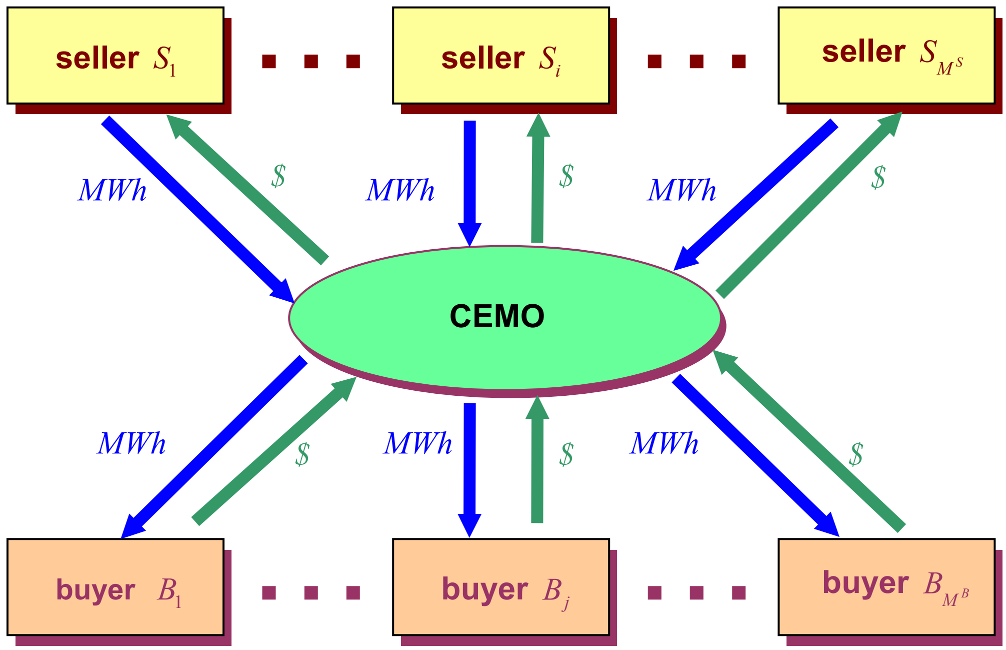


Figure 2.3: The centralized electricity market.

We assume that for the double auction mechanism used in electricity markets, the offer (bid) function is a nondecreasing (nonincreasing) function. Note that the offers and bids may not reflect the actual costs and benefits of the buyers and sellers, respectively, since the true costs and benefits are private information [16]. The CEMO aggregates and sorts all the offers (bids) by price in ascending (descending) order to create the *aggregated supply (demand) curve* [17].

Definition 2.6: The *costs* $\mathcal{C}^{S_i}(P^{S_i})$ of seller S_i for the sale of P^{S_i} *MWh/h* in hour *h* are given by

$$\mathcal{C}^{S_i}(P^{S_i}) = \int_0^{P^{S_i}} \sigma^{S_i}(\xi)d\xi \, . \tag{2.15}$$

■

Figure 2.4(a) shows an offer curve and the costs of seller S_i. We denote by $\mathscr{C}_n^S(P_n^S)$ the aggregated costs of the sellers located at bus *n*, where

$$P_n^S = \sum_{\substack{S_i \in \mathbb{S}, \\ S_i \text{ is at node } n}} P^{S_i} . \tag{2.16}$$

Definition 2.7: The *benefits* $\mathscr{B}^{B_j}(P^{B_j})$ of buyer B_j for the purchase of P^{B_j} *MWh/h* in hour *h* are given by

$$\mathscr{B}^{B_j}(P^{B_j}) = \int_0^{P^{B_j}} \nu^{B_j}(\xi) d\xi . \tag{2.17}$$

■

Figure 2.4(b) shows a bid curve and benefits of buyer B_j. We denote by $\mathscr{B}_n^B(P_n^B)$ the aggregated benefits of the buyers located at bus *n*, where

$$P_n^B = \sum_{\substack{B_j \in \mathbb{B}, \\ B_j \text{ is at node } n}} P^{B_j} . \tag{2.18}$$

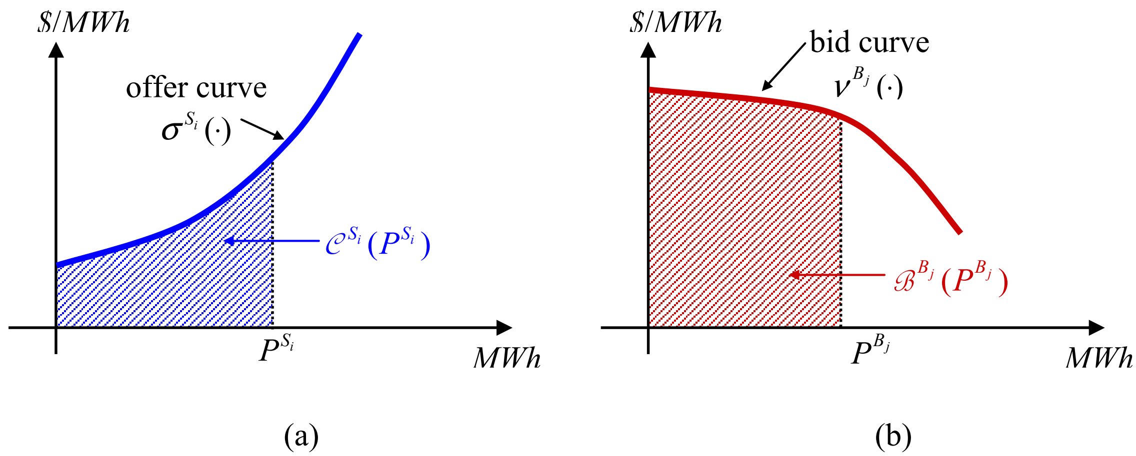


Figure 2.4: Seller and buyer information provided to the CEMO: (a) seller S_i offer curve, and (b) seller B_j bid curve.

Definition 2.8: The *social welfare* $\mathscr{S}$ is a measure of the net benefits of the market sales and purchases, i.e., the difference between the total benefits of the buyers and the total costs of the sellers

$$\mathscr{S} \; = \; \sum_{j=1}^{M^B} \mathscr{B}^{B_j}(P^{B_j}) \; - \; \sum_{i=1}^{M^S} \mathscr{C}^{S_i}(P^{S_i}) \,. \tag{2.19}$$

■

We next rewrite the social welfare in (2.19) in terms of the aggregated costs and the aggregated benefits $\mathscr{C}_n^S(P_n^S)$ and $\mathscr{B}_n^B(P_n^B)$, respectively, at each node n:

$$\mathscr{S} \; = \; \sum_{n=0}^{N} \Big[\mathscr{B}_n^B(P_n^B) \; - \; \mathscr{C}_n^S(P_n^S) \Big] . \tag{2.20}$$

We may view the social welfare $\mathscr{S}$ of the system as being the sum of the nodal social welfare $\mathscr{S}_n$ at each node n of the system, where

$$\mathscr{S}_n \; = \; \mathscr{B}_n^B(P_n^B) \; - \; \mathscr{C}_n^S(P_n^S) \,. \tag{2.21}$$

The social welfare is an important metric since it measures the overall impacts of markets on both sellers and buyers. The double auction market mechanism has as objective the maximization of the social welfare, so as to determine the maximum net benefits for society. Then, for the market without the considerations of the transmission constraints, the successful offers and bids are given by the solution of the following transmission unconstrained optimization problem **(TUP)**:

$$\textbf{(TUP)} \left\{ \begin{array}{l} \max \quad \mathscr{S} = \displaystyle\sum_{n=0}^{N} \Big[\mathscr{B}_n^B(P_n^B) - \mathscr{C}_n^S(P_n^S) \Big] \\ \text{subject to} \\ \displaystyle\sum_{n=0}^{N} P_n^S \; - \; \sum_{n=0}^{N} P_n^B \; = \; 0 \,. \end{array} \right. \tag{2.22}$$

The CEMO solves the **(TUP)** to determine the preferred generation/demand schedule to evaluate the desired quantity sold/bought by each seller/buyer with no transmission constraints considered. This optimization problem contains only one equality constraint and results in a unique clearing price for the market when the solution exists. This is called the *uniform market clearing price* and is denoted by ρ^*.

2.2.1 The unconstrained market

The solution of the **(TUP)** corresponds to the intersection point between the aggregated supply and the aggregated demand curves. Such a point, if it exists, satisfies the equality constraint in (2.22) and also maximizes the social welfare. Graphically, the situation is depicted in Figure 2.5. The solution defines the market clearing price ρ^* and the market clearing quantity P^* that *mediates* the trades. When we say that the solution *mediates* the trades, we mean that once the solution is obtained and ρ^* is established, the corresponding participation in the sale or purchase of every market player – buyer or seller – is determined through its offer or bid. The optimal market solution[2] (ρ^*, P^*) is known as the *market equilibrium* because it represents the economic balance among all the players [14]. For example, consider a price $\rho < \rho^*$. The quantity demanded is greater than the quantity supplied, and so the demand would have to decrease in order to equal the level of the supply. On the other hand, if we consider a price $\rho > \rho^*$, then the quantity supplied is greater than the quantity demanded, and so the supply would have to

[2] The usual order of the coordinates in engineering applications in describing a point in Cartesian coordinates is the ordered pair (ordinate, abscissa). However, in economics, the price is always mentioned before the quantity, and so we adopt the latter convention.

decrease in order to meet the level of the demand. There are no incentives for a seller or a buyer to move from the point given by (ρ^*, P^*).

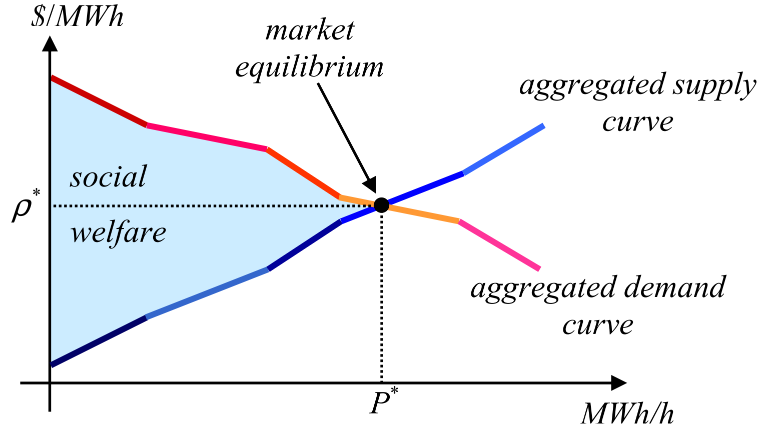


Figure 2.5: Market equilibrium.

The unsuccessful offers and bids cannot be traded in this market. However, they may be traded in other markets or bilaterally. For example, a seller and buyer may agree to a bilateral transaction. Also, a buyer or seller may decide to postpone the trading to a later market such as an hour-ahead market or a real-time market.

The construction of the aggregated supply and the aggregated demand curves allows us to view the uniform-price double auction as a mechanism that systematically matches in sequence the highest bid with the lowest offer. The matching of the pairs of buyers and sellers corresponds to the maximization of the area between the supply and demand curves. We can see this matching between sellers and buyers as fictitious transactions. We can assume that the seller with the lowest offer sells to the buyer with the highest bid. The seller with the next lowest offer sells to the buyer with the next highest bid, successively. These transactions are *fictitious* because there is no agreement between the seller-buyer pairs, the sellers/buyers agree to sell/buy to/from the CEMO.

In what follows, we present an example of an unconstrained market, where the preferred generation/demand schedule and the clearing price are determined.

Example 2.1: We consider the two-bus system of Figure 2.6. The sellers S_1 and S_2, and the buyer B_1 are located at bus 0. The seller S_3 and the buyers B_2 and B_3 are located at bus 1. Figure 2.7(a) shows the offer curve of each seller and Figure 2.7(b) shows how the aggregated supply curve is constructed.

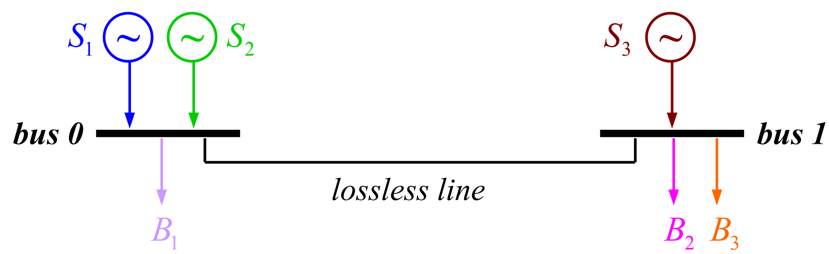


Figure 2.6: The two-bus system of Example 2.1.

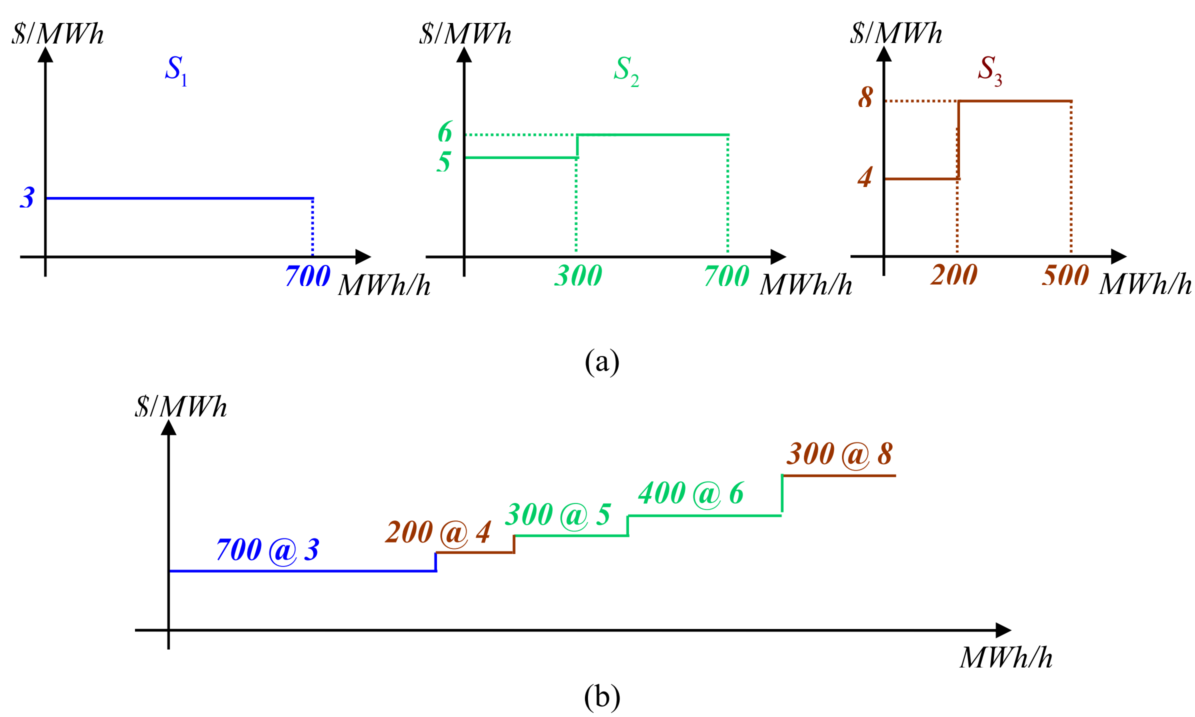


Figure 2.7: Example 2.1: (a) the offers of the sellers, and (b) the aggregated supply curve.

In a similar way, Figure 2.8(a) shows the bid curve of each buyer and Figure 2.8(b) shows how the aggregated demand curve is constructed.

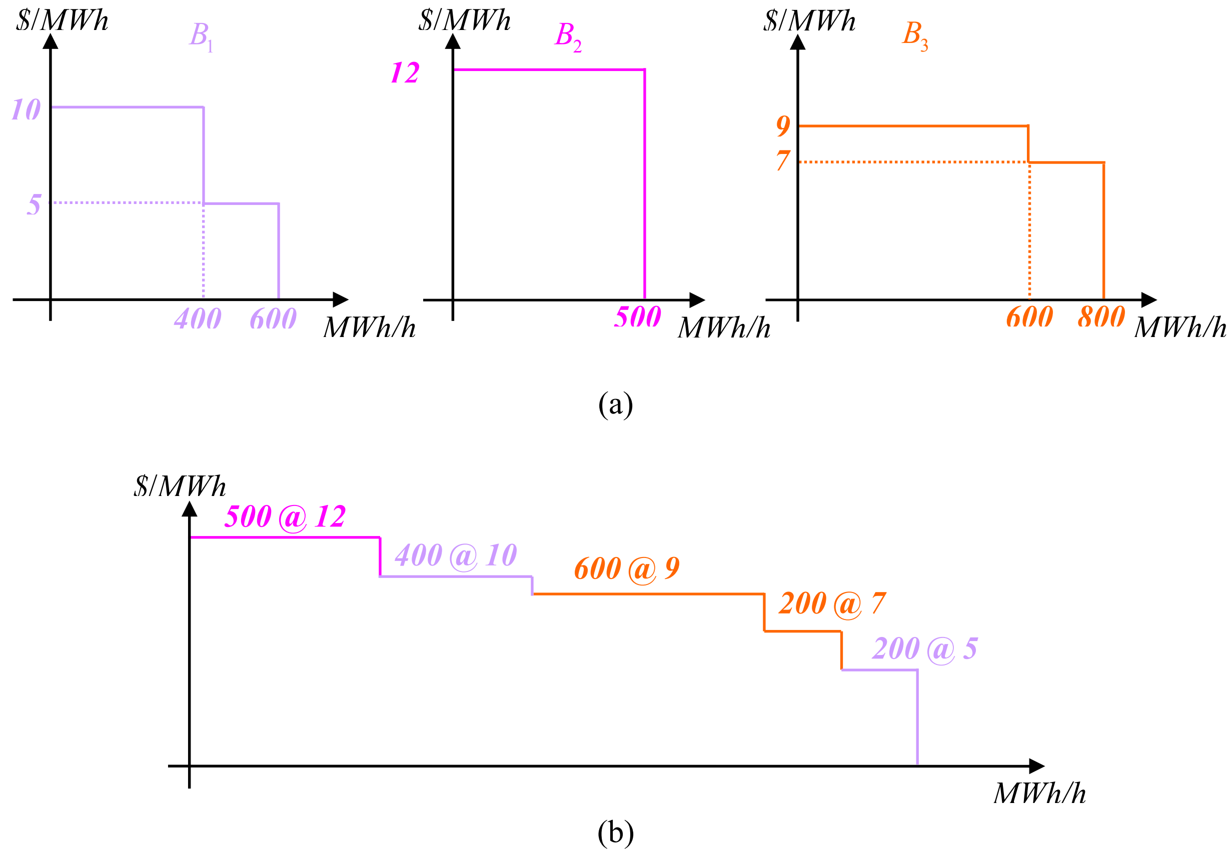


Figure 2.8: Example 2.1: (a) the bids of the buyers, and (b) the aggregated demand curve.

The sequential matching of the highest bid with the lowest offer results in a clearing price ρ^* of 7 $/MWh and a clearing quantity P^* of 1600 MWh/h. Figure 2.9 shows the equilibrium of the market. Table 2.1 shows the corresponding clearing quantity for each player and Table 2.2 shows the revenues or payments of each player.

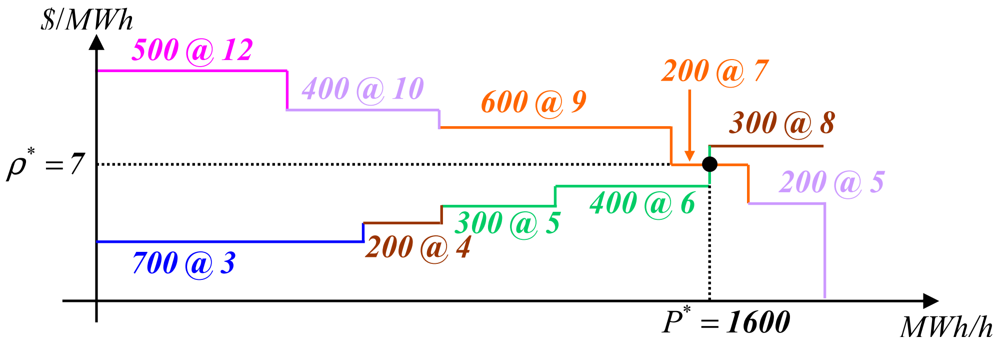


Figure 2.9: The determination of the market equilibrium.

Table 2.1: Example 2.1: Clearing quantities of sellers and buyers.

seller	clearing quantity (MWh/h)
S_1	700
S_2	700
S_3	200
total	1600

buyer	clearing quantity (MWh/h)
B_1	400
B_2	500
B_3	700
total	1600

Table 2.2: Example 2.1: Revenues and payments of sellers and buyers.

seller	revenues ($)
S_1	4900
S_2	4900
S_3	1400
total	11 200

buyer	payments ($)
B_1	2800
B_2	3500
B_3	4900
total	11 200

■

Example 2.1 shows clearly that in an unconstrained market, the clearing price, if it exists, is unique. This clearing price maximizes the social welfare since it sequentially matches the highest bid to the lowest offer at each power level. We also can see how the preferred generation/demand schedule is determined once the clearing price is known.

We notice that the revenues earned by the sellers are exactly equal to the payments made by the buyers.

2.2.2 The transmission-constrained market

The reliable operation of power systems requires that variables be maintained within certain specified limits. For example, the power flow through a line has to be less than or equal to the maximum flow capacity of a line. In this work, we assume that the various constraints can be represented in terms of limits on the active power flow on each line.

When the transmission constraints are considered, the outcomes of the unconstrained market performed by the CEMO may not be feasible for the network. The so-called independent grid operator (IGO) determines the successful offers and bids that the network can accommodate without violating any limits. The IGO is an independent entity, whose responsibility is to provide reliable transmission services to customers and to operate and control the power system. Different implementations of the IGO exist. For example, the PJM ISO (Pennsylvania, New Jersey and Maryland independent system operator) includes the market operator within its structure. This is an example of *maximal* ISO [18]. The restructuring in California created the CAISO (California ISO) and the CAPX (California Power Exchange). The latter served as the CEMO. The California system was an example of a *minimal* ISO [18] since the PX was an independent entity separated from the ISO.

When the transmission considerations are taken into account, we obtain the transmission constrained optimization problem **(TCP)**:

$$
\textbf{(TCP)} \quad \begin{cases} \max \quad \mathscr{S} = \sum_{n=0}^{N} \left[\mathscr{B}_n^B(P_n^B) - \mathscr{C}_n^S(P_n^S) \right] \\ \text{subject to} \\ \qquad \underline{\boldsymbol{P}}^S - \underline{\boldsymbol{P}}^B = \underline{\boldsymbol{B}}\underline{\boldsymbol{\theta}} \\ \qquad P_0^S - P_0^B = \underline{\boldsymbol{b}}_0^T \underline{\boldsymbol{\theta}} \\ \qquad \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}} \underline{\boldsymbol{\theta}} \le \underline{\boldsymbol{f}}^{max} . \end{cases} \tag{2.23}
$$

With the transmission constraints considered, the market may no longer have a unique clearing price [19]. Such a situation arises because we explicitly consider the supply-demand balance at each of the buses of the system in the equality constraints of the **(TCP)**. We analyze the characteristics of the optimal solution of the **(TCP)** in Appendix B. The different clearing prices at each node of the system are called *locational marginal prices* (LMPs). Their definitions are given in Appendix B. Each seller/buyer sells/buys energy at its nodal LMP. The discussion in Appendix B makes clear that the solution of **(TCP)** corresponds to the solution of the optimal power flow. We note that since there are limits on the line flows that have to be met, the optimal solution may no longer be equal to the preferred generation/demand schedule determined in the **(TUP)**. However, the optimal solution of the **(TCP)** is the most economic solution that takes explicitly into account the transmission constraints of the system. In no way can this situation be considered uneconomic [20], even though this characterization is frequently used in the industry.

Next, we present an example of a transmission-constrained market, where the clearing prices at the different buses of the system are determined together with the clearing quantities for each player with the transmission constraint explicitly considered.

Example 2.2: Consider the two-bus system of Example 2.1, with an active flow line limit of 800 MW as shown in Figure 2.10.

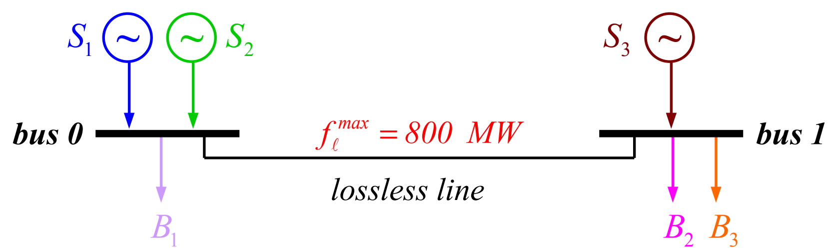
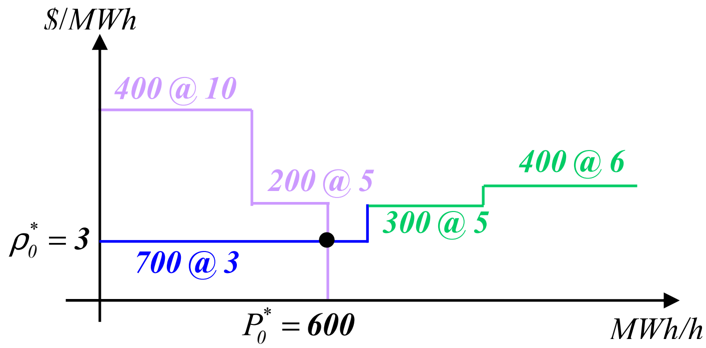
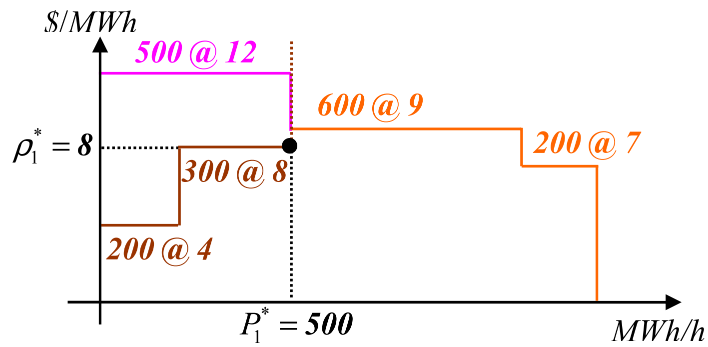


Figure 2.10: The two-bus system of Example 2.1 with the line flow limit explicitly indicated.

Now consider the solution determined in Example 2.1. For that solution, the corresponding active power flow from bus 0 to bus 1 is 1000 MW, which exceeds the line flow limit by 200 MW. Therefore, this solution is not feasible for the **(TCP)**. In fact, since the line is congested, it splits the system into two markets, one at each bus. We can discuss the situation step by step by starting with the market at each of the two buses without the tie line connection. We show the equilibrium of each isolated market in Figure 2.11. We can see that if the markets are not connected, seller S_2 is unable to sell his energy, and buyer B_3 is unable to purchase any energy. This situation is not sustainable over time since seller S_2 and buyer B_3 have no benefits in this market.



(a)



(b)

Figure 2.11: Disconnected markets in Example 2.2: (a) bus 0, and (b) bus 1.

If the two buses 0 and 1 are now connected through a transmission line and no constraint on the maximal flow is considered, we get the solution obtained in Example 2.1. However, we know that this solution is unfeasible when we consider the maximal line flow limit. Then, the market for bus 0 has the same supply curve as that in Figure 2.11(a), but the demand curve considers the demand of bus 1 that can be supplied without violating the line constraint as shown in Figure 2.12(a). In a similar way, the market at bus 1 has the same demand curve as that in Figure 2.11(b), but the supply curve considers the supply of bus 0 that can be imported through the transmission system without violating the flow limit, as shown in Figure 2.12(b).

Table 2.3 summarizes the corresponding clearing price and quantity of each seller/buyer. Table 2.4 indicates the revenues and payments of each of the players.

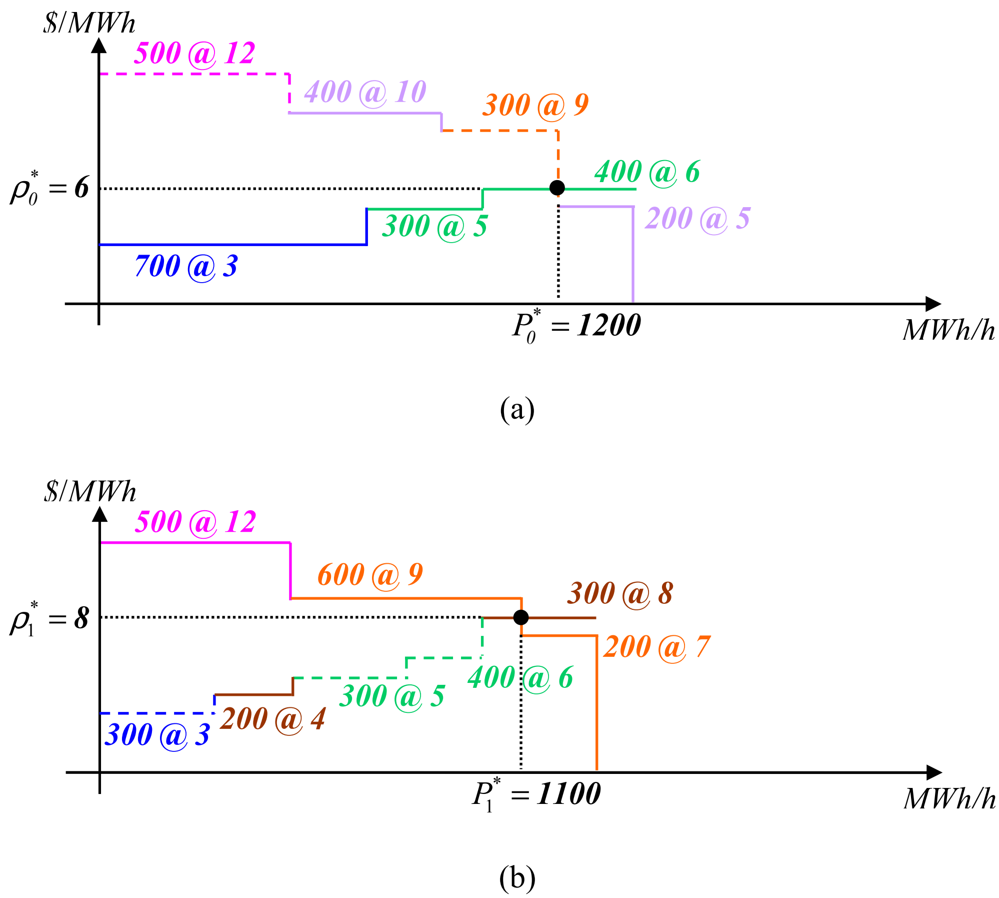


Figure 2.12: Constrained market in Example 2.2: (a) bus 0, and (b) bus 1.

Table 2.3: Example 2.2: Market clearing prices and quantities.

seller	clearing quantity (MWh/h)	clearing price ($)
S_1	700	6.0
S_2	500	6.0
S_3	300	8.0
total	1500	-

buyer	clearing quantity (MWh/h)	clearing price ($)
B_1	400	6.0
B_2	500	8.0
B_3	600	8.0
total	1500	-

Table 2.4: Example 2.2: Revenues and payments of sellers and buyers.

seller	revenues ($)
S_1	4200
S_2	3000
S_3	2400
total	9600

buyer	payments ($)
B_1	2400
B_2	4000
B_3	4800
total	11 200

This example illustrates the increasing complexity due to the explicit consideration of the flow limits.

■

Under the dispatch of Table 2.3, the flow though the line is exactly 800 MW which is at the line limit. As such, the system is congested as stated in Definition 2.3. For this simple two-bus system, we can see that congestion leads to different nodal prices and the redispatch of the generation and demand away form the preferred generation/demand schedule of the unconstrained problem. We can see that the total energy trade decreases, from 1600 MWh/h in the unconstrained case to 1500 MWh/h in the constrained case. We can also notice that the amount paid by the buyers differs from the amount collected by the sellers. This difference of $1600 corresponds to the congestion rents. We introduce this concept more formally in the next chapter.

In this chapter we have provided a mathematical framework that integrates into a single framework the operation of the power system and the electricity markets. We have shown that, due to the consideration of physical constraints, the solution of the **(TCP)** represents the effects of the interaction between the market players and the network layer. The IGO ensures that the nodal injection vector $\underline{\boldsymbol{P}}^S$ and the nodal withdrawal vector $\underline{\boldsymbol{P}}^B$

determined from economic considerations are also physically feasible. Once the feasibility is established, the congested lines are identified and the LMPs are evaluated. Then the quantity sold/bought by each seller/buyer can be determined. Examples 2.1 and 2.2 serve to clearly illustrate the differences between the unconstrained and the constrained systems. We can see how the system has to be redispatched to ensure that the flow limit is not violated, and how congestion in this system leads to different nodal prices. This situation creates a difference between the payments of the buyers and the revenues of the sellers.

3. MARKET PERFORMANCE MEASURES

In this chapter, we introduce the various metrics we use to measure the market performance and the impacts of congestion [21]. We consider the metrics for measuring the performance of the individual players. These metrics are useful for the analysis of both the unconstrained and constrained markets. Then we discuss the metrics we require for constrained markets. Again, we illustrate the various concepts with examples.

3.1 Metrics for Measuring Individual Player Performance

The social welfare provides a measure of the performance of the market as a whole. However, it gives no insights about how each individual player fares. We define measures to evaluate the performance of each seller and buyer and also that of their respective aggregated groups. The relationship of the aggregated group measures to the social welfare is discussed.

Definition 3.1: The *individual producer surplus* $\mathscr{S}^{S_i}$ of seller S_i measures the difference between the revenues that the seller receives for his clearing quantity at the market clearing price and those that he would receive at his offer prices. For seller S_i located at node *n*,

$$\mathscr{S}^{S_i} = \rho_n^* \cdot \left[P^{S_i} \right]^* - \mathscr{C}^{S_i}\left(\left[P^{S_i} \right]^* \right). \tag{3.1}$$

Here, ρ_n^* is the clearing market price (LMP) at bus *n*, and $\left[P^{S_i}\right]^*$ is the market clearing quantity sold by seller S_i.

■

We can think of $\mathscr{P}^{S_i}$ as the profits of seller S_i for the sale of the quantity $\left[P^{S_i}\right]^*$ as a result of the submitted offer. The individual producer surplus for seller S_i, when his offer function is affine, is shown in Figure 3.1.

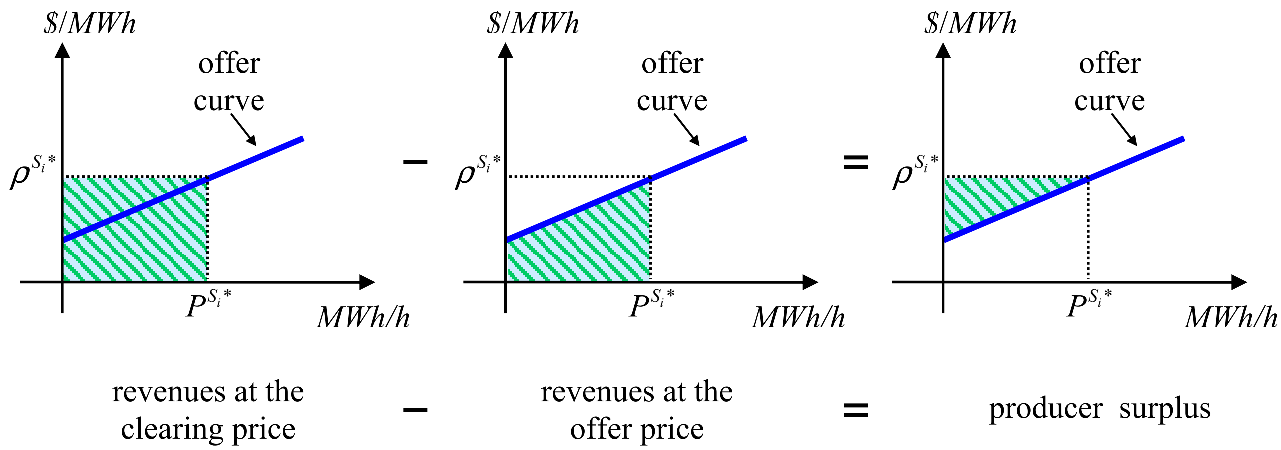


Figure 3.1: The individual producer surplus of seller S_i for the case of the offer function being affine.

Definition 3.2: The *individual consumer surplus* $\mathscr{S}^{B_j}$ of a buyer B_j measures the difference between the value of the energy purchased at the bid prices and that at the market clearing price. For buyer B_j located at bus *n*

$$\mathscr{S}^{B_j} = \mathscr{B}^{B_j}\left(\left[P^{B_j}\right]^*\right) - \rho_n^* \cdot \left[P^{B_j}\right]^*. \tag{3.2}$$

Here, $\left[P^{B_j}\right]^*$ is the market clearing quantity bought by buyer B_j.

■

We can view $\mathscr{S}^{B_j}$ as the savings realized by the buyer B_j in the market with respect to his willingness to pay. The individual consumer surplus for buyer B_j, when his bid function is affine, is shown in Figure 3.2.

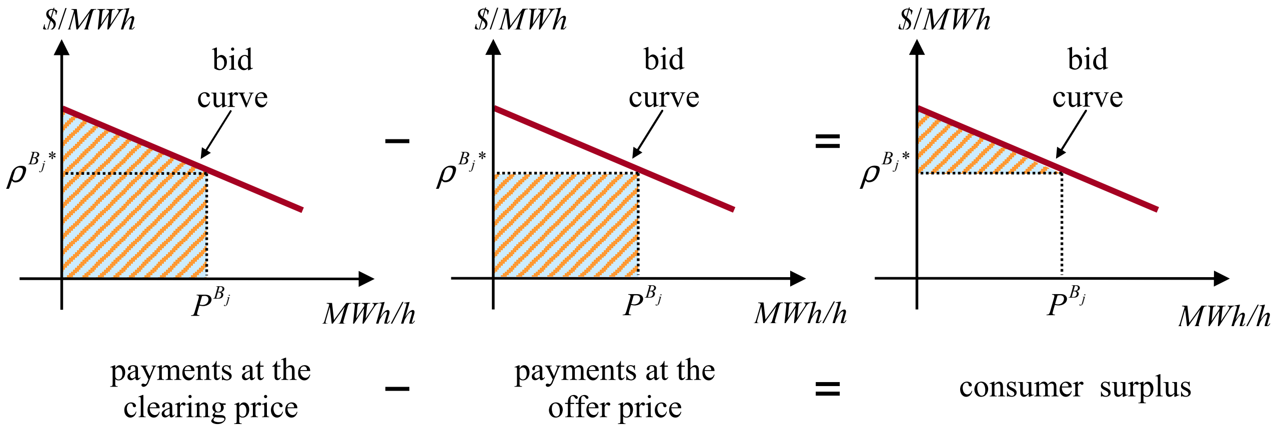


Figure 3.2: The individual consumer surplus of buyer B_j for the case of the bid function being affine.

Definition 3.3: The *nodal producer surplus* $\mathscr{S}_n^S$ at node n is the sum of the individual producer surpluses of each seller located at bus n

$$\mathscr{S}_n^S = \sum_{\substack{S_i \in \mathbb{S}, \\ S_i \text{ is at node } n}} \mathscr{S}^{S_i} . \tag{3.3}$$

■

We can express $\mathscr{S}_n^S$ in terms of the aggregated costs function $\mathscr{C}_n^S(P_n^S)$ of all the sellers located at bus n:

$$\mathscr{S}_n^S = \rho_n^* \cdot \left[P_n^S\right]^* - \mathscr{C}_n^S\left(\left[P_n^S\right]^*\right), \tag{3.4}$$

where P_n^S is given in (2.16).

Definition 3.4: The *nodal consumer surplus* $\mathscr{S}_n^B$ at node *n* is the sum of the *individual consumer surpluses* of each buyer located at bus *n*

$$\mathscr{S}_n^B = \sum_{\substack{B_j \in \mathbb{B}, \\ B_j \text{ is at node } n}} \mathscr{S}^{B_j} . \tag{3.5}$$

■

We can express $\mathscr{S}_n^B$ in terms of the aggregated benefits function $\mathscr{B}_n^B(P_n^B)$ of all the buyers located at bus *n*:

$$\mathscr{S}_n^B = \mathscr{B}_n^B\left(\left[P_n^B\right]^*\right) - \rho_n^* \cdot \left[P_n^B\right]^*, \tag{3.6}$$

where P_n^B is given in (2.18).

Definition 3.5: The *producers' surplus* $\mathscr{S}^S$ of the market corresponds to the sum of all the individual producer surpluses of all the sellers in the market

$$\mathscr{S}^S = \sum_{i=1}^{M^S} \mathscr{S}^{S_i} = \sum_{n=0}^{N} \left\{ \sum_{\substack{S_i \in \mathbb{S} \\ S_i \text{ is at node } n}} \mathscr{S}^{S_i} \right\} = \sum_{n=0}^{N} \mathscr{S}_n^S . \tag{3.7}$$

■

Definition 3.6: The *consumers' surplus* $\mathscr{S}^B$ of the market corresponds to the sum of all the individual consumer surpluses of all the buyers in the market

$$\mathscr{S}^B = \sum_{j=1}^{M^B} \mathscr{S}^{B_j} = \sum_{n=0}^{N} \left\{ \sum_{\substack{B_j \in \mathbb{B} \\ B_j \text{ is at node } n}} \mathscr{S}^{B_j} \right\} = \sum_{n=0}^{N} \mathscr{S}_n^B . \tag{3.8}$$

■

For the case of electricity markets in which brokers/marketers play an important role, the terms *producer surplus* and *consumer surplus* are, in some sense, misleading. It would be therefore more appropriate to use the terms *seller surplus* and *buyer surplus*. Nevertheless, we will use the standard terminology [22]. We henceforth use the term *seller* to represent generation entities and brokers/marketers, and the term *buyer* to represent consumers, brokers/marketers, distribution entities, and generation entities.

Let us evaluate the expression for the social welfare in (2.20) at the solution of the **(TCP):**

$$\mathscr{S} \;=\; \sum_{n=0}^{N}\left[\mathscr{B}_n^B\left(\left[P_n^B\right]^*\right) \;-\; \mathscr{C}_n^S\left(\left[P_n^S\right]^*\right)\right].$$

We may replace each $\mathscr{C}_n^S\left(\left[P_n^S\right]^*\right)$ by the expressions in (3.4) so that

$$\mathscr{C}_n^S\left(\left[P_n^S\right]^*\right) \;=\; \rho_n^* \cdot \left[P_n^S\right]^* \;-\; \mathscr{S}_n^S .$$

Similarly, we replace each $\mathscr{B}_n^B\left(\left[P_n^B\right]^*\right)$ by the expression in (3.6), resulting in

$$\mathscr{B}_n^B\left(\left[P_n^B\right]^*\right) \;=\; \mathscr{S}_n^B \;+\; \rho_n^* \cdot \left[P_n^B\right]^* .$$

Therefore,

$$\mathscr{S} \;=\; \sum_{n=0}^{N}\left(\mathscr{S}_n^B \;+\; \rho_n^* \cdot \left[P_n^B\right]^*\right) \;-\; \sum_{n=0}^{N}\left(\rho_n^* \cdot \left[P_n^S\right]^* \;-\; \mathscr{S}_n^S\right),$$

and then using the relation (3.7) and (3.8), we obtain

$$\mathscr{S} \;= \mathscr{S}^S \;+\; \mathscr{S}^B \;+\; \left[\sum_{n=0}^{N}\rho_n^* \cdot \left[P_n^B\right]^* \;-\; \sum_{n=0}^{N}\rho_n^* \cdot \left[P^S\right]^*\right]. \tag{3.9}$$

Thus, the total social welfare is the sum of the producers' surplus, the consumers' surplus, and a term that we investigate in more detail in the next two sections.

3.2 The Transmission-Unconstrained Market Metrics

For the unconstrained market we have a unique market clearing price. Then, $\rho_n^* = \rho^*$ at each bus $n = 0,1,...,N$. Therefore, (3.9) becomes

$$\mathscr{S} = \mathscr{S}^S + \mathscr{S}^B + \rho^* \cdot \left[\sum_{n=0}^{N} \left[P_n^B \right]^* - \sum_{n=0}^{N} \left[P_n^S \right]^* \right].$$

However, at the market equilibrium there is a supply-demand balance, i.e.,

$$\sum_{n=0}^{N} \left[P_n^B \right]^* = \sum_{n=0}^{N} \left[P_n^S \right]^* .$$

It follows that

$$\mathscr{S}|_u = \mathscr{S}^S + \mathscr{S}^B , \qquad (3.10)$$

where $\mathscr{S}|_u$ denotes the social welfare in the transmission-unconstrained market. Since the social welfare in the unconstrained market is the sum of the producers' and consumers' surpluses, it is sometime called the *social surplus* [21].

We illustrate the concepts of producer surplus, consumer surplus and social welfare in the transmission-unconstrained market for the simple two-bus system used in Chapter 2.

Example 3.1: We consider the unconstrained market of Example 2.1. We calculate the individual producer surplus for each seller and the individual consumer surplus for each buyer. The results are summarized in Table 3.1. Figure 3.3 shows these surpluses graphically. Note that the block of demand that determines the market clearing price has no surplus. From (3.10) it follows that the social welfare for this example is

$$\mathscr{S}|_c = \$4400 + \$4900 = \$9300 .$$

Table 3.1: Example 3.1: Producer and consumer surplus for the market.

Seller	surplus ($)
S_1	1000
S_2	2800
S_3	600
producers' surplus	4400

buyer	surplus ($)
B_1	1200
B_2	2500
B_3	1200
consumers' surplus	4900

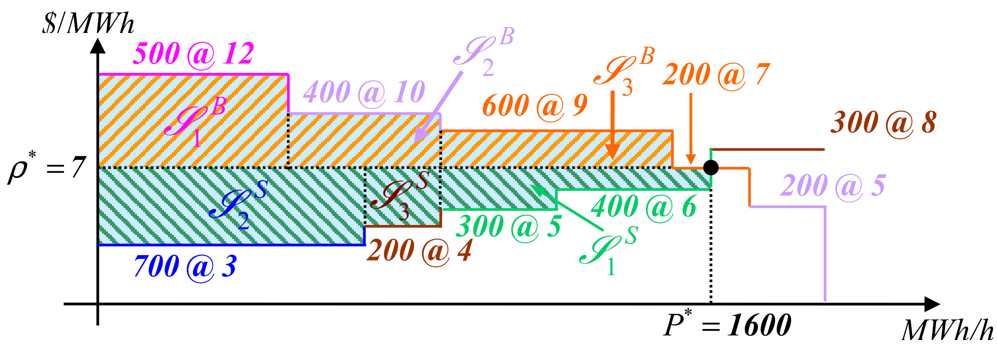


Figure 3.3: Example 3.1: the individual producer and consumer surpluses for the transmission-unconstrained market.

■

In this section we defined the measures to evaluate the performance of individual players. We used the two-bus system of the previous chapter to exemplify these metrics. Example 3.1 clearly shows that in the unconstrained market, the social welfare achieved corresponds to the maximum available, i.e., the area between the aggregated demand curve and the aggregated supply curve.

3.3 The Transmission-Constrained Market Measures

When the transmission constraints are introduced, the measures defined in the previous section are modified since we no longer have a uniform market clearing price. Moreover, we need to introduce a measure to evaluate the impacts of the congestion in the system.

We have seen that congestion leads to the change in the preferred generation/demand schedule which, in turn, may imply a curtailment in production or/and consumption. This change in the schedule is known as *redispatch*. We can evaluate the impacts of congestion in terms of both the *MW* quantity that has to be rescheduled and the costs of the redispatch.

Definition 3.7: The *redispatch power* r_M is given by the increase, if any, of the sales of the sellers in the transmission-constrained system to ensure that there are no constraint violations. Let Δ^{S_i} be the difference between the energy $\left.\left[P^{S_i}\right]^*\right|_c$ sold in hour h by seller S_i in the constrained market and energy $\left.\left[P^{S_i}\right]^*\right|_u$ sold in the unconstrained market, i.e.,

$$\Delta^{S_i} = \left.\left[P^{S_i}\right]^*\right|_c - \left.\left[P^{S_i}\right]^*\right|_u . \tag{3.11}$$

Then, the redispatch power is the nonnegative quantity

$$r_M = \sum_{i=1}^{M^S} \max\left\{0, \Delta^{S_i}\right\}. \tag{3.12}$$

■

The redispatch power measures the energy that has to be supplied by a possibly more expensive seller due to congestion. Absent transmission considerations, the same energy would be supplied by less costly sellers as established by the preferred generation/demand schedule. We associate with the solution of the **(TCP)** the aggregated quantity

$$\left[P^{S}\right]^{*}\Big|_{c} = \sum_{i=1}^{M^{S}} \left[P^{S_i}\right]^{*}\Big|_{c} . \tag{3.13}$$

Definition 3.8: The *redispatch costs* of the system $\mathscr{C}_r$ are given by the difference between the costs of the energy $\left[P^{S}\right]^{*}\Big|_{c}$ for the constrained market and the corresponding costs for the supply of the same energy in the unconstrained market. We denote the total costs of $\left[P^{S}\right]^{*}\Big|_{c}$ as $\mathcal{C}_c\left(\left[P^{S}\right]^{*}\Big|_{c}\right)$ and those that would be incurred in the unconstrained market for the same amount of energy as $\mathcal{C}_u\left(\left[P^{S}\right]^{*}\Big|_{c}\right)$.

Then, the redispatch costs $\mathscr{C}_r$ are given by

$$\mathscr{C}_r = \mathcal{C}_c\left(\left[P^{S}\right]^{*}\Big|_{c}\right) - \mathcal{C}_u\left(\left[P^{S}\right]^{*}\Big|_{c}\right). \tag{3.14}$$

■

The redispatch costs are incurred when the rescheduling of the supply-side resources becomes necessary to avoid violations of the transmission constraints. Since redispatch power is nonnegative consequently, redispatch costs are also nonnegative. To illustrate the redispatch power and costs we again use the two-bus system of Chapter 2.

Example 3.2: We consider the market equilibriums of Examples 2.1 and 2.2. Tables 2.1 and 2.3 show the power sold by each seller in the unconstrained and constrained case, respectively. To calculate the *redispatch power*, we first calculate Δ^{S_i} for each of the seller,

$$\Delta^{S_1} = \left[P^{S_1}\right]^*\Big|_c - \left[P^{S_1}\right]^*\Big|_u = 700 - 700 = 0.$$

$$\Delta^{S_2} = \left[P^{S_2}\right]^*\Big|_c - \left[P^{S_2}\right]^*\Big|_u = 500 - 700 = -200.$$

$$\Delta^{S_3} = \left[P^{S_3}\right]^*\Big|_c - \left[P^{S_3}\right]^*\Big|_u = 300 - 200 = 100.$$

Then, we have that

$$\begin{aligned} r_M &= \max\left\{0,\Delta^{S_1}\right\} + \max\left\{0,\Delta^{S_2}\right\} + \max\left\{0,\Delta^{S_3}\right\} \\ &= \max\left\{0,0\right\} + \max\left\{0,-200\right\} + \max\left\{0,100\right\} \\ &= 100 \text{ MWh/h.} \end{aligned}$$

The total energy sold in the constrained market is

$$\left[P^{S}\right]^*\Big|_c = 700 + 500 + 300 = 1500 \text{ MWh/h}$$

then, the total costs are

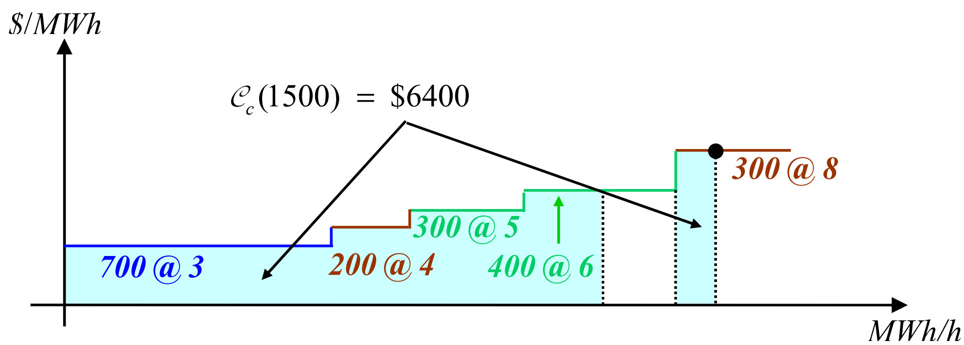
$$\mathcal{C}_c(1500) = \$6400$$

as can be ascertained from Figure 3.4(a). But, the corresponding costs of the same amount of energy in the unconstrained market are

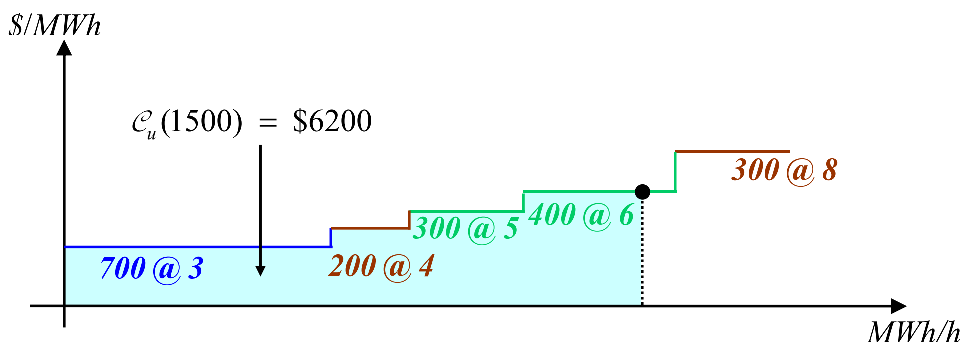
$$\mathcal{C}_u(1500) = \$6200$$

as shown in Figure 3.4(b). Then, the redispatch costs for the transmission-constrained market are

$$\mathscr{C}_r = \mathcal{C}_c(1500) - \mathcal{C}_u(1500) = \$6400 - \$6200 = \$200.$$



(a)



(b)

Figure 3.4: Example 3.2: Redispatch costs with (a) the costs of selling 1500 MWh/h in the constrained market, and (b) the costs of selling 1500 MWh/h in the unconstrained market.

The implications of congestion in this market are that seller S_2 sells 200 MWh/h less than in the transmission-unconstrained market. Then, seller S_3 could make up the sales of the reduction of 200 MWh/h. However buyer B_3's willingness to pay is high enough only for the first 100 MWh/h offered by seller S_3. Consequently, seller S_3 can only sell 100 MWh/h more in the transmission-constrained market. This means that there is a redispatch power of 100 MWh/h and the resulting redispatch costs of \$200. Note that the constrained market outcomes implicitly imply a decrease of the purchases of 100 MWh/h due to the higher redispatch costs.

■

We discuss further the nature of the redispatch costs in Example 4.1 in the next chapter. The redispatch metrics, redispatch power and redispatch costs, are principally related to the supply-side players. But, the redispatch affects the surpluses of all the players in the system and consequently the social welfare. We now focus our attention on the third term of the social welfare given in (3.9), and for this purpose we introduce the following definition:

Definition 3.9: The *congestion rents* of the system κ are given by the difference between the total payments for the buyers' purchases and the total revenues of the sellers:

$$\kappa = \sum_{n=0}^{N} \rho_n^* \cdot \left[P_n^B \right]^* - \sum_{n=0}^{N} \rho_n^* \cdot \left[P_n^S \right]^* . \qquad (3.15)$$

■

The congestion rents arise from the different clearing prices at each bus and measure the impacts of these differences. We consider the relationship in (3.9) and we recognize the quantity in the brackets to be κ. Therefore,

$$\mathscr{S} = \mathscr{S}^S + \mathscr{S}^B + \kappa . \qquad (3.16)$$

While some authors use the term *merchandising* (or *merchandise*) *surplus* for the congestion rents [1], [20], [21], [23], this terminology is not commonly used in the economics literature and so we do not adopt it for this thesis.

The congestion rents can be seen to measure how the trade between different players is mediated; in essence, we have that the energy is bought from the sellers at prices different from the prices paid by the buyers. This results in the mediation costs

κ [20]. Congestion rents may be positive, zero or negative, depending on the nature of the grid constraints [21]. The congestion rents are collected by the IGO.

We next illustrate the evaluation of the congestion rents for the simple two-bus system used in Chapter 2.

Example 3.3: Consider the constrained market discussed in Example 2.2. We calculate each individual producer surplus, consumer surplus, congestion rents, and social welfare for this example. The results are presented in Table 3.2. The producers' and consumers' surpluses are also depicted in Figure 3.5. The congestion rents obtained using (3.15) are

$$\kappa = 1600 .$$

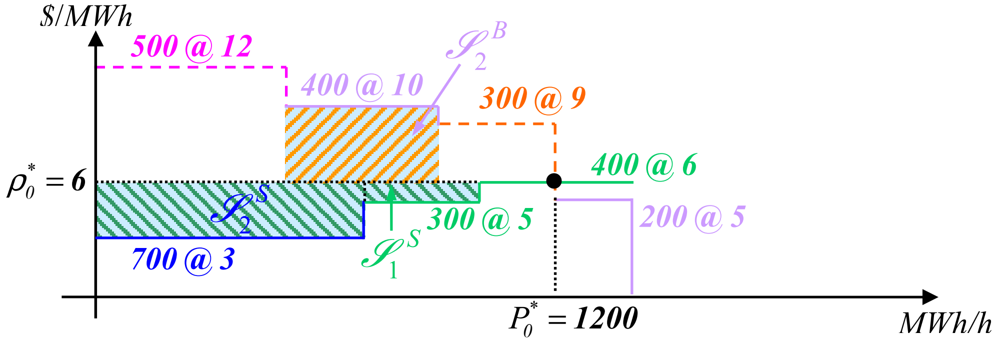
From (3.16) it follows that the social welfare for this example is

$$\mathscr{S}|_u = \$3200 + \$4200 + \$1600 = \$9000 .$$

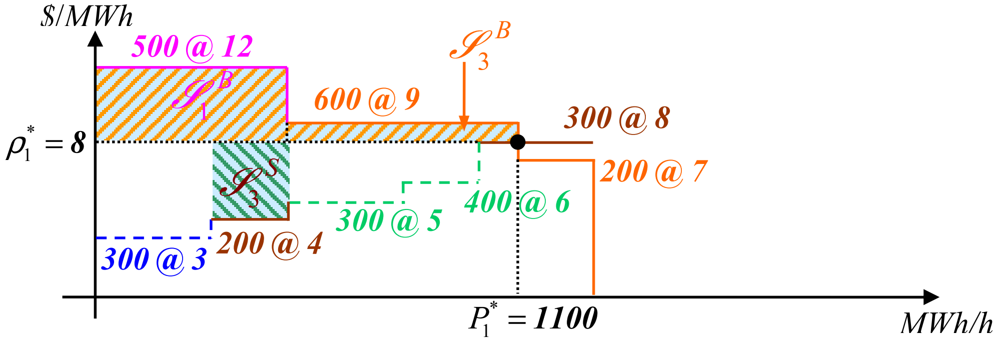
Table 3.2: Example 3.2: the individual surplus of each player, the producers' surplus, and the consumers' surplus

seller	surplus ($)
S_1	300
S_2	2100
S_3	800
producers' surplus	3200

buyer	surplus ($)
B_1	1600
B_2	2000
B_3	600
consumers' surplus	4200



(a)



(b)

Figure 3.5: Example 3.3: (a) the individual player surplus for each player at bus 0, and (b) the individual player surplus for each player at bus 1.

■

This simple example clearly shows that the social welfare of the constrained market is bounded from above by that of the unconstrained market. A comparison of the formulation of the **(TUP)** and **(TCP)** implies that such a result always holds. This reduction in the social welfare represents the market efficiency loss due to congestion.

Definition 3.10: The *market efficiency loss* $\mathcal{E}$ is the reduction in the social welfare caused by congestion

$$\mathcal{E} \triangleq -(\mathscr{S}|_c - \mathscr{S}|_u). \tag{3.17}$$

■

In general, the market efficiency loss measures the value of the energy that is neither sold nor bought due to the presence of congestion in the system [16], [22]. The loss of market efficiency is known in the economic literature as the *dead-weight loss* [14], [16]. We define $\mathscr{E}$ in such a way as to always be a nonnegative quantity. We illustrate $\mathscr{E}$ for Example 3.3.

Example 3.4: We calculate the market efficiency loss using the results of Examples 3.1 and 3.2. Thus,

$$\mathscr{E} = -(\$9300 - \$9000) = \$300.$$

■

We provide a more detailed discussion of the market efficiency loss in the next chapter.

In Chapter 2, we saw that the congestion impacts the system by changing the preferred generation/demand schedule and shifting from the single market equilibrium point to two distinct nodal equilibrium points. In the examples of this chapter, we see the impacts of congestion in monetary, energy and efficiency loss terms through the metrics defined. Congestion, in general, leads to a market efficiency loss, changes – increases or decreases – of the producer surplus of some of the sellers and the consumer surplus of some of the buyers, a new congestion rents term, and an overall reduction in the social welfare.

3.4 Summary of the Market Performance Measures

We have defined three basic market performance measures: the social welfare, the producers' surplus, and the consumers' surplus. The social welfare is a key indicator of the performance of the market as a whole. The producers' surplus and the consumers' surplus serve to evaluate how all the sellers and all the buyers fare, respectively. We can also assess the performance of the individual players through the individual producer surplus and the individual consumer surplus.

We have also defined measures to evaluate the impacts of congestion. We have the redispatch power and the redispatch costs that basically evaluate the impacts of rescheduling including the additional costs incurred due to a shift away from the preferred generation/demand schedule. The congestion rents correspond to the difference between the amounts paid by the buyers and the amounts collected by the sellers. The market efficiency loss represents the reduction in the social welfare due to congestion. It is important to note that each of these measures is zero in the case that there is no congestion in the network.

Figure 3.6 illustrates conceptually how to view the metrics introduced in this chapter in terms of the supply and demand curves. In effect, this conceptual diagram is valid only for a system with the generation and load at a single node. In the presence of congestion, there is a difference in the nodal sales price ρ^S and nodal purchase price ρ^B. This difference leads to a shift away from the unconstrained market equilibrium to the constrained equilibrium characterized by the different sales/purchase prices. The resulting decrease in the producers' surplus, consumers' surplus, and overall market efficiency are clearly visible. Under congestion, the individual producer surplus of a seller and the individual consumer surplus of a buyer change positively or negatively.

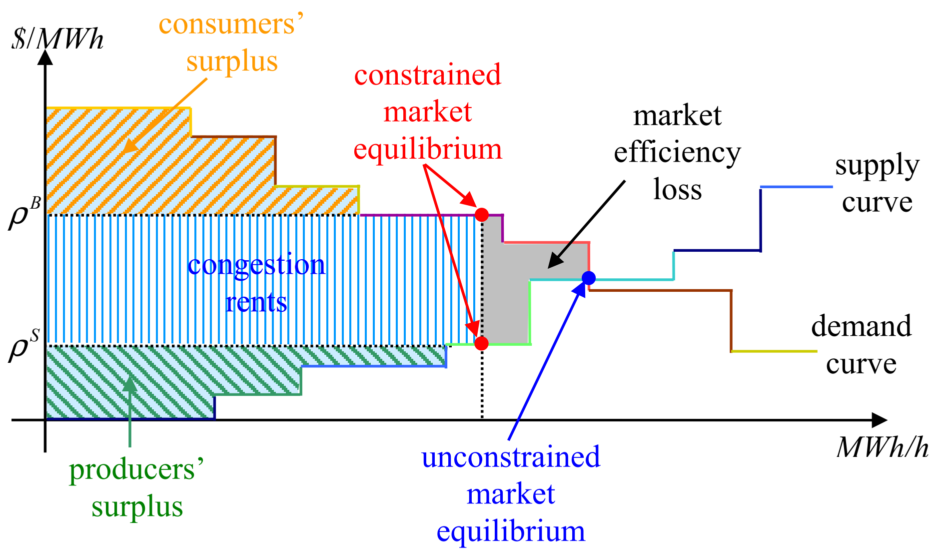


Figure 3.6: Market performance measures.

The congestion rents may be interpreted as the short-run costs of the transmission services. This statement should not be taken to imply that the transmission services have no costs when congestion does not occur, but when it does occur, the congestion rents represent the additional costs that the transmission customers incur due to the constraints of the of transmission network. The presence of congestion makes markets lose efficiency because of the inability of the transmission system to accommodate the preferred generation/demand schedule that would lead to larger social welfare than that of the constrained market. The market efficiency loss captures the loss in efficiency due to the lack of capability of accommodating the preferred generation/demand schedule.

4. CHARACTERIZATION AND MEASUREMENT OF CONGESTION IMPACTS

Congestion occurs whenever the preferred generation/demand schedule requires the provision of transmission services beyond the capability of the transmission system. In such a case, the preferred schedule cannot be accommodated without violating the physical limits of the transmission network. Congestion is a real fact-of-life phenomenon whose presence does impact the outcomes of the markets. Consequently, the benefits foreseen through restructuring may not be fully realized since the presence of congestion does introduce unavoidable losses in efficiency. Therefore, the effective management of congestion is a critically important contributor into the smooth functioning of competitive electricity markets through its key role of minimizing the impacts of congestion on both the individual players and the market as a whole. In this chapter, we examine the characterization of congestion and its impacts. We provide additional insights into the measures defined in the previous chapter. The analysis in this chapter serves to lay the foundation for the motivation of the simulation studies that we report in Chapter 5.

4.1 Congestion

Congestion is a phenomenon that affects the entire system and, as such, all the players in electricity markets. Once a line becomes congested, the entire system suffers the resulting impacts: the market shifts from a single equilibrium point to possibly different nodal equilibrium points, the individual surplus of each player changes and the market outcomes are, consequently, changed. Congestion has come to play an important role in power systems in the restructuring of the electricity industry [2]. Transmission congestion existed of course, even before restructuring, but was discussed in the context of constrained system operations [4]. The optimal operation of the system requires the optimization of a specified objective function subject to ensuring that no violation of the constraints occurs. The utilities were vertically integrated, owning and controlling both generation and transmission, so any conflict between security and economics was resolved by the single decision-making entity, which both owned and controlled all the facilities. However, in competitive markets, where generation and transmission are unbundled, there are many players vying for transmission services. The IGO, therefore, faces the challenge of maximizing the social welfare of the market while ensuring the reliability and integrity of the system [24]. Congestion thus becomes a key issue in the effective operation of power systems under competitive conditions [25].

Congestion may occur at any point in time, so its study may be undertaken over different time frames. Over the short term, congestion conditions must be managed in the day-ahead market as well as over the shorter term in the hour-ahead and the real-time markets. Several congestion management schemes have been proposed by and implemented in various jurisdictions around the world [1]-[4], [26], [27]. These schemes differ in terms of their objectives, the approach for congestion relief and the pricing mechanism. For example, some schemes have as primary objective the redispatch of the least amount of megawatts, in order to minimize the change in the preferred generation/demand schedule. Other schemes have the objective to minimize the costs incurred in the redispatch. The set of rules involved in each congestion management scheme need to be robust, fair and clear to each market participant to facilitate the competition in the market [2]. Over time, the increased use of the grid exacerbates the congestion situation. Then, new investments in the expansion/improvement of the transmission network become necessary to alleviate the congestion bottleneck impacts and to accommodate the changing patterns over time of regional energy transfers [28].

Our focus is the evaluation of congestion and its impacts in the day-ahead market. The measures we discuss and the analysis we undertake are appropriate for such markets. The discussion of the long-term congestion issues and the analysis of real-time congestion are beyond the scope of this report.

The IGO ensures the reliability of the system by verifying the capability of the network to provide transmission services for the forecasted reference case condition as well as under set of postulated contingencies. Congestion is said to occur if, in any of these cases including each contingency case considered, the network is unable to accommodate the desired generation/demand schedule. For the purpose of simplicity, we only consider the forecasted reference case in this thesis. However, the analysis used here can be extended in a straight forward way to include also all the postulated contingency cases.

The problem of congestion lies in the insufficient transfer capability of the system [25]. Under the ideal conditions of the transmission-unconstrained markets, the buyers try to purchase energy from the sellers with the lowest offer prices. When the physical constraints of the transmission network are also considered, the constrained transfer capabilities of the network may be unable to accommodate the preferred unconstrained market schedule without violating the physical constraints. Therefore, congestion is caused by the lack of sufficient transfer capabilities to simultaneously transfer energy between the various selling and buying entities. The congestion conditions may be alleviated if the lower priced sellers are located at areas from which additional transfers may be carried out without stressing the limited transfer capabilities of the transmission network.[1]

Congestion affects virtually each market player either in a positive or in a negative way. For example, a seller (buyer) located at an importing node may suffer an (a) increase (decrease) in its producer (consumer) surplus. This situation arises because such a buyer is limited in his ability to access the energy from other locations due to system transfer constraints. Therefore, the only choice is to buy the energy from the higher-priced seller located at the importing node. This lack of choice may result in the higher payment prices and/or reduced purchases of energy. Therefore, the buyer's consumer surplus decreases. However, a seller located at the importing node may offer his output at higher prices, and if there is willingness to pay on the part of the buyers, he may sell more energy, resulting in an increased producer surplus. Conversely, a seller (buyer) located at the exporting node may suffer a (an) decrease (increase) in his producer (consumer) surplus. In the presence of congestion, the clearing price at the exporting node may decrease, resulting in this seller's lower energy sales than in the unconstrained market; consequently, its producer surplus decreases. The buyer, on the other side, may increase its energy purchases if the buyer's willingness to pay is sufficiently high and in this way its consumer surplus increases over that of the constrained market. In addition to the change in the surplus of each individual player, the producers' surplus and the consumers' surplus also change under congestion. These

[1] Therefore, it is possible to view generation and transmission as substitutable goods.

changes imply that the social welfare reached at the unconstrained case is redistributed in the constrained case. The reductions in the producers' surplus or consumers' surplus or both make up the congestion rents and the market efficiency loss.

The problems caused by congestion do not necessarily lead to efforts for its total elimination since the costs of removing congestion may be far greater that the benefits attainable. It is unrealistic to design a network that would have no congestion. Strategic investments in transmission facilities may result in lowering the congestion rents and the market efficiency loss. The congestion rents and market efficiency loss constitute short-term signals. How they may be used to create long-term signals to incentive transmission investments is still an open research area.

We use the discussion on congestion and its impacts of this section to develop the analysis in the next sections in order to explain in more detail the deployment of the metrics we need to measure the quantifiable impacts of congestion.

4.2 Redispatch Impacts

We discussed in Section 3.2 that the occurrence of congestion may result in a change of the preferred generation/demand schedule in order to avoid any violations of the transmission network limits. Therefore, in the supply-demand balance of the transmission constrained market, the quantity sold/bought by each seller/buyer may change. These changes have additional impacts in terms of the energy that may have to be supplied by out-of-merit-order[2] blocks and the loss of benefits due to the inability, be it physical or economic, to supply some of the buyers' demand.

In Section 3.2, we discussed the calculation of the redispatch power and the redispatch costs. We analyze the nature of the redispatch costs in more detail in this section. In order to do so, we first revisit the results of Example 3.2, in which we computed the redispatch metrics for the simple two-bus system of Chapter 2.

Example 4.1: We consider the unconstrained dispatch of the two-bus system in Example 2.1, with the resulting flow through the line being 1000 MW. To reduce this flow to the 800 MW maximum allowed through the line, there is a need to unload 200 MWh/h. From the economic point of view, such unloading should come from the most expensive seller at bus 0, leading to the unloading of 200 MWh/h from seller S_2 as shown in Figure 4.1(a). The resulting situation provides seller S_3 an opportunity to replace the unloaded capacity of 200 MWh/h as shown in Figure 4.1(b). However, buyer B_3's willingness to pay exceeds the offer price only for the first 100 MWh/h. Then, due to the lack of willingness to pay on the part of buyer B_3, seller S_3 is able to sell 100 MWh/h as indicated in Figure 4.1(c). The cross-hatched area of Figure 4.1(c) represents the costs of purchasing the replacement energy from seller S_3 for the redispatched 100 MWh/h. If the 100 MWh/h were supplied by seller S_2, the costs would be \$600, but since they are, in fact, supplied by seller S_3 the costs become

[2] The merit order list corresponds to the ordered list of blocks used to construct the aggregated supply curve for the transmission unconstrained market.

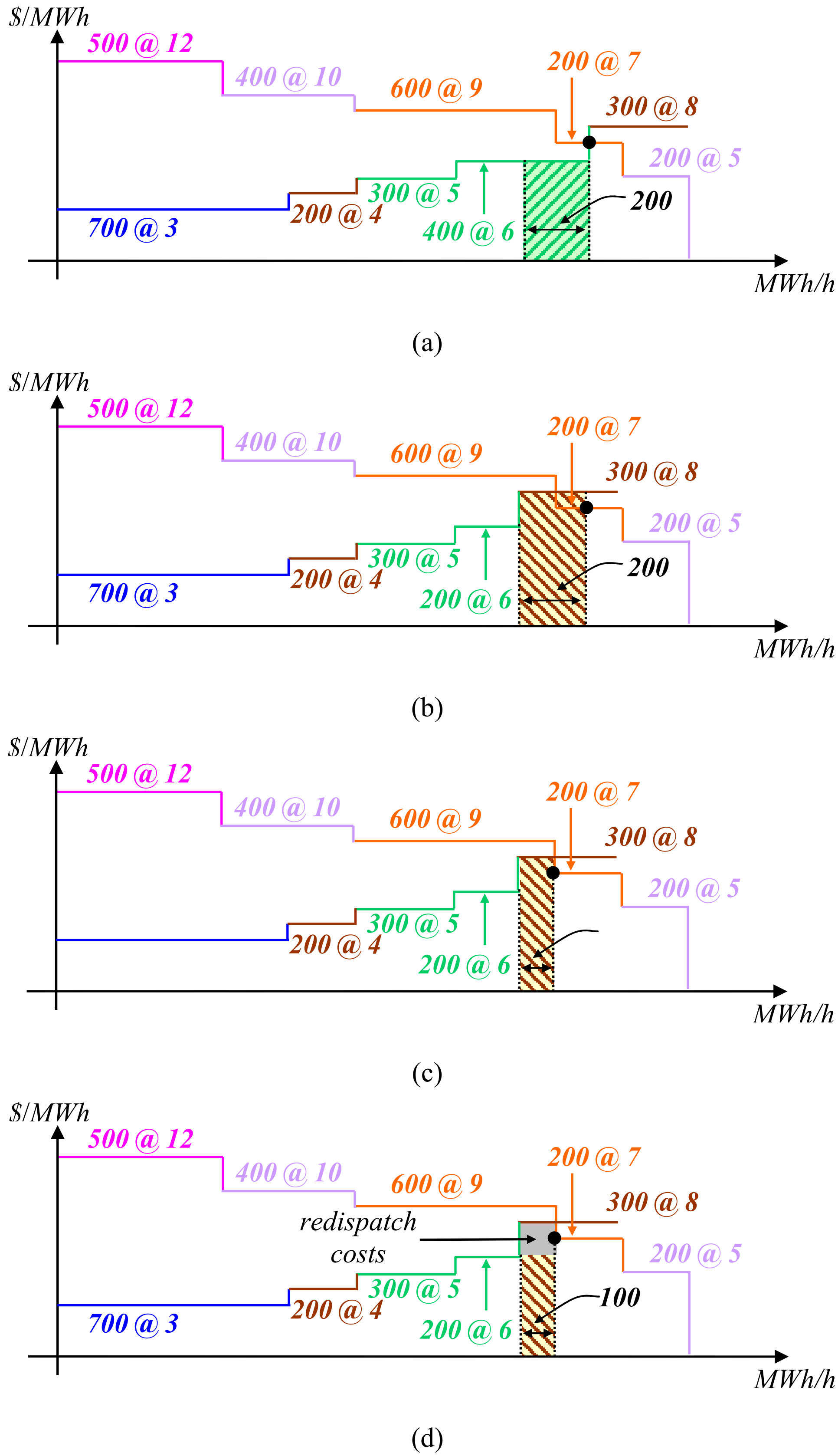


Figure 4.1: Example 3.2: The steps used to evaluate the redispatch costs.

$800. The additional costs of $200 are needed in the redispatch to avoid transmission constraint violations.

■

In this two-bus system case, it is easy to identify which block is unloaded and which block is loaded due to congestion. In larger systems, this identification process is not as straightforward. In general, we can identify the sellers who suffer a decrease in their sales and those who have an increase, but we cannot determine uniquely the replacement block for each unloaded capacity block.

The two redispatch metrics measure the impacts of the constrained market, but only in the supply side since the redispatch in the demand side is not explicitly considered. We do not evaluate the change in the demand schedule to identify the buyers who decrease their purchases and the associated changes in the benefits. The definition of redispatch costs presented in this thesis is not one that is universally used. There are other definitions used in different jurisdictions. For example, in the New England ISO (ISO-NE), redispatch costs are defined to be the additional costs incurred by the out-of-merit-order blocks measured with respect to the clearing price of the unconstrained market [25]. For the two-bus system of Example 2.2, the redispatch costs according to the ISO-NE definition are shown in Figure 4.2. The definition is then important since very different outcomes are measured.

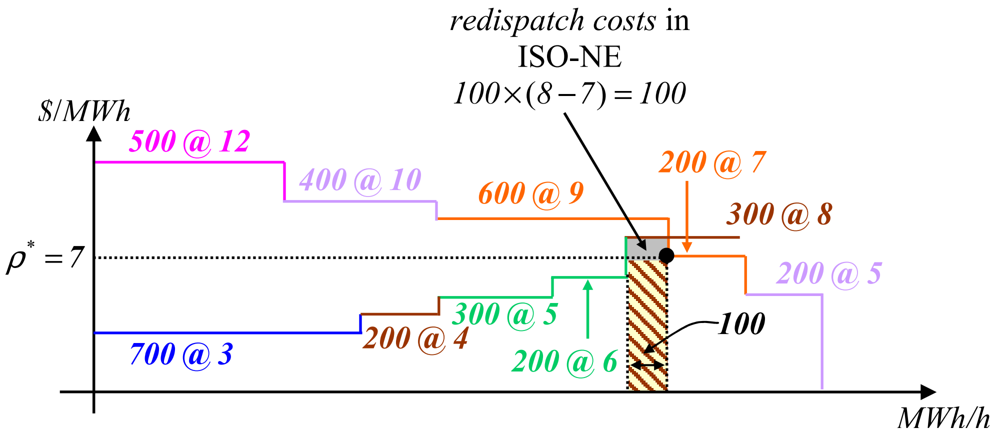


Figure 4.2: Redispatch costs for the two-bus system of Chapter 2 using the definition of the ISO-NE

The presence of congestion in the transmission network may require the redispatch of both the supply and the demand sides in the system. The resulting redispatch impacts both the sellers and the buyers. In general, the consideration of the transmission constraints results in a shift away from the desired schedule and may lead to lowering power purchases and/or the dispatch of more expensive energy. The redispatch metrics presented provide us the capability to measure the impacts of congestion in the entire market. However, these metrics are unable to provide measures of the impacts of congestion on each individual player.

4.3 The Nature of Congestion Rents

In Chapter 3, we defined the *congestion rents* as the difference between the payments of the buyers and the revenues of the sellers. These rents are a direct function of the LMP differences between the receiving and sending nodes, and the amounts of power transfers. The congestion rents provide a measure of the impacts of congestion on the entire market.

We analyze congestion rents by making use of the fictitious transactions concept discussed in Section 2.2. The matching process that established the preferred generation/demand schedule may be viewed in effect, as a set of fictitious transactions between the selling entities and the buyers. The aggregated supply and demand curves are constructed by ordering the offers and bids, respectively, so that the seller with the lowest offer prices is systematically matched with the buyer with the highest bid prices in sequence. In this way, the matches become the fictitious transactions. The presence of congestion brings about a change in the schedule of these fictitious transactions. We view the changed schedule as a modified set of fictitious transactions for the congested network. The congestion rents for each *f*ictitious transaction in the modified set is a simple multiplication. A fictitious transaction between nodes *n* and *m* for *t* MWh/h, injects *t* MWh/h at bus *n* and withdraws *t* MWh/h at node *m*. Then, the fictitious transaction congestion rents are $t(\rho_m^* - \rho_n^*)$ \$/h where $\rho_m^*(\rho_n^*)$ is the LMP at node *n*(*m*) [10], [29]. This value may be positive, negative or zero depending on the LMP difference. Positive rents are collected for $\rho_m^* > \rho_n^*$. Negative fictitious transaction congestion rents are collected for the opposite case. The rationale behind the sign of the fictitious transaction congestion rents is the contribution of each fictitious transaction to the network congestion. If $\rho_m^* > \rho_n^*$, the fictitious transaction aggravates the existing congestion, otherwise, the fictitious transaction flows from a higher-priced node to a lower priced node. The sum of the congestion rents of all the fictitious transactions equals the congestion rents for the entire market given in (3.15). The congestion rents are the costs of providing transmission service for the modified set of fictitious transactions by the congested network.

4.4 Market Efficiency Loss

The overall performance of the market is measured by the social welfare. We have seen that

$$\mathscr{S}\big|_c \;\le\; \mathscr{S}\big|_u \,.$$

The implication of this inequality is that congestion reduces the social welfare reached in the transmission-unconstrained market. The difference between the social welfare of the unconstrained market and that of the constrained market is the market efficiency loss [2], [21]. This inefficiency is due to the inability of the network to accommodate the preferred generation/demand schedule.

The market efficiency loss defined by (3.17) is not meaningful for the case of markets with inelastic demand. In such markets, the social welfare is undefined since it goes to infinity due to the fact that the consumer surplus goes to infinity. In Figure 4.3, we illustrate this situation. When transmission considerations are taken into account and congestion occurs, the social welfare of the market still tends to infinity for the same reason. Consequently, we cannot define the market efficiency loss in such markets [30]. One way to get around this situation is by redefining the social welfare as the negative of the payments by the buyers [2]. In this way, we can still use (3.17) to compute the market efficiency loss, and since the payments of buyers in the constrained market are always greater or equal than the payments in the unconstrained market, the market efficiency loss is nonnegative.

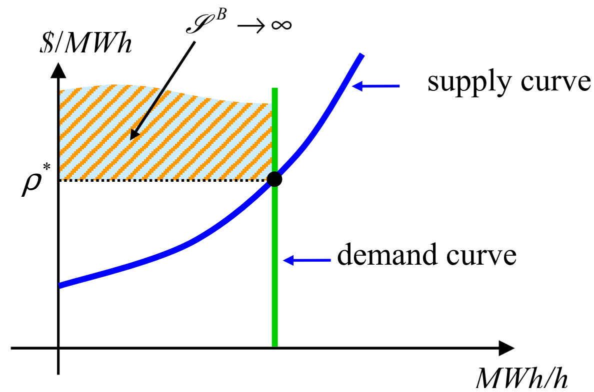


Figure 4.3: Market equilibrium for a general supply curve and an inelastic demand curve.

In general, it is difficult to pinpoint how congestion results in the reassignment of the individual surplus changes into congestion rents and market efficiency loss from the unconstrained market outcomes. In Figure 4.4, we illustrate how the social welfare in the unconstrained market gets redistributed in the constrained market. We note that we cannot identify exactly what portion of the surplus of a player is taken by the congestion rents, market loss of efficiency or even by the surplus of other players. It remains an open question to determine the pattern in the reassignment of the individual players surplus into the market efficiency loss, if any.

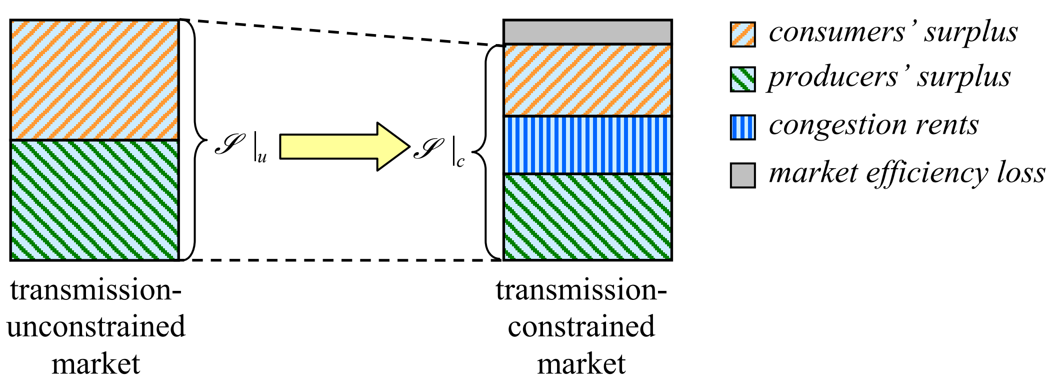


Figure 4.4: Redistribution of the social welfare from the transmission-unconstrained market in the transmission constrained market.

The market efficiency loss is principally due to the physical limitations of the transmission network. The constraints imposed on the market because of the network considerations result in the change of the preferred generation/demand schedule. The impacts of the change away from the preferred schedule – the fact the network is unable to accommodate all transaction that economically would be desirable – are measure by the market efficiency loss.

4.5 Congestion and Market Power

A seller is said to exercise market power when he reduces his output or raises the price of his offer [5]. Such actions lead to a change in the market clearing price [5]. Figure 4.5 illustrates how the clearing price of a market with inelastic demand increases when a seller withholds a block of his offer. In this way, the seller controls the clearing price of the market.

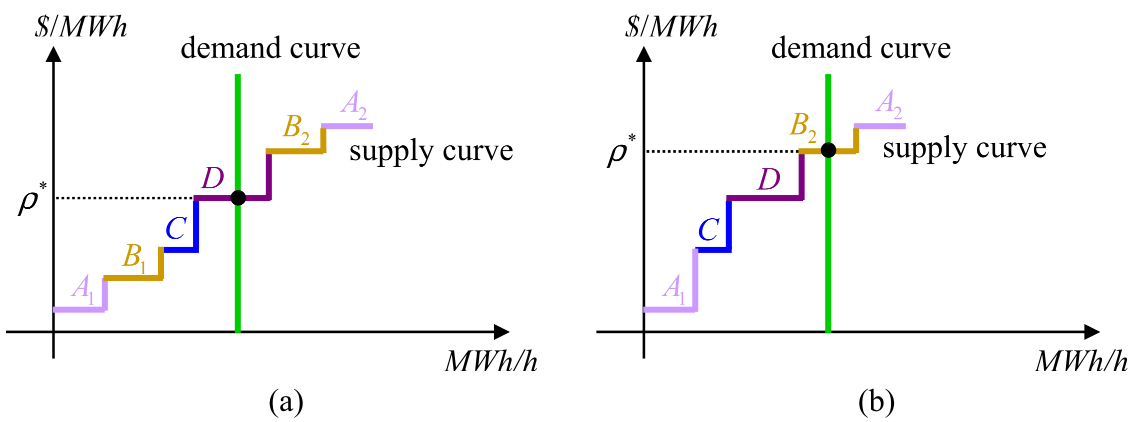


Figure 4.5: Effect of the reduction of the output of seller *B*:
(a) the original supply curve with blocks B_1 and B_2 of seller *B*, and
(b) the modified supply curve with block B_1 removed.

A seller who is unable to exercise market power is known as a *price taker* [22]. A seller who is a price taker can maximize his profits by offering his energy at his marginal costs [31]. On the other hand, a seller who successfully offers his energy at a price above his marginal costs is said to exercise market power and is a *price maker* [2]. We illustrate a simple example of market power exercise in an unregulated monopoly and a price inelastic demand in Figure 4.6. As the monopoly supplier increases his offer prices, the market clearing price increases. Such increases of the offer prices are exercises of market power [5].

There are many situations which can facilitate the exercise of market power. One important situation arises in the presence of congestion. To illustrate how congestion facilitates market power exercise, we consider two examples. These examples provide conceptual illustrations of the role congestion can play in facilitating the exercise of market power.

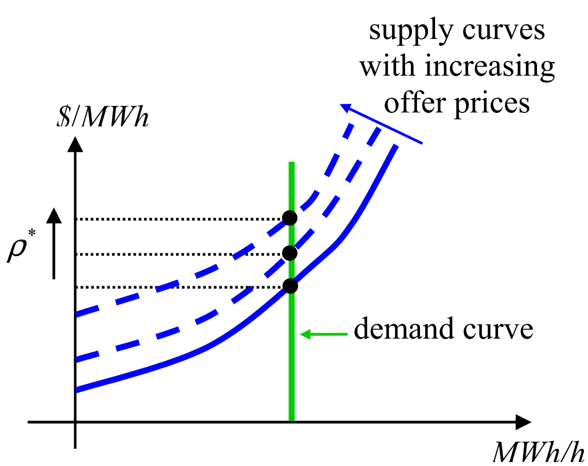


Figure 4.6: Market equilibrium for different supply curves with inelastic demand.

Example 4.2: We consider the two-bus system of Example 2.2 depicted in Figure 2.10. The sellers' offers are given in Figure 2.7(a). We consider that the buyers are not price responsive and have fixed demand with buyer B_1's demand being 200 MWh/h, that of buyer B_2 600 MWh/h, and that of buyer B_3 600 MWh/h. Then, the total demand of the system is 1400 MWh/h. Figure 4.7 shows the market equilibrium for the unconstrained case with a market clearing price of 6 \$/MWh. Due to competition among the sellers, seller S_3 cannot increase the offer price of his 200 MWh/h block above 6 \$/MWh since seller S_2 is offering 200 MWh/h to sell at 6 \$/MWh. Next, we consider the line flow constraint of 800 MW and its impacts. Figure 4.8 shows the resulting nodal market equilibriums for the constrained market. The market clearing price at bus 0 is 5 \$/MWh and the market clearing price at bus 1 is 8 \$/MWh. In this case we observe that seller S_3 can increase its price above 8 \$/MWh and still succeed since buyers B_2 and B_3 are willing to pay any price to meet their 1200 MWh/h demand. Therefore, congestion *enables* seller S_3 to exercise market power at bus 1, since the buyers at bus 1 have no choice.

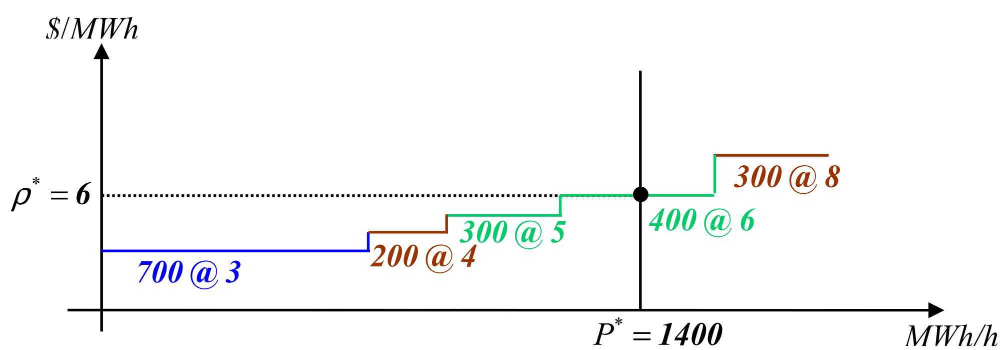
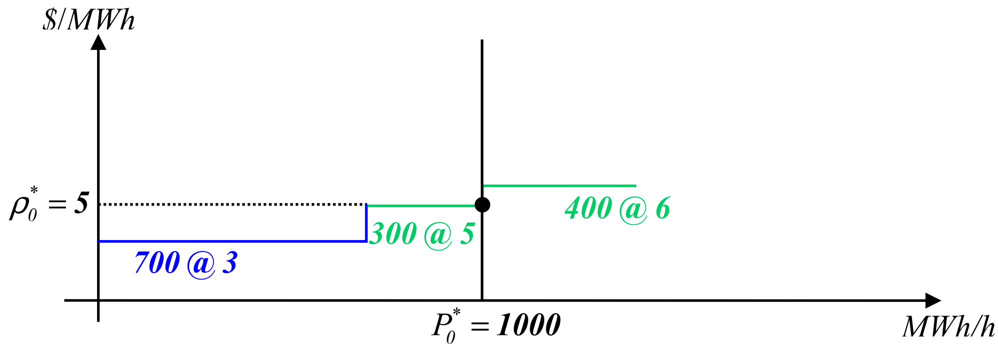
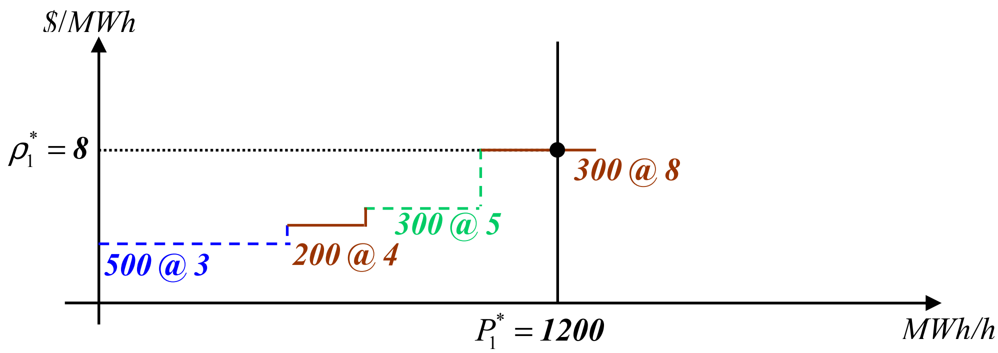


Figure 4.7: Example 4.2: market equilibrium for the unconstrained case.



(a)



(b)

Figure 4.8: Example 4.2: the equilibrium of the constrained markets at (a) bus 0 and at (b) bus 1.

■

Example 4.3: We consider again the system of Example 2.2. The seller's offer and the buyers' bids are identical to those given in Figures 2.7 and 2.8. In Figure 2.12(b) the market clearing price for bus 1 is 8 \$/MWh. If seller S_3 increases the offer price of his last block from 8 \$/MWh to 8.5 \$/MWh, the market clearing price at bus 1 increases from 8 \$/ MWh to 8.5 \$/MWh, as is shown in Figure 4.9. We see again that congestion allows seller S_3 to exercise market power at bus 1.

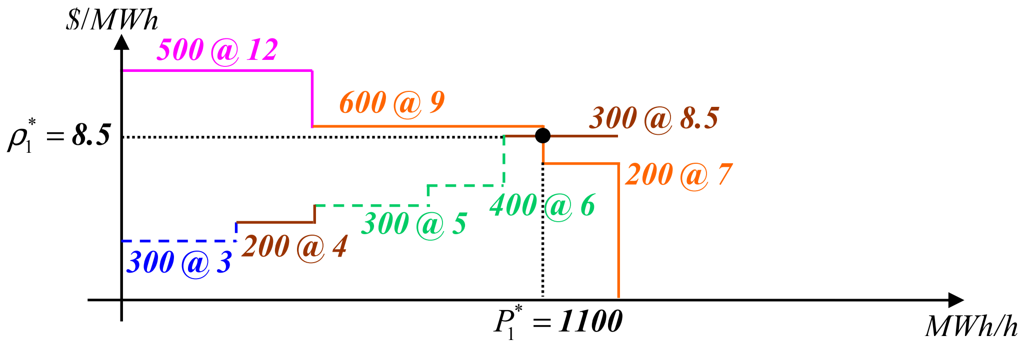


Figure 4.9: Example 4.3: market equilibrium at bus 1 corresponding to the offer of S_3 at an increased price of 8.5 $/MWh.

However, the fact that there is no longer a price insensitive demand, results in a rather different situation. We observe that seller S_3 cannot increase his price unilaterally beyond the willingness to pay of buyer B_3. Figure 4.10 shows how the clearing price at bus 1 varies when the offer price of seller S_3 varies from 8 $/MWh to 9.2 $/MWh. The clearing price cannot exceed 9 $/MWh since buyer B_3's willingness to pay is no more than 9 $/MWh. Thus, the ability of seller S_3 to exercise market power is curbed by the price-responsive demand. In fact, the attempt of seller S_3 to impose a price above 9 $/MWh fails and the seller S_3 foregoes any sales at a price above 9 $/MWh. This ability of price-responsive demand to curb the exercise of market power by seller S_3 is a critically important factor in the smooth operation of competitive markets.

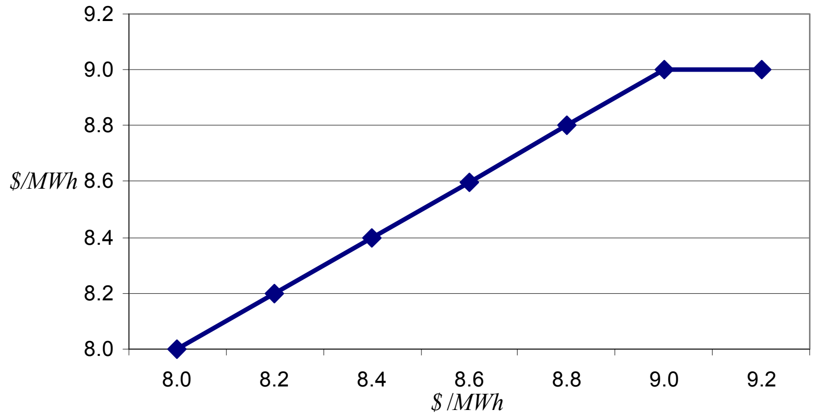
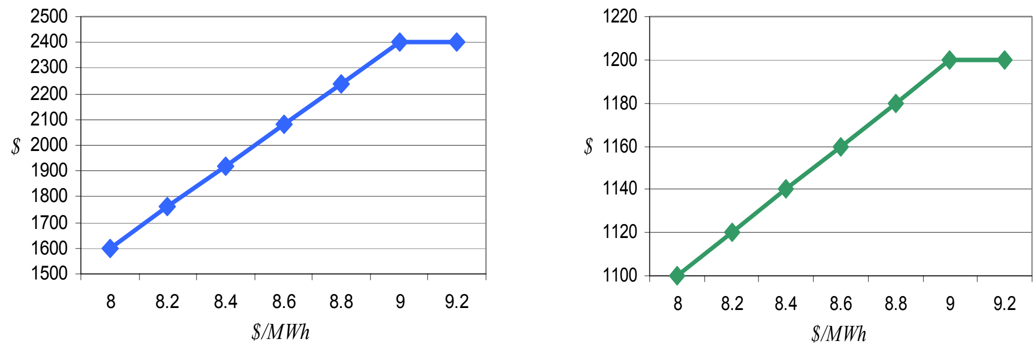


Figure 4.10: Example 4.3: plot of the clearing price at bus 1 versus the offer price of S_3.

We also investigate the impacts of seller S_3's attempts to raise his prices on the market as measured by the other metrics we discussed. We observe in Figure 4.11(a) that the congestion rents are bounded for increases in offer prices of seller S_3. The market efficiency loss is also bounded as shown in Figure 4.11(b). Then, the impacts of congestion and the exercise of market power by seller S_3 are limited in this market due to the price-responsive demand. We see this situation as a consequence of the impossibility of seller S_3 to control the price at bus 1. Since the clearing price at this bus will not exceed the 9 $/MWh, the difference in the amount paid by the buyers and the amount collected by the sellers will not exceed the $2400. In the same way, the loss in the efficiency of the market will not exceed the $1200. Again, the nature of the impacts measured by these two metrics reflects the effects of the price-responsive demand on the constrained market outcomes.



(a) (b)

Figure 4.11: Example 4.3: (a) Plot of the congestion rents versus the offer price of S_3, and (b) plot of the market efficiency loss versus the offer price of S_3.

■

Examples 4.2 and 4.3 provide us with some insights into the impacts of congestion on the competitiveness of the system in cases when some seller wishes to exploit a perceived advantage due to the seller's location. We observe that the impacts measured by the market efficiency loss and the congestion rents are bounded due to the presence of price-responsive demand. These results give rise to the following question: how are the impacts of price-responsive demand measured in electricity markets? The two simple examples motivate the studies reported in the next chapter. We are interested in assessing the capabilities of price-responsive demand to control the exercise of market power in different networks.

In this chapter, we have characterized congestion and studied different measures for its impacts in power networks. These metrics evaluate both the energy and the monetary aspects of congestion. We have seen also that congestion may be conducive to the exercise of market power. The impacts of the market power exercise are, however, limited if the demands are price responsive leading to limiting the losses in market efficiency. This sets the basis for the simulation studies we discuss in the next chapter.

5. SIMULATION-STUDY RESULTS

The discussion in Section 4.2 provides a conceptual understanding of how congestion can facilitate the exercise of market power and the importance of the role of price-responsive demand in its mitigation. In this chapter, we illustrate quantitatively the notions introduced in the previous chapter. Our focus is on the study of the impacts of congestion on the various players, in general, and on the behavior of sellers who may wish to exercise market power, in particular. We carried out an extensive set of simulations to study the market impacts in terms of the metrics we introduced in Chapter 3 for cases in which some sellers change their offers in an attempt to increase their profits. In this study, sellers who change their offers are allowed to vary systematically their offer prices over a specified range. In this way, the results provide a parametric analysis of these metrics, for the particular sellers' offer prices parameters.

We have seen that congestion shifts the market from a unique equilibrium point to different nodal equilibrium points. When there is congestion in the network, the flow from an exporting area to an importing area becomes constrained. As a result, the players at the importing node may fare very differently from those at the exporting node. This asymmetric effect is manifested through the changes in the individual surplus of the players at the importing and exporting nodes. Among the players whose surplus improves are the sellers at the importing node(s). Such sellers may also have conditions that are more inducive for the exercise of market power. We analyze the conduciveness of such conditions and examine the impacts that accrue due to congestion on these sellers as well as the other players and the market, as a whole. From the previous chapter, we know that the case of markets with inelastic demand is not interesting since a seller in the importing area may exercise market power at will and thereby increase his profits. Therefore, in this study we limit our consideration to markets with price-responsive demand.

We report the simulation results for five test systems of varying size. We describe each test system in Section 5.1. For each system, we study cases with at least one congested line. The objective of the studies is to assess the impacts of congestion when a seller located at an importing bus varies his offer prices, in an attempt to take advantage of the constrained situation. We analyze the quantifiable impacts on the system and on the individual players. We discuss the objectives and the scope of these studies in Section 5.2. In the section that follows, we focus on the analysis of the individual player impacts in each test system to systematically assess the variation of the producer and consumer surplus in response to changes in the offer prices of a particular seller. In Section 5.4, we discuss the redispatch aspects of the congestion impacts as a function of the variation of the offer prices of the particular seller. For each system, we also investigate the congestion impacts in terms of congestion rents and market efficiency loss in Section 5.5, as the parametric variation is undertaken. Section 5.6 summarizes the range of results discussed in this chapter.

5.1 The Test Systems

We report the simulation results on five tests systems varying in size from 2 buses, the smallest, to 57 buses, the largest. The other three systems are 3-, 7-, and 14-bus networks. We label the test systems with letters from system A (the 2-bus system) to E (the 57-bus system) and C, D, and E for other three test systems, respectively, in order of increasing number of nodes. For each system, we describe the characteristics of the network and the market aspects.

5.1.1 The network characteristics of the test systems

We assume each system is lossless. Each line in each system has a flow limit. System A represents the very simple two-bus network we used in the previous chapters to illustrate the key aspects of congestion and the metrics introduced to measure its impacts. The network description for test system A is given in Example 2.2. The simplest system with a loop flow is the three-bus network of test system B. The one-line diagram of the system is given in Section C.1 of Appendix C. The three other systems have network topologies whose complexity increases with the size of the network. Test system C is based on a modification of the simple system used in [4]. Test systems D and E are obtained from the standard IEEE test systems [32]. The one-line diagrams of test systems C, D, and E are given in Appendix C. The reactance of each line of each system and its corresponding flow limit are also provided in Appendix C.

5.1.2 The market characteristics of the test systems

For the construction of the market layer of the test system, we deploy quadratic expressions for the costs and benefits functions for each player. In general, we use for a) seller S_i the form

$$\mathcal{C}^{S_i}(P^{S_i}) = \beta^{S_i} P^{S_i} + \gamma^{S_i} (P^{S_i})^2, \qquad i = 1, 2, \ldots, M^S \tag{5.1}$$

and for a buyer B_j the expression

$$\mathscr{B}^{B_j}(P^{B_j}) \;=\; \beta^{B_j} P^{B_j} \;-\; \gamma^{B_j}(P^{B_j})^2, \quad j = 1, 2, ..., M^B . \tag{5.2}$$

It follows, then, that the offer function of seller S_i has the form

$$\sigma^{S_i}(P^{S_i}) \;=\; \beta^{S_i} \;+\; \gamma^{S_i} P^{S_i}, \quad i = 1, 2, ..., M^S \tag{5.3}$$

and the bid function of buyer B_j is given by

$$\nu^{B_j}(P^{B_j}) \;=\; \beta^{B_j} \;-\; \gamma^{B_j} P^{B_j}, \quad j = 1, 2, ..., M^B . \tag{5.4}$$

We refer to β^{S_i} and γ^{S_i} [β^{B_j} and γ^{B_j}] as the offer[bid] parameters of seller S_i [buyer B_j].

There is a single buyer and a single seller at each bus of the test systems A and B. In test system C, there is a single buyer at each of the seven buses, but only 5 sellers. The situation of having more buyers than sellers is also true in test systems D and E. There are 3 sellers and 12 buyers in test system D and 7 sellers and 42 buyers in test system E. The offer parameters of the buyers and sellers of each system are found in Appendix C. For each player, the maximum amount of energy that he can sell or buy is also specified. No other market descriptors are used.

5.2 The Nature of the Simulations

The objective of the simulation studies is to investigate the impacts of congestion on each player and on the market as a whole. In particular, we are interested in exploring how the various market participants fare when one seller attempts to exercise market power. We quantify the impacts of congestion in terms of the metrics defined in Chapter

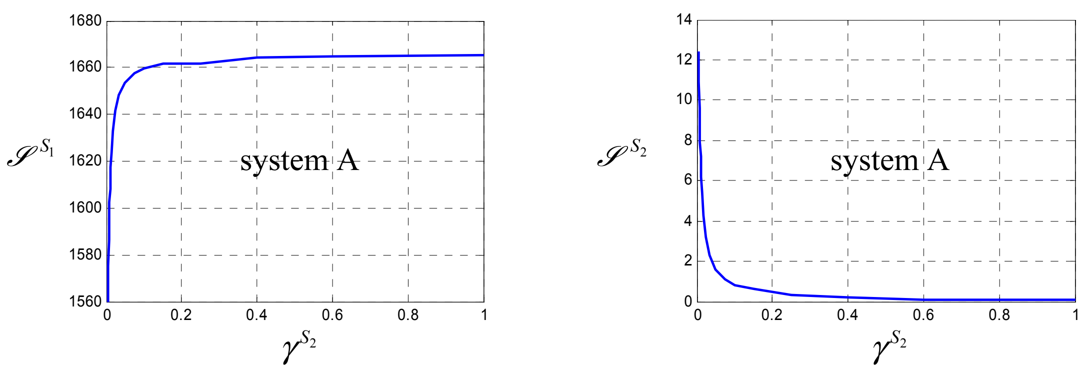
3 and studied in Chapter 4. The critical role of the transmission grid in electricity markets is emphasized throughout. We evaluate the impacts of the transmission considerations with respect to the outcome of the unconstrained market. We simulate the constrained markets whose results are the key focus and analyze the relationship between a particular seller's actions and congestion. A seller at an importing node varies his offer prices, and we study the impacts of such variations on the transmission constrained markets. Specifically, the particular seller $S_{\hat{i}}$ varies the $\gamma^{S_{\hat{i}}}$ parameter of his offer. For the test systems A, B, C, and E the offer parameter that varies is that of seller S_2 and for the test system D the variation is of the offer parameter of seller S_3. In the sections that follow, we present the results of the simulations for the various test systems. We analyze the effects on the individual players through the study of the individual surpluses. Then, we assess the impacts on the market: the redispatch, the congestion rents, and the market efficiency loss.

5.3 Individual Player Impacts

We examine the impacts on the individual players in the market of the variation of the offer parameters of the seller located at the bus in the constrained importing region of each network of the test systems. We evaluate the producer and consumer surplus for each seller/buyer and classify the impacts of the price variation of seller $S_{\hat{i}}$ on each individual player.

5.3.1 The impacts on sellers

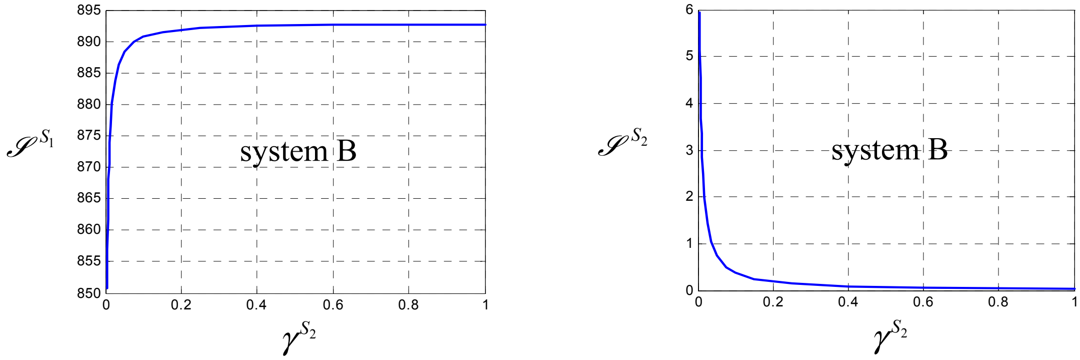
For the unconstrained market reference case the two sellers in test system A experience opposite effects as seller S_2 varies his offer prices. Figure 5.1 shows the producer surplus for the two sellers in the test system A. We observe that the producer surplus of seller S_1 increases as the seller S_2 increases his offer prices, while the producer surplus of seller S_2 decreases. In this case, seller S_2 cannot exercise market power, because any increase of his offer prices leads the buyers to purchase from seller S_1. Thus, absent the consideration of network effects and, consequently, congestion, seller S_2 cannot exercise market power.



(a) (b)

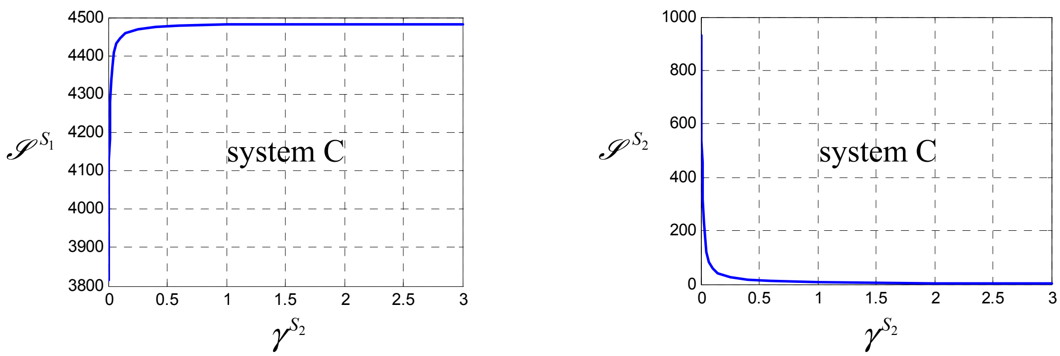
Figure 5.1: The unconstrained market outcomes for the test system A: the plot of the producer surplus as a function of γ^{S_2} for (a) seller S_1, and (b) seller S_2.

We observe precisely the same effects in the other test systems, i.e., the seller $S_{\hat{i}}$, who increases its offer prices, suffers a resulting decrease in its producer surplus while the producer surplus of each other seller in the market increases. Figures 5.2 to 5.5 depict the unconstrained market outcomes of the test systems B-E. For each system, each figure shows the variations of the producer surplus of the seller who experiences the maximum increase in its producer surplus and that for the seller $S_{\hat{i}}$.



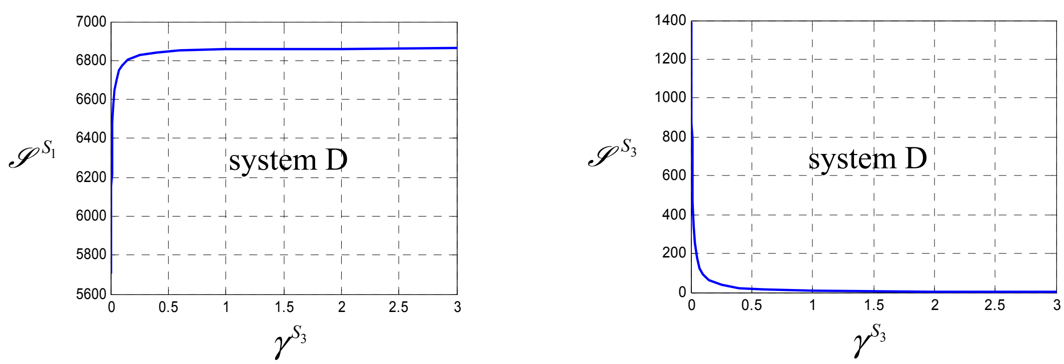
(a) (b)

Figure 5.2: The unconstrained market outcomes for the test system B: the plot of the producer surplus as a function of γ^{S_2} for (a) seller S_1, and (b) seller S_2.



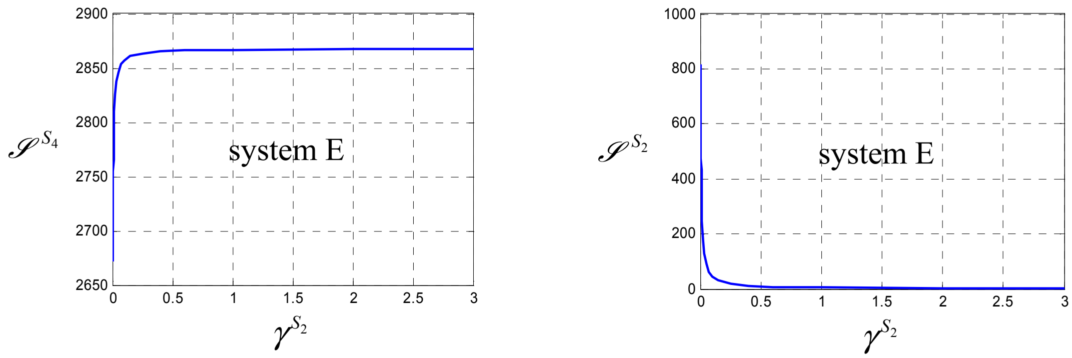
(a) (b)

Figure 5.3: The unconstrained market outcomes for the test system C: the plot of the producer surplus as a function of γ^{S_2} for (a) seller S_1, and (b) seller S_2.



(a) (b)

Figure 5.4: The unconstrained market outcomes for the test system D: the plot of the producer surplus as a function of γ^{S_3} for (a) seller S_1, and (b) seller S_3.



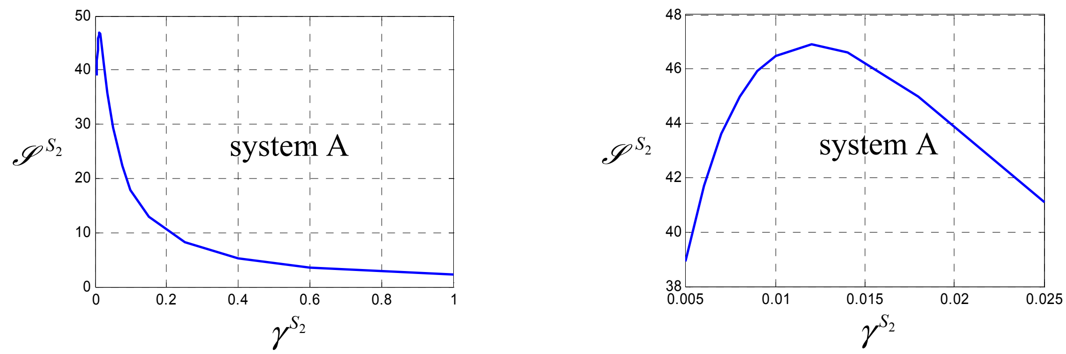
(a) (b)

Figure 5.5: The unconstrained market outcomes for the test system E: the plot of the producer surplus as a function of γ^{S_2} for (a) seller S_4, and (b) seller S_2.

An important feature is that for each system, the producer surplus has an asymptotic behavior. Therefore, the changes in the producer surplus as a result of the variation of the offer parameter of a particular seller $S_{\hat{i}}$ are bounded. Such a phenomenon is due to the price responsiveness of the demand that *enables* buyers to effectively curb the exercise of any market power by seller $S_{\hat{i}}$ when the transmission considerations are not taken into account.

We next study the explicit consideration of the network constraints. In each test system, at least one line becomes congested. There is considerably greater diversity on the impacts on the various sellers when seller $S_{\hat{i}}$ increases his offer prices in the presence of congestion. For the test system A, we notice that the producer surplus of seller S_1 does not vary when seller S_2 varies his offer prices because the amount of energy that seller S_1 can transfer from bus 0 to bus 1 is limited by the flow limit of the line. Moreover, the bid prices of buyer B_1 are fixed. Then, the energy sold by seller S_1 does not vary as a result of the changes in the offer prices of seller S_2. However, the presence of congestion impacts seller S_2 differently. Figure 5.6 shows that the producer surplus of seller S_2 increases for small values of γ^{S_2}, as is seen in Figure 5.6(b). However, Figure 5.6(a) shows that the producer surplus of seller S_2 monotonically decreases for larger values of γ^{S_2}.

To understand this impact, we need to study the market equilibrium at bus 1, the constrained importing node. This is shown in Figure 5.7. Note that as seller S_2 increases its offer prices, the clearing price at bus *1* increases. However, since the demand is price responsive, the energy sold at this bus decreases. For small values in γ^{S_2}, the increase in the clearing price results in the increase of producer surplus of seller S_2 despite the decrease in the quantity of energy sold. But, for larger values in γ^{S_2} the increases in the clearing prices become smaller and then, the producer surplus of seller S_2 decreases because of the decrease in energy sold. In this case, the exercise of market power by seller S_2 only allows an increase in seller S_2's producer surplus over a limited range of values of γ^{S_2}. In this test system, the elasticity of the demand allows the buyer to curb the exercise of market power by seller S_2. Since the increase in the producer surplus of seller S_2 occurs over a small range of γ^{S_2}, this seller has little incentive to change its offer prices.



(a) (b)

Figure 5.6: The outcomes for the transmission constrained market for the test system A: the plot of the producer surplus of seller S_2 versus γ^{S_2} with (a) $\gamma^{S_2} \in [0,1]$, and (b) $\gamma^{S_2} \in [0.005, 0.0025]$.

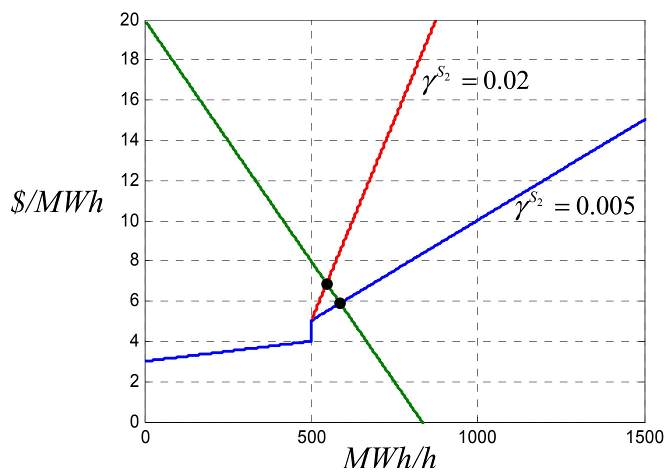
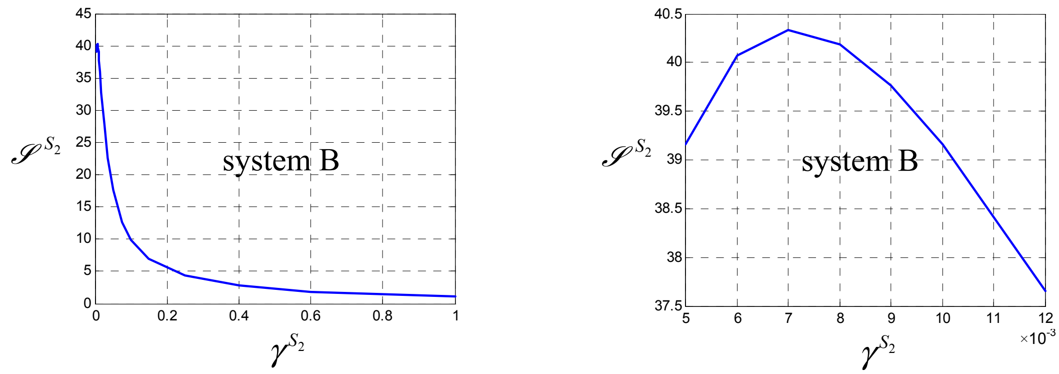


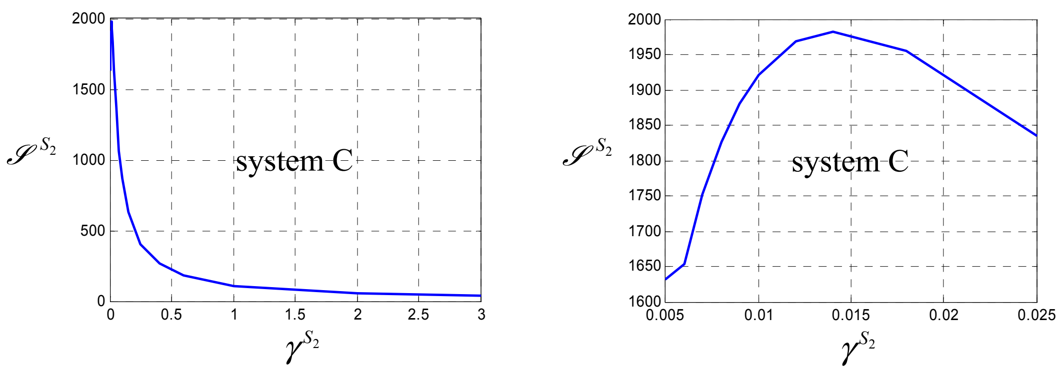
Figure 5.7: The bus 1 market equilibrium for the test system A.

We observe similar results in the other test systems. In each system, the seller $S_{\hat{i}}$ has increased producer surplus only for small values of the $\gamma^{S_{\hat{i}}}$ parameter and suffers decreases in its producer surplus for larger values of $\gamma^{S_{\hat{i}}}$. The asymptotic nature of the variation of this producer surplus ensures that the decrease in the producer surplus is bounded for each such seller in each system. The plots in Figures 5.8 to 5.11 provide the variations of the producer surplus of each seller $S_{\hat{i}}$ in response to the changes in its offer prices for the test systems B-E.



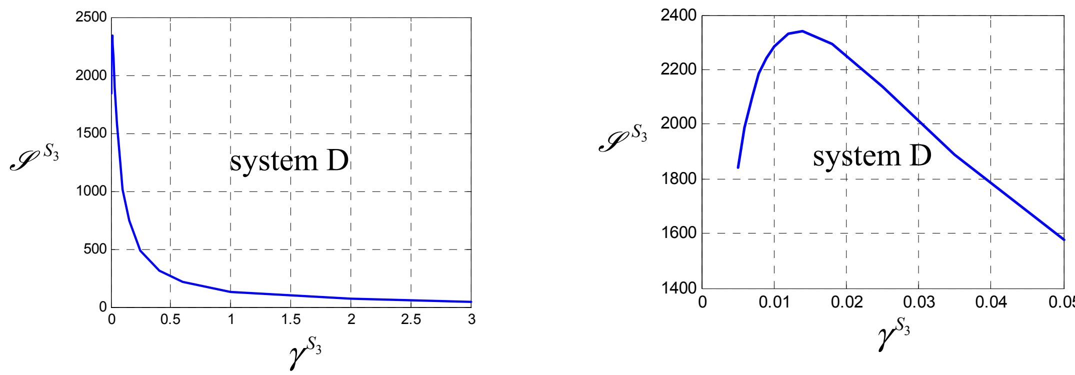
(a) (b)

Figure 5.8: The outcomes of the transmission-constrained market for the test system B: the plot of the producer surplus of seller S_2 versus γ^{S_2} with (a) $\gamma^{S_2} \in [0,1]$, and (b) $\gamma^{S_2} \in [0.005, 0.0012]$.



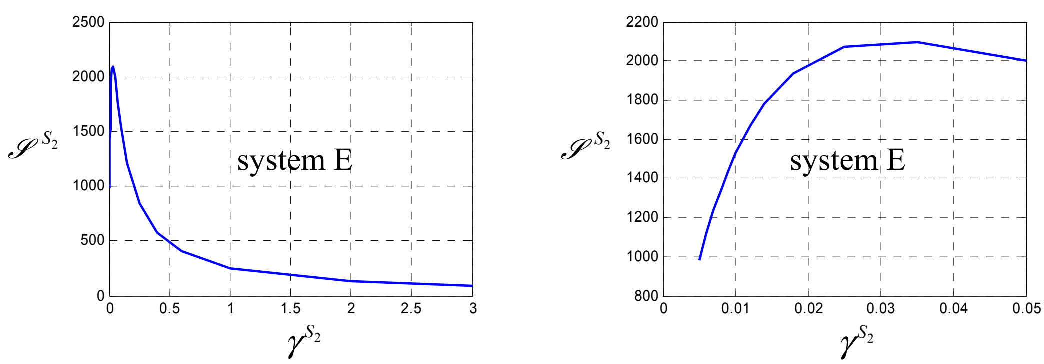
(a) (b)

Figure 5.9: The outcomes of the transmission-constrained market for the test system C: the plot of the producer surplus of seller S_2 versus γ^{S_2} with (a) $\gamma^{S_2} \in [0,3]$, and (b) $\gamma^{S_2} \in [0.005, 0.0025]$.



(a) (b)

Figure 5.10: The outcomes of the transmission-constrained market for the test system D: the plot of the producer surplus of seller S_3 versus γ^{S_3} with (a) $\gamma^{S_3} \in [0,3]$, and (b) $\gamma^{S_3} \in [0.005, 0.050]$.



(a) (b)

Figure 5.11: The outcomes of the transmission-constrained market for the test system E: the plot of the producer surplus of seller S_2 versus γ^{S_2} with (a) $\gamma^{S_2} \in [0,3]$, and (b) $\gamma^{S_2} \in [0.005, 0.050]$.

The sellers in test system E other than seller S_2 have similar impacts to those of seller S_1 in the test system A. For the test system E sellers, the producer surplus of each seller whose offer remains unchanged remains unaffected by the variation of the offer prices of seller S_2. For the other three systems, the performance of the sellers whose offers remain unchanged fare in different ways. There are sellers whose producer surplus decreases and others whose producer surplus increases as a result of the variation of the offer prices of seller $S_{\hat{i}}$. In the test system B, the producer surplus of seller S_1 decreases monotonically when the offer prices of seller S_2 increase. However, the producer surplus of seller S_3 increases under the same situation. We can see these effects in Figure 5.12. For this system, seller S_3 benefits from the increases in the offer prices of seller S_2 and from the network topology in a way that allows seller S_3 to sell more energy at the higher clearing prices. We refer to seller S_3 as a *free rider* because this seller benefits from an increase in the offer prices of seller S_2 without taking any action.

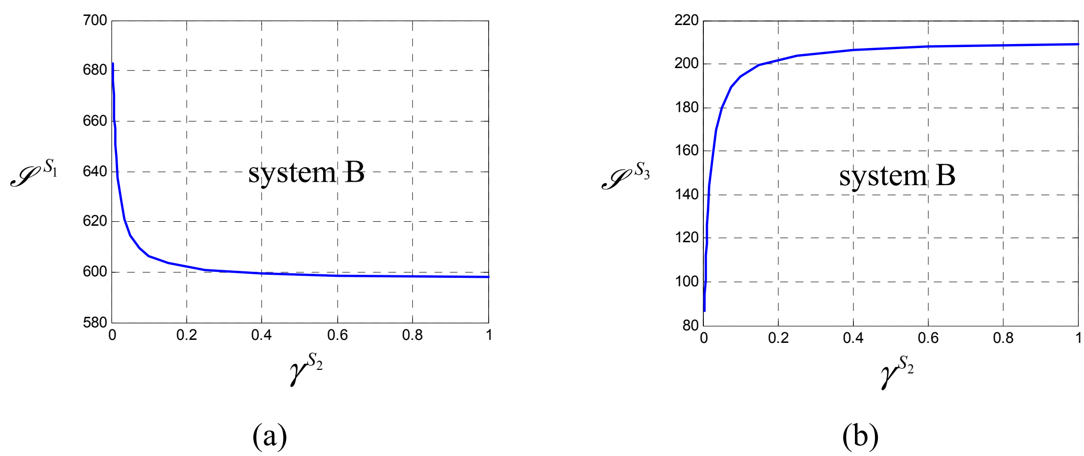
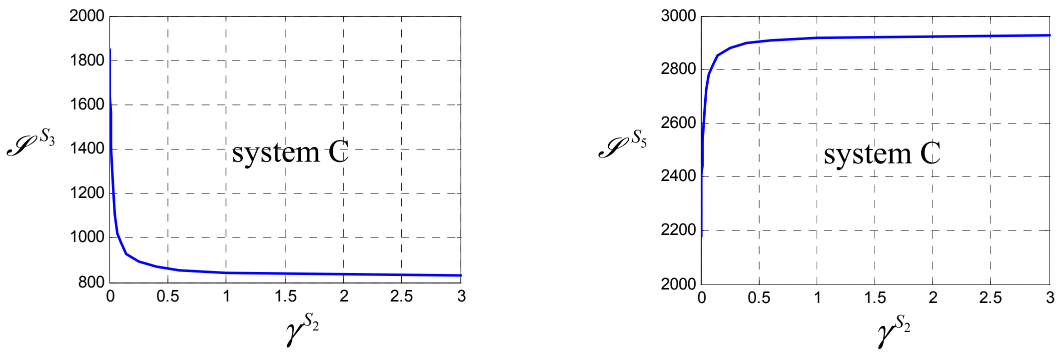


Figure 5.12: The outcomes for the transmission-constrained market for the test system B: the plot of the producer surplus as a function of γ^{S_2} for (a) seller S_1, and (b) seller S_3.

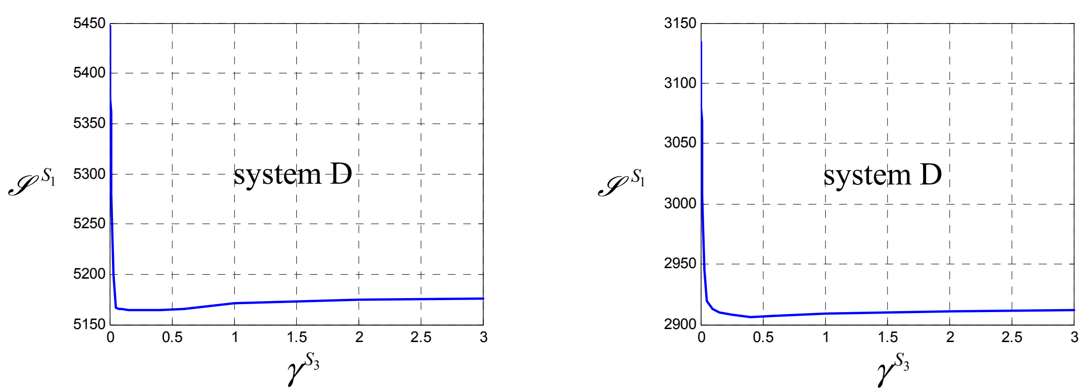
We observe this free rider phenomenon also in the test system C. We show in Figure 5.13 one of the free rider sellers in this system and one seller who suffers a decrease in the producer surplus as a result of the variation of the offer prices of seller S_2. For each seller of the test system B and C, the variation of the producer surplus has asymptotic nature so that the change of the producer surplus of the free riders and also of the negatively impacted seller is bounded. We conclude that the price responsiveness of the demand preclude sellers from increasing their producer surplus at will.



(a) (b)

Figure 5.13: The outcomes for the transmission-constrained market for the test system C: the plot of the producer surplus as a function of γ^{S_2} for (a) seller S_3, and (b) seller S_5.

In the test system D, there is no free rider effect since each seller who does not increase its offer prices suffers a decrease in its producer surplus, as shown in Figure 5.14. This situation is due to the network topology of system D and the location of the sellers with respect to the buyers. For this network, no seller can take advantage of the presence of congestion and so has no incentives to exercise market power.



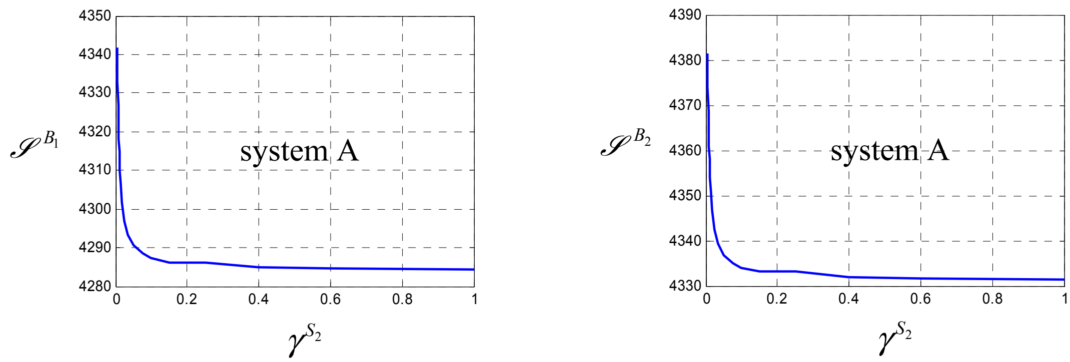
(a) (b)

Figure 5.14: The outcomes for the transmission-constrained market for the test system D: the plot of the producer surplus as a function of γ^{S_3} for (a) seller S_1, and (b) seller S_2.

We conclude from the results reported here on the five test system that as a particular seller $S_{\hat{i}}$ in an importing area increases its offer prices, the producer surplus of another seller in an exporting area typically decreases due to congestion. This is also true for the other seller being located far from the buyer because of the inability to transfer more energy to buyers. However, if another seller is located at a bus from which additional energy transfer is possible, such a seller becomes a free rider and may experience an increase in its producer surplus. The presence of price-responsive demand results in the asymptotic nature of the variation of the producer surplus of each seller.

5.3.2 The impacts on buyers

Next, we study the impacts on the buyers of the variation of the offer prices the seller $S_{\hat{i}}$ for each test system. For the unconstrained market of test system A, the two buyers experience similar effects, as the seller S_2 varies his offer prices. Figure 5.15 shows the consumer surplus of the two buyers in the test system A. We observe that the consumer surplus of each buyer decreases as the offer prices of seller S_2 increase. This occurs because the market clearing price in the unconstrained market increases as the offer prices of seller S_2 increase. Consequently, each buyer ends up in a decrease of his energy purchases, and therefore, the consumer surplus of each buyer decreases.



(a) (b)

Figure 5.15: The unconstrained market outcomes for the test system A: the plot of the consumer surplus as a function of γ^{S_2} for (a) buyer B_1, and (b) buyer B_2.

We observe precisely the same results for the unconstrained market of the other test systems. Each buyer in every system suffers a decrease in his consumer surplus when a particular seller increases his offer prices. We observe that we are able to identify a unique buyer in every system, with the characteristic that this buyer has the most adverse impact due to the seller $S_{\hat{i}}$ variation of his offer prices. In Figure 5.16, the consumer surpluses plots of these unique buyers in each system are shown. We note that the consumer surplus also has an asymptotic behavior. Therefore, the decreases in the consumer surplus due to the variation of the offer prices of seller $S_{\hat{i}}$ are bounded. Such a phenomenon is due to the price-responsive demand that *enables* buyers to curb the exercise of market power.

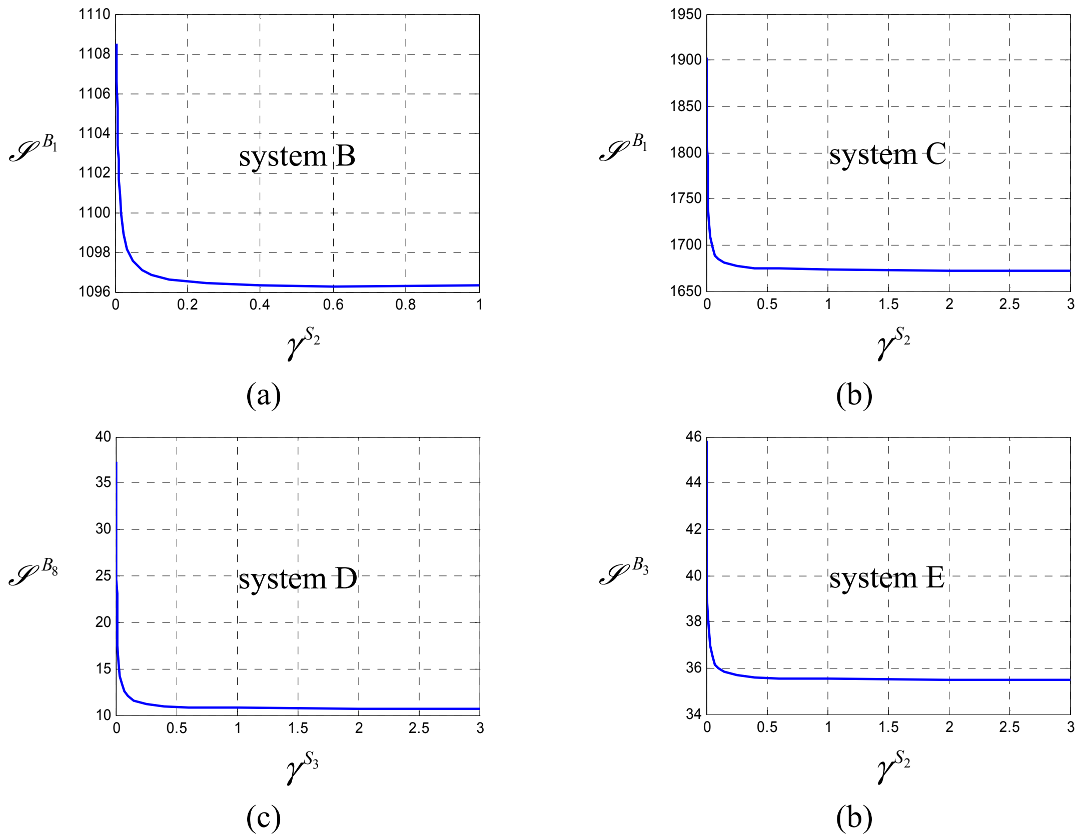
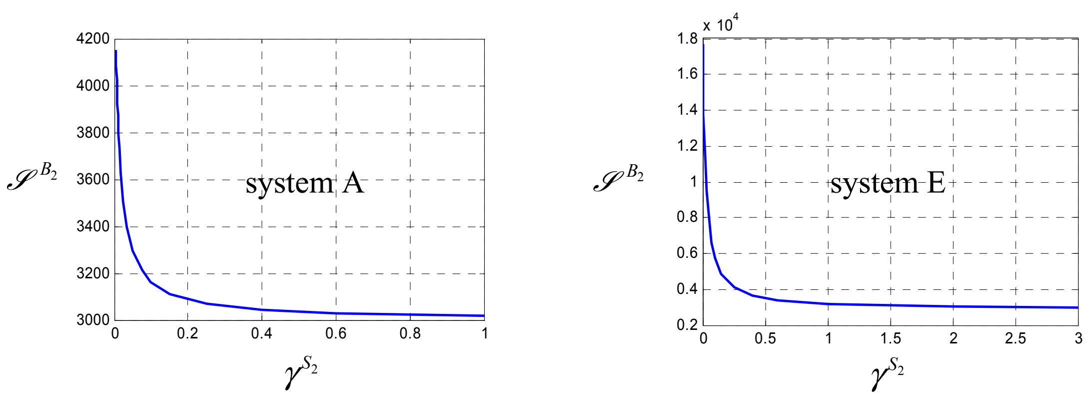


Figure 5.16: The unconstrained market outcomes: the plots of the consumer surplus of the buyer with the most adverse impacts, as a function of γ^{S_i}, for (a) buyer B_1 in test system B, (b) buyer B_1 in test system C, (c) buyer B_8 in test system D, and (d) buyer B_{11} in test system E.

We study also the transmission constrained market, where at least one line becomes congested in each system. The greater diversity observed for the impacts on the producer surplus of the various sellers is also seen as it applies to the consumer surplus impacts of the various buyers. For the test system A, we note that the consumer surplus of the buyer B_1 located at the exporting bus does not vary when seller S_2 varies its offer prices. The reason is the same as that given for the producer surplus. As the offer prices of seller S_1 and the bid prices of buyer B_1 are fixed and the flow through the transmission line is fixed at its limit, the energy purchased by buyer B_1 is also fixed.

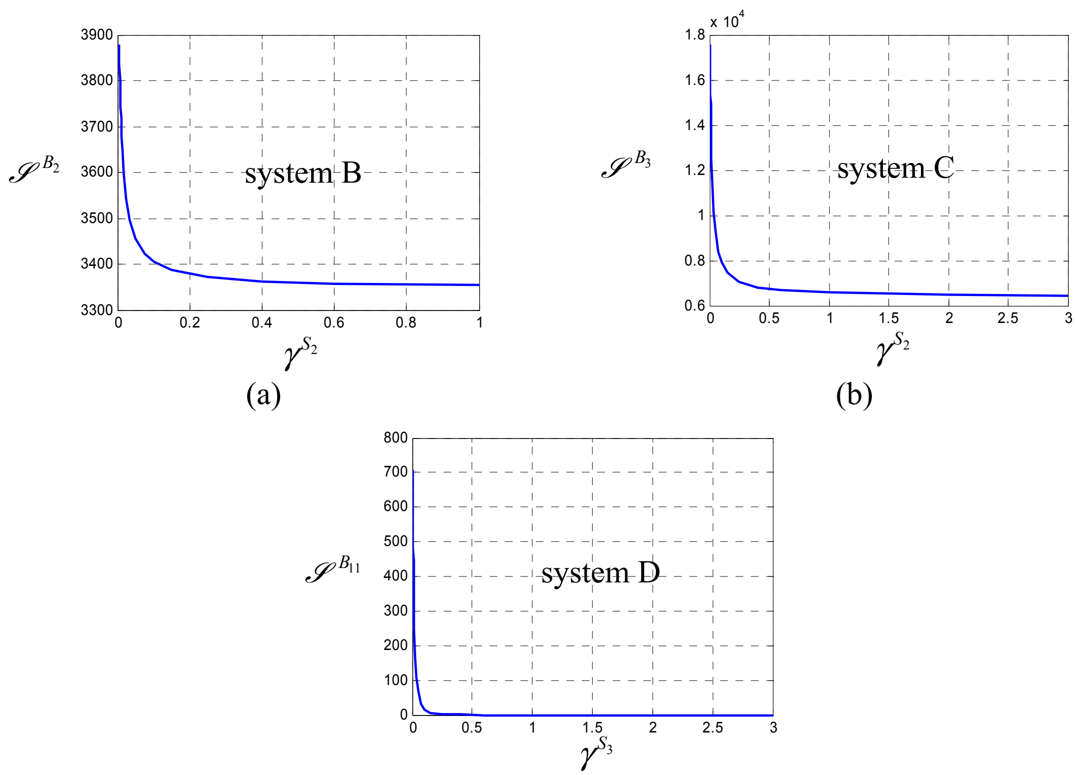
Consequently, the consumer surplus of buyer B_1 does not vary as a result of the changes in the offer prices of seller S_2. However, the presence of congestion impacts buyer B_2 differently. Figure 5.17(a) shows that the consumer surplus of buyer B_2 decreases as the seller S_2 raises its offer prices. This is because the higher offer prices lead to higher nodal prices and, consequently, buyer B_2 purchases less energy. Therefore, buyer B_2's consumer surplus decreases. We obtain similar results for test system E, where the consumer surplus of all the buyers other than B_2 are invariant with the increase of the offer prices of S_2, but buyer B_2 located at the same bus as seller S_2 suffers a decrease in its consumer surplus as shown in Figure 5.17(b).



(a) (b)

Figure 5.17: The outcomes of the transmission-constrained market: the plot of the consumer surplus of buyer B_2 as a function of γ^{S_2} for (a) test system A, and (b) test system E.

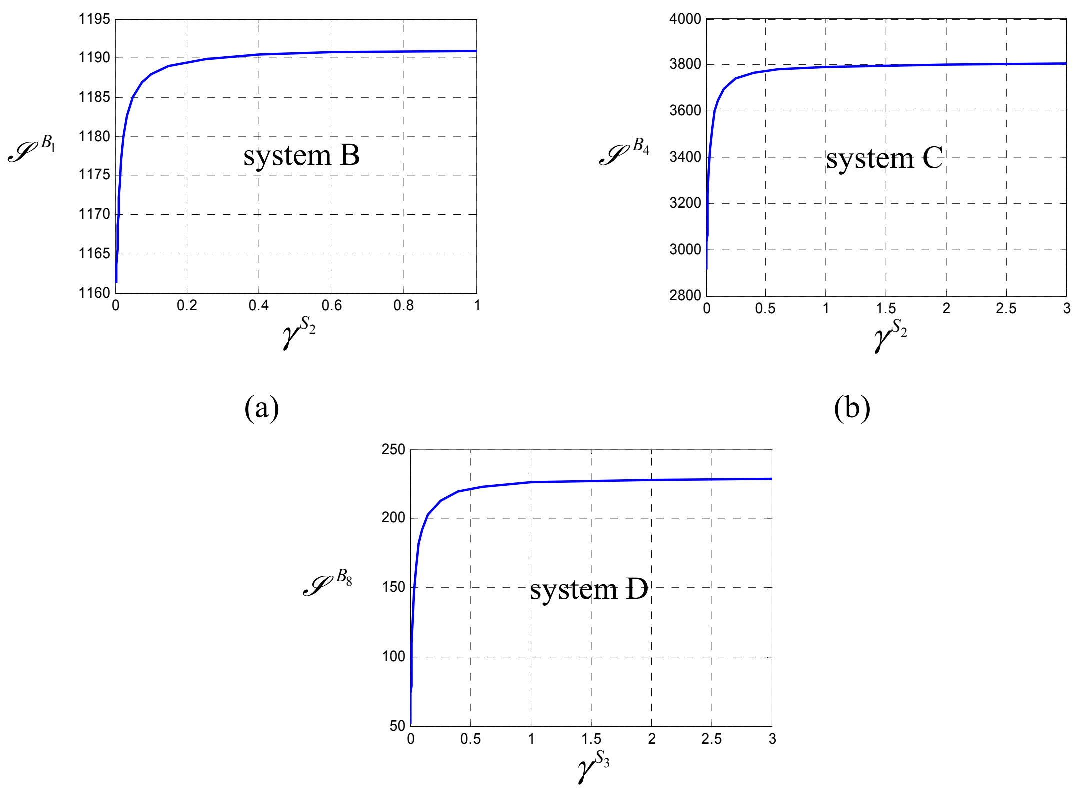
The results observed for systems B, C, and D are different. In these systems, some buyers experience increases in their consumer surpluses while others suffer decreases. No buyer in these test systems remains unaffected by the action of the seller $S_{\hat{i}}$ who varies its offer prices. For these three test systems, the buyer located at the same bus as the seller $S_{\hat{i}}$ suffers a decrease in its consumer surplus. The reason for this is the same as that given in our discussion of the results of test systems A and E. In each test system B, C, and D, the buyer at the same node as seller $S_{\hat{i}}$ purchases less energy at the resulting higher nodal price. Figure 5.18 shows the resulting consumer surplus of these buyers in these three test systems. We observe that the monotonically decreasing behavior of the consumer surplus has an asymptotic nature. Hence, the decrease in the consumer surplus of each buyer located at the same node as seller $S_{\hat{i}}$ is bounded. Again, this behavior is due to the price responsiveness of each such buyer. These buyers are able, therefore, to curb the exercise of market power of seller $S_{\hat{i}}$. Each test system B, C, and D, has at least



(c)

Figure 5.18: The outcomes of the transmission-constrained market: the plot of the consumer surplus of the buyer located at the same bus as seller $S_{\hat{i}}$, as a function of $\gamma^{S_{\hat{i}}}$, for (a) buyer B_2 in test system B, (b) buyer B_3 in test system C, and (c) buyer B_{11} in test system D.

one buyer whose consumer surplus increases as seller $S_{\hat{i}}$ increases its offer prices. This is true due to the location of this buyer relative to that of seller $S_{\hat{i}}$ and the network topology. The plots in Figure 5.19 display the consumer surplus of these buyers for the three systems. We pick for each system, the buyer who experiences the maximum increase as a result of the variation of the offer prices of the seller $S_{\hat{i}}$. We also call these free riders because they benefit from the increase in the offer prices of seller $S_{\hat{i}}$ without taking any action of their own. For each buyer who is a free rider, the variation of the consumer surplus is also asymptotic; therefore, the increase in the consumer surplus of the free-rider buyer is bounded.



(c)

Figure 5.19: The outcomes of the transmission-constrained market: the plot of the consumer surplus of a free-rider buyer, as a function of $\gamma^{S_{\hat{i}}}$ for (a) buyer B_1 in test system B, (b) buyer B_4 in test system C, and (c) buyer B_8 in test system D.

We conclude that there are commonalities in the results reported here on the five test systems. On the unconstrained market in each system, the consumer surplus of the buyer located at the same node as that of seller $S_{\hat{i}}$ decreases as the offer prices of seller $S_{\hat{i}}$ increase. Such a decrease becomes more marked for the transmission-constrained market of these test systems. In the transmission-unconstrained markets, the decrease in the consumer surplus as a result of the increase of the offer prices of seller $S_{\hat{i}}$ is experienced by each buyer in each system. However, in the transmission-constrained markets, there are buyers in the system that are negatively affected by the variation of the offer prices of seller $S_{\hat{i}}$, and buyers that become free riders and experience an increase in their consumer surplus. The price responsiveness of the demand results in the asymptotic nature of the variations of the consumer surplus of each buyer.

In this section, we quantify the impacts on the sellers and the buyers of the variation of the offer prices of a particular seller $S_{\hat{i}}$ in the presence of congestion. The results clearly indicate the critical role of the network since the network creates losers and gainers as a result of the offer price variations. The effects of the price-responsive demand in the effective curb of exercise of market power are also proved.

5.4 Redispatch Metrics

In this section, we quantify the redispatch impacts of congestion as the particular seller $S_{\hat{i}}$ varies his offer prices. We use the redispatch power and the redispatch costs metrics for this purpose. For each test system, we evaluate the two metrics for these variations. Figure 5.20 shows the plots of the redispatch power and redispatch costs for the test system A when seller S_2 varies its offer prices. There is an interesting up-and-down behavior of the plots of these two metrics: they increase for the small values of γ^{S_2}, but monotonically decrease for larger γ^{S_2} values above a particular value which is not the same for the two metrics. This situation occurs because, for smaller values of the offer price of seller S_2, the redispatch of the system involves an increase in the energy sales of seller S_2 with respect to those in the unconstrained market. However, as the seller S_2 further increases its prices, the seller's energy sales decrease and in the limit go to zero. This decrease is due to the price responsiveness of the demand. The difference of the two metrics results in different values of the γ^{S_2} parameter which optimize the value of each metric.

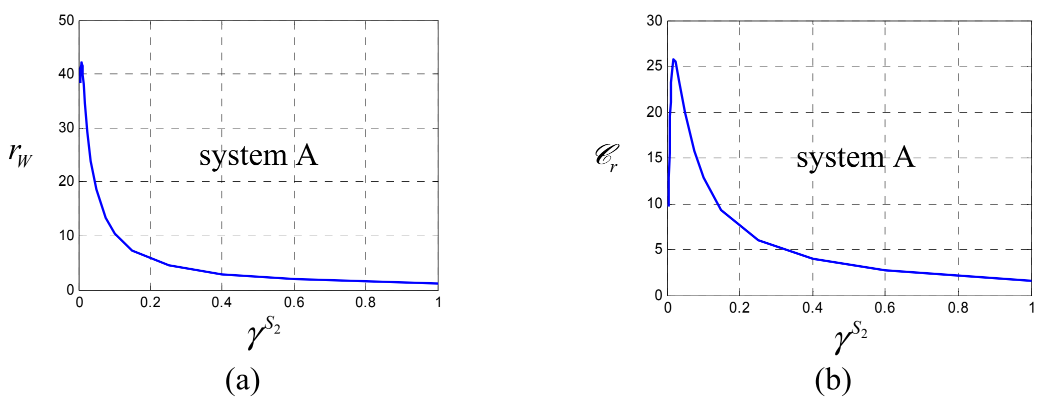
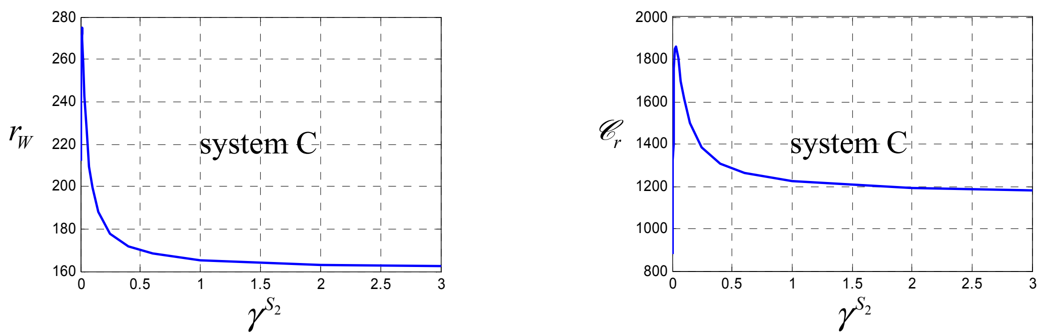


Figure 5.20: The outcomes of the transmission-constrained market for the test system A: the plot of (a) the redispatch power versus γ^{S_2}, and (b) the redispatch costs versus γ^{S_2}.

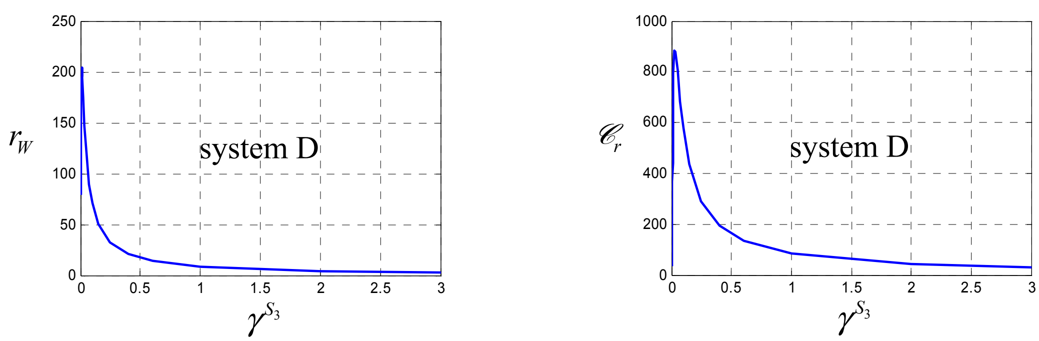
The results for the redispatch metrics in the test systems C, D, and E are similar to those observed in test system A. The redispatch metrics in these systems increase for small value of the γ^{S_i} parameter, but decrease for larger values above the critical value of

$\gamma^{S_{\hat{i}}}$, which is the turn-around point. In each system, the turn around points of r_W and $\mathscr{C}_r$ are different, as we can see in Figures 5.21 to 5.23. The test system C differs from the other test systems in that the value of r_W and $\mathscr{C}_r$ tend to a positive value rather than zero as γ^{S_2} increases. This difference arises because in system C, there is also a seller other than S_2 who is redispatched while in the test systems, A, D, and E, only seller $S_{\hat{i}}$ is redispatched.



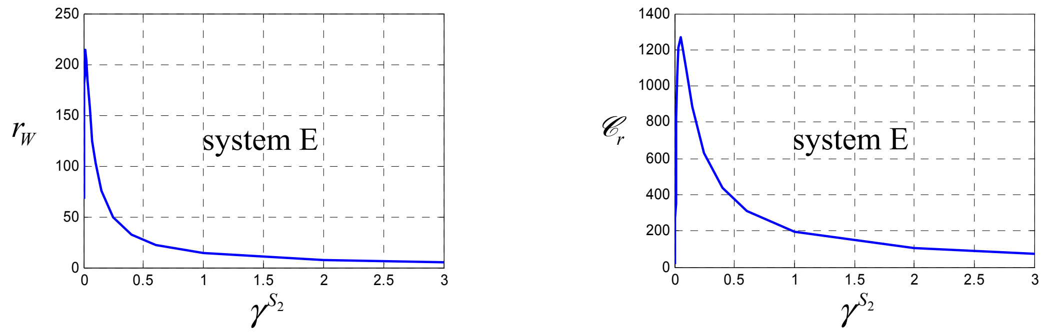
(a) (b)

Figure 5.21: The outcomes of the transmission-constrained market for the test system C: the plot of (a) the redispatch power versus γ^{S_2}, and (b) the redispatch costs versus γ^{S_2}.



(a) (b)

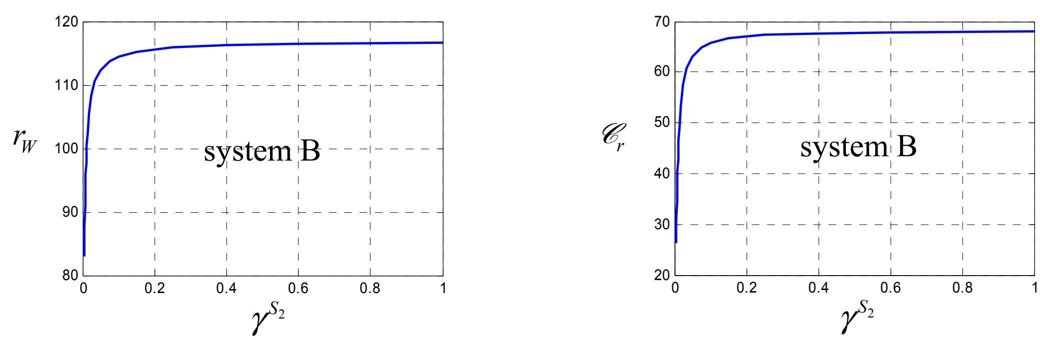
Figure 5.22: The outcomes of the transmission-constrained market for the test system D: the plot of (a) the redispatch power versus γ^{S_3}, and (b) the redispatch costs versus γ^{S_3}.



(a) (b)

Figure 5.23: The outcomes of the transmission-constrained market for the test system E: the plot of (a) the redispatch power versus γ^{S_2}, and (b) the redispatch costs versus γ^{S_2}.

For test system B, the redispatch as seller S_2 increases his offer parameter γ^{S_2} results in very different effects, as we can see in Figure 5.24. The redispatch power and the redispatch costs for test system B increase monotonically as γ^{S_2} increases. Such a behavior was not seen in other test system. These results are due to the network topology of this simple loop system. As seller S_2 increases his offer prices, his sales decrease. On the other hand, the sales of seller S_3 increase, consequently, the redispatch metrics does not tend to zero. Due to the price-responsive demand, the increases of energy sold by seller S_3 have also an asymptotic shape. As such, the redispatch power and the redispatch costs have and asymptotic shape, resulting in the boundedness of the two metrics.



(a) (b)

Figure 5.24: The outcomes of the transmission-constrained market for the test system B: the plot of (a) the redispatch power versus γ^{S_2}, and (b) the redispatch costs versus γ^{S_2}.

The redispatch impacts are not uniform for the five test systems. Some of the systems have the up-and-down behavior of both the redispatch power and the redispatch costs, tending to a value of zero as the offer prices of seller $S_{\hat{i}}$ reach higher values. But, one system had asymptotic values of the redispatch power and the redispatch costs different than zero. Also, there is one system with monotonically increasing redispatch power and redispatch costs. Thus, no general conclusions on the redispatch impacts, other than the asymptotic nature, can be done.

5.5 Congestion Rents and Market Loss of Efficiency

The congestion rents of the test systems have a common behavior in response to the variation of the offer prices of seller $S_{\hat{i}}$. These are shown in Figure 5.25 . We can see that the congestion rents increase for each system when the seller $S_{\hat{i}}$ increases its offer prices. Since the price-responsive demand can curb the exercise of market power by seller $S_{\hat{i}}$, the increases in the congestion rents have an asymptotic nature, and therefore, they are bounded. The bounded nature of the congestion rents is due to the bounded nature of the individual surplus and the fact that the social welfare is bounded by its value in the unconstrained market $\mathscr{S}|_u$.

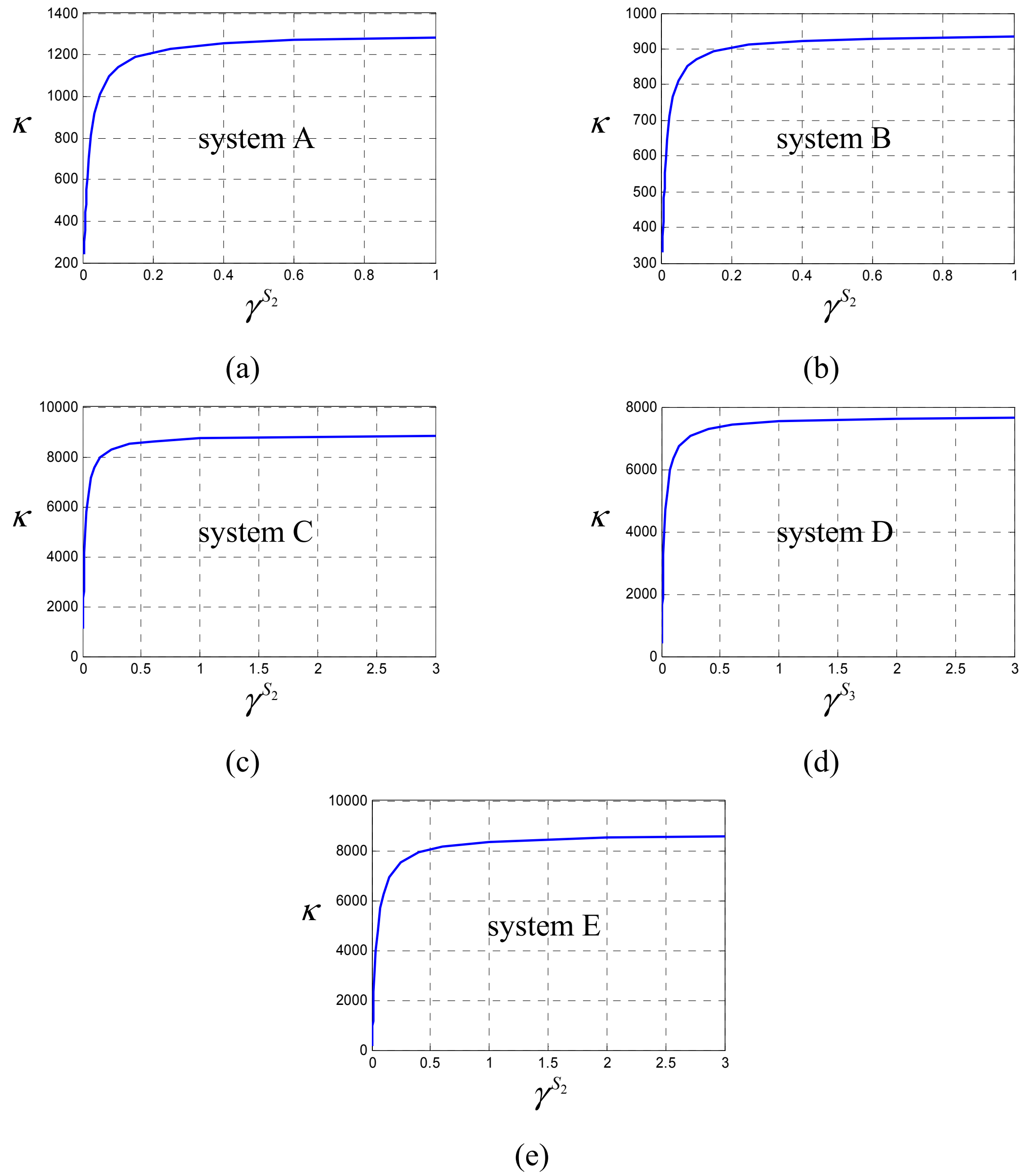


Figure 5.25: The plot of the congestion rents versus the $\gamma^{S_{\hat{i}}}$ for (a) test system A, (b) test system B, (c) test system C, (d) test system D, and (e) test system E.

Figure 5.26 shows the market loss of efficiency for each of the five test systems, we observe similar effects as those noticed in the congestion rents. The market loss of efficiency increases monotonically as the offer prices of seller $S_{\hat{i}}$ increase. They also shown an asymptotic nature, therefore, the increase in the market efficiency loss is bounded for each test system.

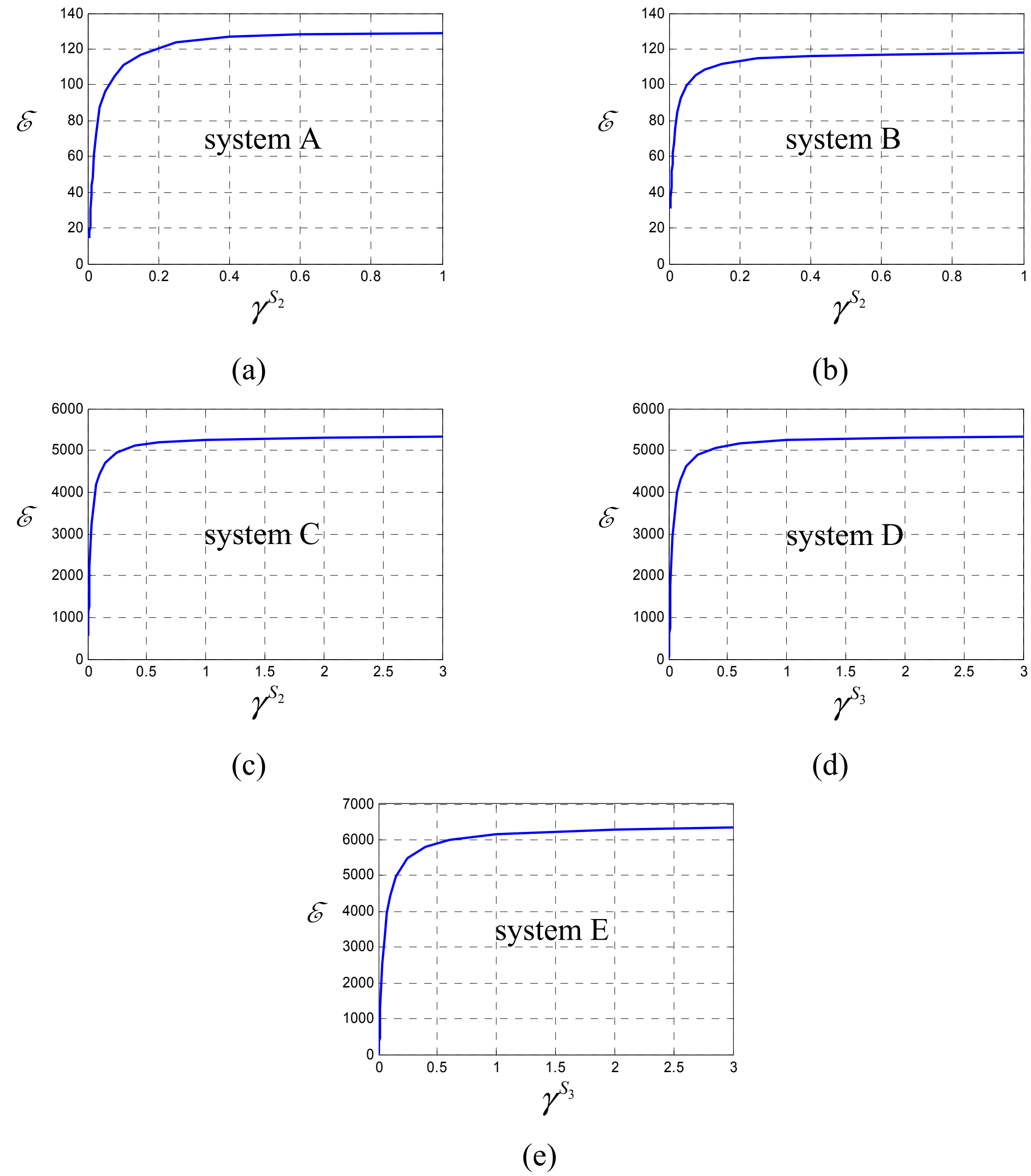


Figure 5.26: The plot of the market efficiency loss versus $\gamma^{S_{\hat{i}}}$ for (a) test system A, (b) test system B, (c) test system C, (d) test system D, and (e) test system E.

5.6 Concluding Remarks

We reported the results of our simulation studies on the impacts of congestion on the individual players and on the market as a whole in the different test systems. We focused on how each market player and the market itself fare when a particular seller $S_{\hat{i}}$

at an importing node increases its offer prices in an attempt to exercise market power. We found that the seller $S_{\hat{i}}$, who attempts to exercises market power, may increase his producer surplus only under the presence of congestion. However, this increase is in a limited range since the demand responds to prices and so the incentives to change his prices are small. Depending of the topology of the network and of the relative location of the various players, some sellers and buyers are favored when a particular seller exercises market power, and so are free riders. The remaining buyers and seller are negatively impacted.

We observed a nongeneralizable behavior of the redispatch power and the redispatch costs as the particular seller $S_{\hat{i}}$ increases his offer prices. For most test system, the redispatch metrics showed an up-and-down behavior, however, for one test system the redispatch power and the redispatch costs increase monotonically to an asymptotic value. For the system with the up-and-down behavior, the redispatch metrics tend to zero, except for one of these systems, where we noticed that the redispatch power and the redispatch costs tend to a positive value rather than zero. The congestion rents are monotonic increasing to an asymptotic positive value. We observe a similar phenomenon with the market loss of efficiency. The nature of the impacts of congestion on the individual players and the market as a whole is such that in the presence of price-responsive demand the ability of any single seller to exercise market power is limited.

6. CONCLUSIONS

This thesis studies the impacts of congestion on the individual market players and the market as a whole, in general, and the quantification of these impacts when a seller attempts to exercise market power by varying its offer prices, in particular. Throughout the study, we explicitly consider the critical role of the network in electricity markets so as to be able to measure its impacts. We also investigate the role of price-responsive demand in the mitigation of possible market power exercise. For this study, we make use of a two-layer analytical framework that integrates the physical characteristics of power systems with the appropriate economic aspects of electricity markets. We discuss the characterization of congestion using this framework and define a comprehensive set of measures to evaluate the impacts of congestion on both the individual players and the entire market. We report simulation results on five test systems of various sizes to illustrate the nature of the impacts on each player and the market as a whole.

The extensive simulations we conducted are useful not only in illustrating the nature of results that can be analytically determined but also in quantitatively assessing the impacts arising from the attempt of a particular seller $S_{\hat{i}}$ to take advantage of the congestion situation, seeking to exercise market power. These simulation results lead to the conclusion that the impacts of congestion on the individual players and on the entire market are bounded due to the asymptotic nature of each metric used in the studies. The asymptotic behavior of the producer and the consumer surplus, the redispatch metrics, the congestion rents, and the market efficiency loss is due to the price-responsive characteristics of the demand. We observe that the attempt of seller $S_{\hat{i}}$ to exercise market power fails to produce the desired increase in its profits since an increase in its producer surplus occurs only over a small range of its offer prices. Therefore, there are limited incentives for seller $S_{\hat{i}}$ to change its offer prices on the seller's own initiative. We also observe the existence of free riders in the market on both the supply and the demand sides. The individual surplus of each free rider increases monotonically to an asymptotic value. However, some players are negatively impacted by the attempt of $S_{\hat{i}}$ to exercise market power. Each of these players suffers a monotonic reduction in its individual surplus, but the asymptotic nature of the reduction bounds the decrease experienced. The simulation studies are useful to underline the critical role of the network topology and the relative location of the market participants in determining who are the losers and the gainers as a result of the market power exercise attempt of seller $S_{\hat{i}}$.

In terms of the overall market impacts of congestion, we cannot establish general conclusions for the redispatch effects. The results for the redispatch power and the redispatch costs vary for the different test systems in response to the increases in the offer prices of seller $S_{\hat{i}}$. The only common feature for all the systems is the asymptotic nature of the two metrics. On the other hand, the congestion rents and the market efficiency loss display a similar behavior in the various test systems. These two metrics have a monotonically increasing nature when seller $S_{\hat{i}}$ attempts to exercise market power and are bounded because of the price-responsiveness of the demand. In this way, the loss in efficiency that appears in the market due to congestion and that is exacerbated by the attempt to exercise of market power is limited.

This thesis has focused on a limited range of analysis on the various impacts of congestion. While we extensively evaluated the congestion impacts in response to a particular seller varying his offer prices, we did not undertake a similar analysis of the variation of the bid prices of a particular buyer. This was beyond the scope of this thesis and is a topic for further research.

There are a number of natural extensions of the work presented here. These are in addition to the topics previously mentioned as being beyond the scope of this thesis. We group these topics in the areas of modeling, parametric analysis, interrelationships of the commodity megawatthour market with other markets and the study of the market efficiency loss composition. In the modeling area, the key topics of interest are the incorporation of real power losses into the network layer model, the detailed representation of various additional transmission considerations and the incorporation of contingency case evaluation in the simulations. This work requires both analytic development and computational implementation.

The parametric analysis carried out in this study needs to be extended to evaluate the impacts of congestion when multiple players vary their offer/bid prices simultaneously. Such a study requires the careful interplay between the various players. In particular, the study should consider the possibility of collusion among the players.

The study of the interrelationships between the commodity and ancillary service markets is an important area. There is a major need to understand and quantify the impacts of congestion in all the markets and their interrelationships. This is a major undertaking and will require considerable effort.

We have not studied the contributions of the changes of the individual surpluses of the market players and the impacts of redispatch on the composition of the market efficiency loss. However, this is a very challenging topic that may have an analytical formulation. The question arises as to the existence of possible patterns in the composition of the market efficiency loss. The investigation of this problem needs also to be explored via simulations on various test systems.

APPENDIX A
ACRONYMS AND NOTATION

A.1 Acronyms

CAISO	:	California independent system operator
CALPX	:	California power exchange
CEMO	:	centralized electricity market operator
IGO	:	independent grid operator
ISO	:	independent system operator
ISO-NE	:	New England independent system operator
LMP	:	locational marginal price
PJM	:	Pennsylvania, New Jersey, and Maryland Interconnection
TCP	:	transmission constrained problem
TUP	:	transmission unconstrained problem

A.2 Notation

The following are the key aspects of the notation used:

- All variable are in *italics*
- All vectors and matrices are in ***bold*** and <u>underlined</u>
- The subscript u refers to the unconstrained market
- The subscript c refers to the constrained market
- The superscript * refers to the optimal solution
- The superscript S refers to sellers

- The superscript *B* refers to buyers

The list of indices is

i : seller index

j : buyer index

ℓ : line index

n, m : bus index

These indices come from the following sets

$\mathbb{B} = \{B_1, B_2, ..., B_{M^B}\}$: set of buyers

$\mathcal{L} = \{\ell_1, \ell_2, ..., \ell_L\}$: set of lines

$\mathcal{N} = \{0, 1, ..., N\}$: set of buses

$\mathbb{S} = \{S_1, S_2, ..., S_{M^S}\}$: set of sellers

The network and market parameters are

$\underline{\hat{\boldsymbol{A}}}$: augmented branch-to-node incidence matrix

$\underline{\boldsymbol{A}}$: branch-to-node incidence matrix

$\underline{\hat{\boldsymbol{B}}}$: augmented branch-to-node susceptance matrix

$\underline{\boldsymbol{B}}$: branch-to-node susceptance matrix

$\underline{\boldsymbol{B}}_d$: diagonal branch susceptance matrix

f_ℓ^{max} : maximum flow allowed through line ℓ

β^{S_i}, γ^{S_i} : offer parameters of seller S_i

β^{B_j}, γ^{B_j} : bid parameters of buyer B_j

The variables used in the formulation are

f_ℓ : the real power flow on line ℓ

P : total quantity sold/purchased in the transmission-unconstrained market

P^{B_j} : power purchased by buyer B_j

P_n^B : total quantity of power purchased by the buyers at node *n*

P^{S_i} : power sold by seller S_i

P_n^S : total quantity of power sold by the sellers at node *n*

θ_n : voltage phase angle at node *n*

ρ : market price in the transmission-unconstrained market

ρ_n : market price at node *n*

All power quantities are in megawatts (MW) or since in markets the commodity sold is megawatthour (MWh), the quantities may be expressed as megawatthour per hour (MWh/h). All prices are in dollars per megawatthour ($/MWh).

The functions used in the analysis are:

$\mathscr{B}^{B_j}(P^{B_j})$: benefits of buyer B_j from the purchase of P^{B_j}

$\mathscr{B}_n^B(P_n^B)$: aggregated benefits of the buyers located at bus *n*

$\mathscr{C}^{S_i}(P^{S_i})$: costs of the purchase of P^{S_i} from seller S_i

$\mathscr{C}_n^S(P_n^S)$: aggregated costs of purchases P_n^S from the sellers located at bus *n*

$\nu^{B_j}(\cdot)$: bid function of buyer B_j

$\nu_n^B(\cdot)$: aggregated bid function of the buyers located at bus *n*

$\sigma^{S_i}(\cdot)$: offer function of seller S_i

$\sigma_n^S(\cdot)$: aggregated offer function of the sellers located at bus *n*

The metrics defined and used in the report are:

$\mathscr{C}_r$: redispatch costs

$\mathscr{E}$: market efficiency loss

κ : congestion rents

r_M : redispatch power

$\mathscr{S}$: social welfare

$\mathscr{S}^{B_j}$: consumer surplus of buyer B_j

$\mathscr{S}_n^B$: consumer surplus of the buyers located at bus *n*

$\mathscr{S}^B$: consumers' surplus of all the buyers

$\mathscr{S}^{S_i}$: producer surplus of seller S_i

$\mathscr{S}_n^S$: producer surplus of the sellers located at bus *n*

$\mathscr{S}^S$: producers' surplus of all the sellers

APPENDIX B
CHARACTERIZATION OF THE OPTIMAL SOLUTION OF THE (TCP)

We analyze the characteristics of the optimal solution of the **(TCP)** using the Lagrange multiplier theory [33]. We associate Lagrange multipliers with the optimal solution of the programming problem formulation of the **(TCP)** and interpret them in the context of markets.

We can rewrite the **(TCP)** problem as a minimization problem:

$$\min \quad \sum_{n=0}^{N}\left[\mathscr{C}_n(P_n^S)-\mathscr{B}_n(P_n^B)\right]$$

subject to

$$\left.\begin{array}{lll} \underline{\boldsymbol{P}}^S-\underline{\boldsymbol{P}}^B = \underline{\boldsymbol{B}}\underline{\boldsymbol{\theta}} & \leftrightarrow & \underline{\boldsymbol{\mu}} \quad \text{(B.1)} \\ P_0^S-P_0^B = \underline{\boldsymbol{b}}_0^T\underline{\boldsymbol{\theta}} & \leftrightarrow & \mu_0 \quad \text{(B.2)} \\ \underline{\boldsymbol{B}}_d\underline{\boldsymbol{A}}\underline{\boldsymbol{\theta}} \le \underline{\boldsymbol{f}}^{max} & \leftrightarrow & \underline{\boldsymbol{\lambda}} \quad \text{(B.3)} \end{array}\right\} \textbf{(TCP)}$$

The Lagrange multipliers variables associated with the constraints (B.1), (B.2), and (B.3) are $\underline{\boldsymbol{\mu}}$, μ_0, and $\underline{\boldsymbol{\lambda}}$, respectively.

The Lagrangian of the **(TCP)** is given by

$$\mathscr{L}(\underline{\hat{\boldsymbol{P}}}^S,\underline{\hat{\boldsymbol{P}}}^B,\underline{\boldsymbol{\theta}},\underline{\hat{\boldsymbol{\mu}}},\underline{\boldsymbol{\lambda}}) = \sum_{n=0}^{N}\left[\mathscr{C}_n(P_n^S)-\mathscr{B}_n(P_n^B)\right] + \underline{\boldsymbol{\mu}}^T\left(\underline{\boldsymbol{P}}^S-\underline{\boldsymbol{P}}^B-\underline{\boldsymbol{B}}\underline{\boldsymbol{\theta}}\right) + \mu_0\left(P_0^S-P_0^B-\underline{\boldsymbol{b}}_0^T\underline{\boldsymbol{\theta}}\right) + \underline{\boldsymbol{\lambda}}^T\left(\underline{\boldsymbol{B}}_d\underline{\boldsymbol{A}}\underline{\boldsymbol{\theta}}-\underline{\boldsymbol{f}}^{max}\right). \quad \text{(B.4)}$$

where

$$\underline{\hat{\boldsymbol{P}}}^S = \begin{bmatrix} P_0^S \\ \underline{\boldsymbol{P}}^S \end{bmatrix},$$

$$\underline{\hat{\boldsymbol{P}}}^B = \begin{bmatrix} P_0^B \\ \underline{\boldsymbol{P}}^B \end{bmatrix},$$

$$\underline{\hat{\boldsymbol{\mu}}} = \begin{bmatrix} \mu_0 \\ \underline{\boldsymbol{\mu}} \end{bmatrix}, \text{ with } \quad \underline{\boldsymbol{\mu}} = \left[\mu_1, \mu_2, \ldots, \mu_N\right]^T,$$

and

$$\underline{\boldsymbol{\lambda}} = \left[\lambda_1, \lambda_2, \ldots, \lambda_L\right]^T.$$

The Kuhn-Tucker conditions for this problem are [33]

$$\frac{\partial \mathscr{L}}{\partial P_n^S} = -\frac{\partial \mathscr{C}_n}{\partial P_n^S} - \mu_n^* = 0 \qquad n = 0, 1, 2, \ldots, N \tag{B.5}$$

$$\frac{\partial \mathscr{L}}{\partial P_n^B} = -\frac{\partial \mathscr{B}_n}{\partial P_n^B} - \mu_n^* = 0 \qquad n = 0, 1, 2, \ldots, N \tag{B.6}$$

$$\frac{\partial \mathscr{L}}{\partial \underline{\boldsymbol{\theta}}} = -\underline{\boldsymbol{B}}^T \underline{\boldsymbol{\mu}}^* - \mu_0^* \underline{\boldsymbol{b}}_0^T + \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_d \underline{\boldsymbol{\lambda}}^* = \underline{\boldsymbol{0}} \tag{B.7}$$

$$\frac{\partial \mathscr{L}}{\partial \underline{\boldsymbol{\mu}}} = \left[\underline{\boldsymbol{P}}^S\right]^* - \left[\underline{\boldsymbol{P}}^B\right]^* - \underline{\boldsymbol{B}}\underline{\boldsymbol{\theta}}^* = \underline{\boldsymbol{0}} \tag{B.8}$$

$$\frac{\partial \mathscr{L}}{\partial \mu_0} = \left[P_0^S\right]^* - \left[P_0^B\right]^* - \underline{\boldsymbol{b}}_0^T \underline{\boldsymbol{\theta}}^* = 0 \tag{B.9}$$

$$\left[\underline{\boldsymbol{\lambda}}^T\right]^* \left(\underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}} \underline{\boldsymbol{\theta}}^* - \underline{\boldsymbol{f}}^{max}\right) = \underline{\boldsymbol{0}} \quad . \tag{B.10}$$

(B.10) corresponds to the complementary slackness condition.

From (B.5) and (B.6) we get

$$\mu_n^* = -\frac{\partial \mathscr{C}_n}{\partial P_n^S} = -\frac{\partial \mathscr{B}_n}{\partial P_n^B} \qquad n = 0, 1, 2, \ldots, N . \tag{B.11}$$

Next, we bring in the market information, through $\sigma_n(P_n^S)\left[\nu_n(P_n^B)\right]$, corresponding to the aggregated supply[demand] curve at bus *n*. We obtain that

$$\frac{\partial \mathcal{C}_n}{\partial P_n^S} = \sigma_n(P_n^S) \quad \text{and} \quad \frac{\partial \mathcal{B}_n}{\partial P_n^B} = \nu_n(P_n^B). \tag{B.12}$$

Then, (B.11) becomes

$$\mu_n = -\sigma_n(P_n^S) = -\nu_n(P_n^B) \qquad n = 0,1,2,...,N. \tag{B.13}$$

Therefore, at the optimum μ_n^* is the negative of the marginal price at node *n*. Then, the LMP at node *n* is given by the Lagrange multiplier μ_n^*. We note that we may have a different clearing price at each node *n* of the system; this clearing price is such that there are no incentives for increasing or decreasing the power injected or withdrawn at that node. We interpret the result in (B.13) as follows: the optimal solution of the **(TCP)** at each node *n* sets up a market equilibrium with the nodal demand-supply balance maintained. This characterization then holds for the market equilibrium of the **(TCP)** [34].

The multiplier λ_ℓ^* represent the change in the social welfare $\mathcal{S}$ for a change of 1 MW in the capacity of the line ℓ, $\forall \ell \in \mathcal{L}$. If the system is not congested then (B.10) implies $\underline{\boldsymbol{\lambda}}^* = \underline{\boldsymbol{0}}$. In such case, (B.7) leads us to

$$\underline{\boldsymbol{\mu}}^* = -(\underline{\boldsymbol{B}}^T)^{-1}\underline{\boldsymbol{b}}_0^T \mu_0^*. \tag{B.14}$$

Using (2.10) and the fact that $\underline{\boldsymbol{B}}$ is symmetric we get

$$\underline{\boldsymbol{\mu}}^* = \underline{\boldsymbol{1}}^N \mu_0^*. \tag{B.15}$$

We conclude that if no congestion appears in the system (or the line limits are not considered), the marginal prices at each bus of the system are the same since

$\mu_1^* = \mu_2^* = \cdots = \mu_N^* = \mu_0^*$. When congestion occurs then for at least one $\ell \in \mathcal{L}$ $\lambda_\ell^* > 0$, and consequently, $f_\ell^* = f_\ell^{max}$. In fact, for the system under congestion, we obtain from (B.7) that

$$\underline{\boldsymbol{\mu}}^* = -(\underline{\boldsymbol{B}}^T)^{-1}\underline{\boldsymbol{b}}_0^T\mu_0^* + (\underline{\boldsymbol{B}}^T)^{-1}\underline{\boldsymbol{A}}^T\underline{\boldsymbol{B}}_d\underline{\boldsymbol{\lambda}}^* \tag{B.16}$$

This relationship explicitly expresses the impacts of $\underline{\boldsymbol{\lambda}}^*$ on the nodal prices $\underline{\boldsymbol{\mu}}^*$.

APPENDIX C
SYSTEM DATA

In this appendix, we provide the data of the five test systems studied in Chapter 5. The first section shows the network data of these systems. The market data is provided in the second section.

C.1 Network Data

In this section, Figures C.1 to C.4 provide the one-line diagram of the systems B-E. For each line of each system, its reactance and its flow limit is given in Tables C.1 to C.4.

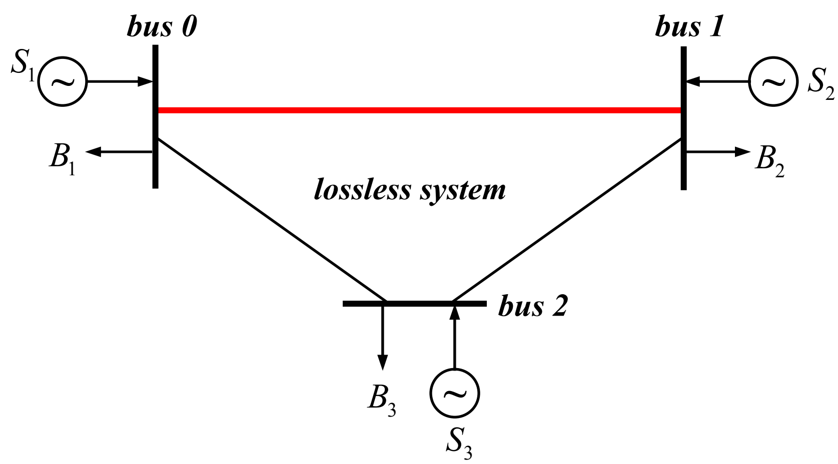


Figure C.1: One-line diagram of test system B.

Table C.1: Reactance and flow limit of each line of test system B.

line $\ell = (i, j)$		x_ℓ	f_ℓ^{max}
i	j	(p.u.)	(MW)
0	1	0.1	300
0	2	0.1	300
2	1	0.1	300

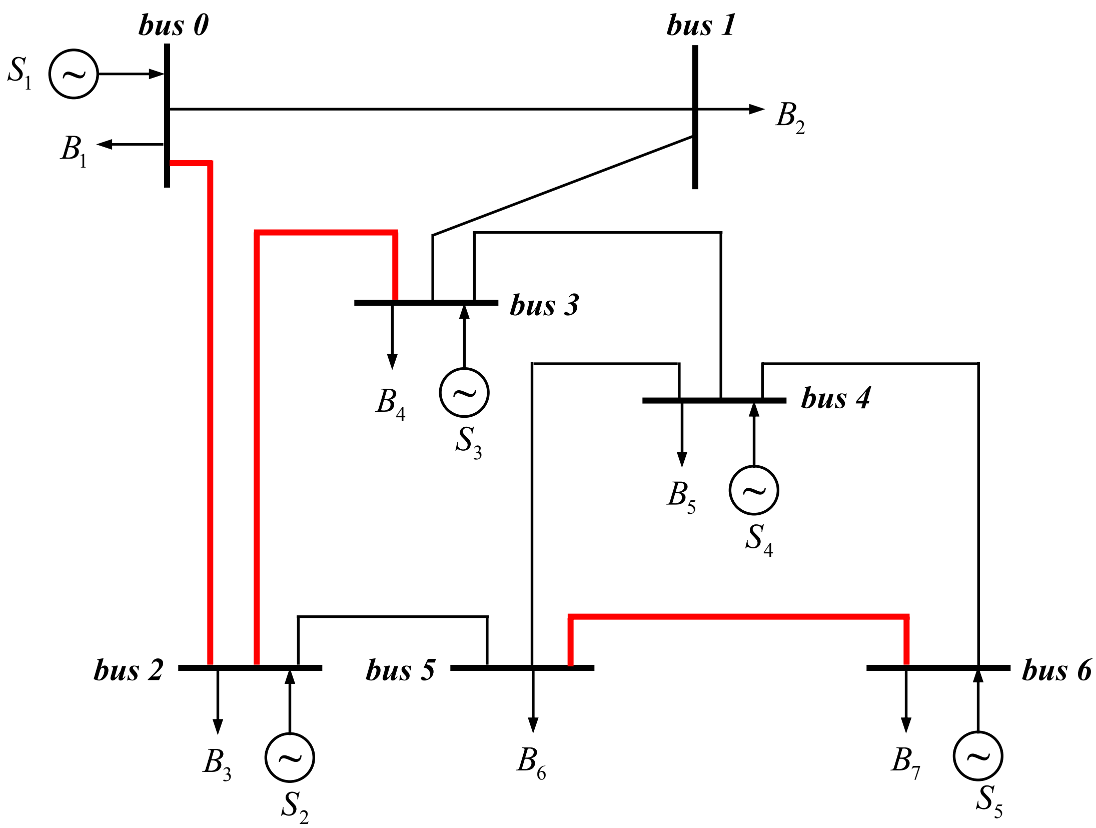


Figure C.2: One-line diagram of test system C.

Table C.2: Reactance and flow limit of each line of test system C.

line $\ell = (i, j)$		x_ℓ	f_ℓ^{max}
i	j	(p.u.)	(MW)
0	1	0.0576	300
0	2	0.0920	200
3	1	0.0586	300
3	2	0.1008	150
4	3	0.0625	300
4	5	0.1610	300
4	6	0.0850	300
5	2	0.1720	300
6	5	0.0856	200

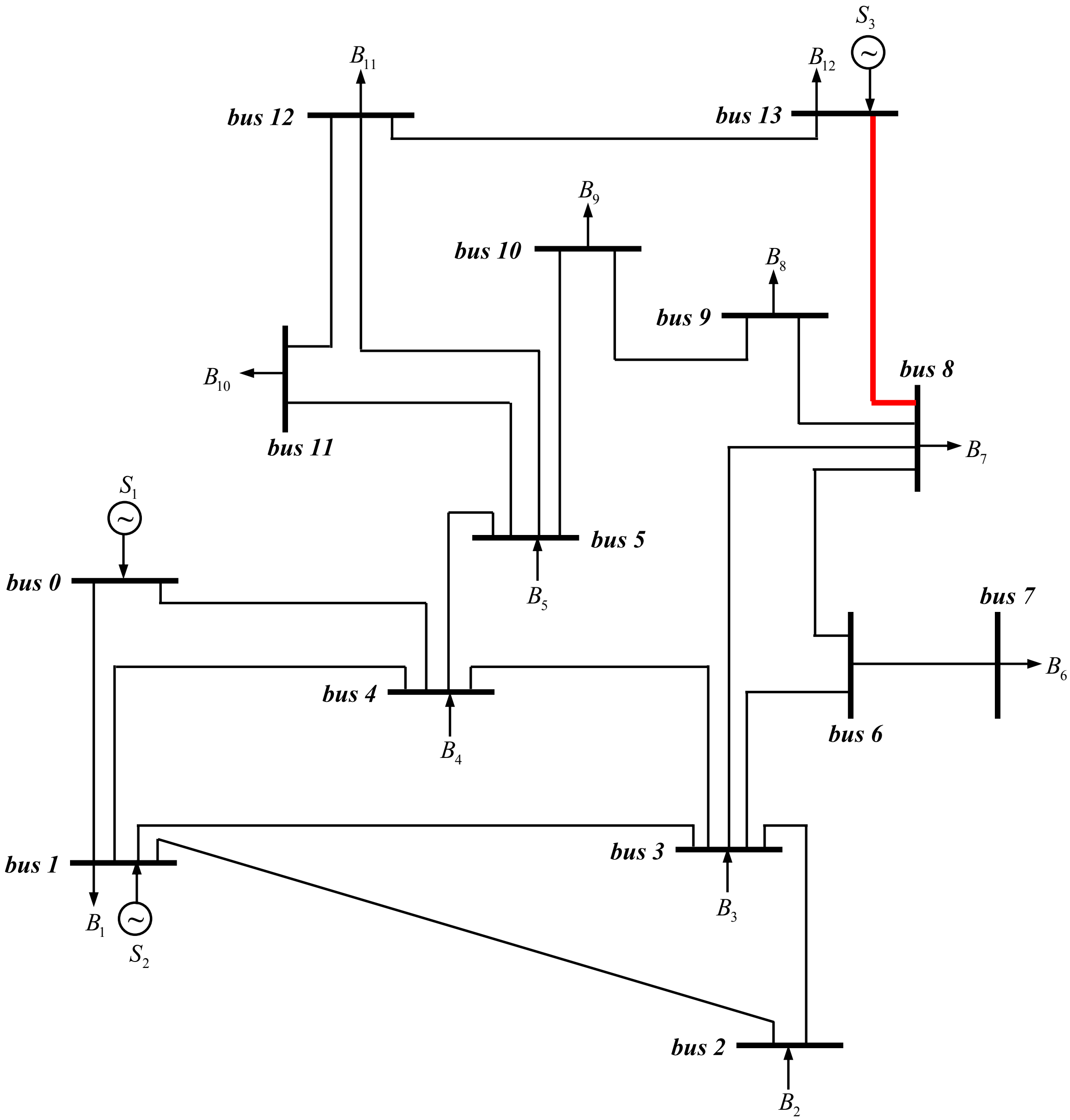


Figure C.3: One-line diagram of test system D.

Table C.3: Reactance and flow limit of each line of test system D.

line $\ell = (i, j)$		x_ℓ	f_ℓ^{max}	line $\ell = (i, j)$		x_ℓ	f_ℓ^{max}
i	j	(p.u.)	(MW)	i	j	(p.u.)	(MW)
0	1	0.0592	800	5	10	0.1989	200
0	4	0.2230	600	5	11	0.2558	200
1	2	0.1980	400	5	12	0.1308	200
1	3	0.1763	600	6	7	0.1762	200
1	4	0.1739	500	6	8	0.1100	200
2	3	0.1710	200	8	9	0.0845	200
3	6	0.2091	500	8	13	0.2704	250
3	8	0.5562	300	10	9	0.1921	200
4	3	0.0421	400	11	12	0.1999	200

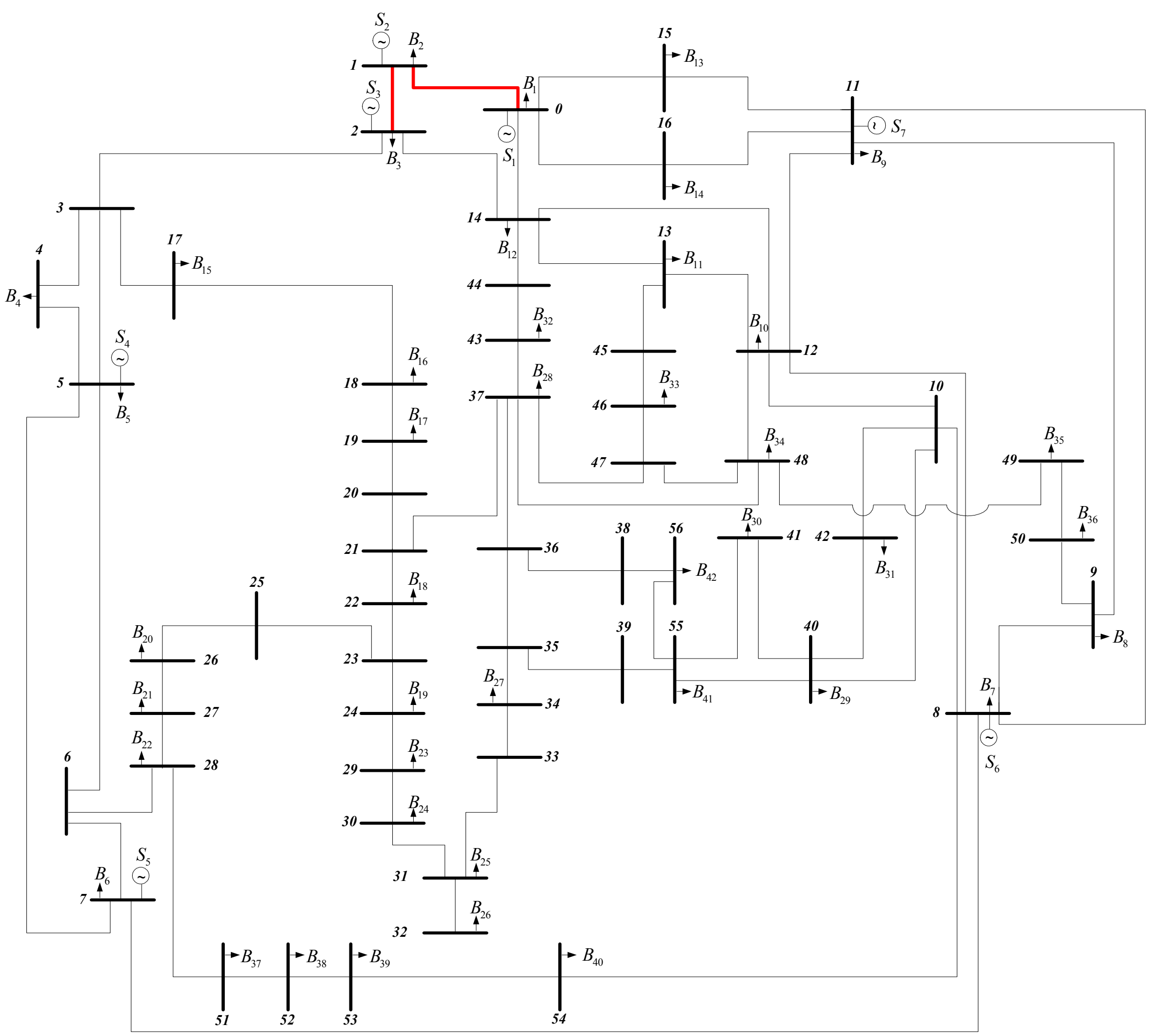


Figure C.4: One-line diagram of test system E.

Table C.4: Reactance and flow limit of each line of test system E.

line $\ell=(i,j)$		x_ℓ	f_ℓ^{max}	line $\ell=(i,j)$		x_ℓ	f_ℓ^{max}
i	j	(p.u.)	(MW)	i	j	(p.u.)	(MW)
0	1	0.0280	150	13	45	0.0735	700
0	14	0.0910	500	14	12	0.0869	200
0	15	0.2060	200	14	13	0.0547	500
0	16	0.1080	200	14	44	0.1042	600
2	1	0.0850	150	17	18	0.6850	400
2	3	0.0366	300	18	19	0.4340	200
2	14	0.0530	900	20	19	0.7767	200
3	17	0.2423	500	21	20	0.1170	200
4	3	0.1320	200	21	22	0.0152	400
5	3	0.1480	200	22	23	0.2560	200
5	6	0.1020	400	23	24	0.6028	400
5	7	0.1730	200	24	29	0.2020	300
5	4	0.0641	200	25	23	0.0473	300
6	28	0.0648	1000	26	25	0.2540	300
7	8	0.0505	700	27	26	0.0954	500
7	6	0.0712	600	28	27	0.0587	800
8	9	0.1679	300	28	51	0.1870	200
8	10	0.0848	500	29	30	0.4970	200
8	11	0.2950	200	30	31	0.7550	200
8	12	0.1580	300	31	32	0.0360	250
8	54	0.1205	300	33	31	0.9530	400
9	50	0.0712	500	34	33	0.0780	400
10	40	0.7490	300	35	34	0.0537	500
10	42	0.1530	400	35	39	0.0466	200
11	9	0.1262	400	36	35	0.0366	500
11	12	0.0580	600	36	38	0.0379	200
11	15	0.0813	200	37	36	0.1009	600
11	16	0.1790	200	37	21	0.0295	500
12	13	0.0434	300	38	56	1.3550	200
12	10	0.0782	200	39	55	1.1950	200
12	48	0.1910	600	40	41	0.3520	200

Table C.4: Continued.

line $\ell=(i,j)$		x_ℓ	f_ℓ^{max}	line $\ell=(i,j)$		x_ℓ	f_ℓ^{max}
i	j	(p.u.)	(MW)	i	j	(p.u.)	(MW)
40	55	0.5490	200	48	47	0.1290	200
41	55	0.3540	200	48	37	0.1770	300
42	40	0.4120	300	49	48	0.1280	200
43	37	0.0585	400	50	49	0.2200	200
44	43	0.1242	600	51	52	0.0984	200
45	46	0.0680	700	53	52	0.2320	200
46	47	0.0233	600	54	53	0.2265	200

C.2 Market Data

In what follow, we provide the values of the offer and bid parameters of each player in the five test systems. We assume for each buyer/seller that the minimum amount of energy that he is willing to sell/purchase is zero. The maximum amount that seller S_i / buyer B_j $\left[P^{S_i}\right]^{max}$ / $\left[P^{B_j}\right]^{max}$ is willing to sell/purchase is given in Tables C.5 to C.14.

Table C.5: Offer parameters of sellers in test system A.

i	β^{S_i} (\$/MWh)	γ^{S_i} (\$/(MW)^{2}h)	$\left[P^{S_i}\right]^{max}$ (MWh/h)
1	3.0	0.001	1500
2	4.0	0.005	1500

Table C.6: Bids parameters of buyers in test system A.

j	β^{B_j} (\$/MWh)	γ^{B_j} (\$/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
1	18	0.009	1500
2	20	0.012	1500

Table C.7: Offer parameters of the seller in the test system B.

i	β^{S_i} (\$/MWh)	γ^{S_i} (\$/(MW)^{2}h)	$\left[P^{S_i}\right]^{max}$ (MWh/h)
1	3.0	0.001	1000
2	4.5	0.005	1000
3	4.0	0.003	1000

Table C.8: Bids parameters of the buyers in test system B.

j	β^{B_j} (\$/MWh)	γ^{B_j} (\$/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
1	13	0.0150	1000
2	23	0.0200	1000
3	16	0.0150	1000

Table C.9: Offer parameters of the seller in test system C.

i	β^{S_i} ($/MWh)	γ^{S_i} ($/(MW)^{2}h)	$\left[P^{S_i}\right]^{max}$ (MWh/h)
1	3.5	0.002	1000
2	5.0	0.005	1000
3	4.5	0.003	1000
4	3.8	0.004	1000
5	3.8	0.004	1000

Table C.10: Bids parameters of the buyers in test system C.

j	β^{B_j} ($/MWh)	γ^{B_j} ($/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
1	20	0.015	1000
2	21	0.018	1000
3	50	0.022	1000
4	20	0.010	1000
5	28	0.017	1000
6	20	0.016	1000
7	27	0.015	1000

Table C.11: Offer parameters of the sellers in test system D.

i	β^{S_i} ($/MWh)	γ^{S_i} ($/(MW)^{2}h)	$\left[P^{S_i}\right]^{max}$ (MWh/h)
1	3.0	0.003	1500
2	4.0	0.004	1500
3	6.0	0.005	1500

Table C.12: Bid parameters of the buyer in test system D.

j	β^{B_j} (\$/MWh)	γ^{B_j} (\$/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
1	15	0.013	1000
2	20	0.018	1000
3	20	0.018	1000
4	22	0.015	1000
5	18	0.015	1000
6	14	0.017	1000

j	β^{B_j} (\$/MWh)	γ^{B_j} (\$/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
7	15	0.018	1000
8	13	0.020	1000
9	14	0.018	1000
10	15	0.010	1000
11	18	0.015	1000
12	50	0.022	1000

Table C.13: Offer parameters of the sellers in test system E.

i	β^{S_i} (\$/MWh)	γ^{S_i} (\$/(MW)^{2}h)	$\left[P^{S_i}\right]^{max}$ (MWh/h)
1	5.0	0.0040	1500
2	8.0	0.0050	1500
3	4.0	0.0030	1500
4	5.5	0.0040	1500
5	3.0	0.0035	1500
6	5.0	0.0045	1500
7	4.5	0.0040	1500

Table C.14: Bid parameters of the buyers in test system E.

j	β^{B_j} (\$/MWh)	γ^{B_j} (\$/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
1	14	0.015	250
2	60	0.032	800
3	14	0.021	250
4	15	0.014	250
5	15	0.010	250
6	14	0.008	250
7	18	0.014	250
8	18	0.019	250
9	15	0.015	250
10	14	0.025	250
11	15	0.015	250
12	16	0.025	250
13	15	0.015	250
14	18	0.015	250
15	15	0.015	250
16	30	0.020	250
17	20	0.020	250
18	25	0.020	250
19	15	0.020	250
20	35	0.018	250
21	25	0.020	250

j	β^{B_j} (\$/MWh)	γ^{B_j} (\$/(MW)^{2}h)	$\left[P^{B_j}\right]^{max}$ (MWh/h)
22	15	0.018	250
23	15	0.020	250
24	15	0.018	250
25	25	0.020	250
26	25	0.020	250
27	15	0.018	250
28	35	0.018	250
29	45	0.018	250
30	15	0.018	250
31	15	0.020	250
32	25	0.018	250
33	15	0.015	250
34	25	0.018	250
35	15	0.018	250
36	25	0.018	250
37	15	0.020	250
38	15	0.018	250
39	15	0.020	250
40	15	0.025	250
41	15	0.018	250
42	15	0.018	250

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