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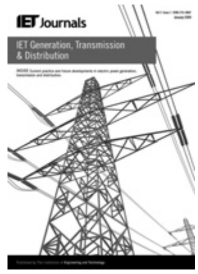

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Comparative analysis of game theory models for assessing the performances of network constrained electricity markets

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Abstract: Competition has been introduced in the electricity markets with the goal of reducing prices and improving efficiency. The basic idea which stays behind this option is that, in competitive markets, a greater quantity of the good is exchanged at a lower and stable price, leading to higher market efficiency. Electricity markets are different from other commodities mainly because of the operational characteristics, perishability and lack of large storage capability, which may impact the market performances. The network structure of the system on which the economic transactions need to be undertaken poses strict physical and operational constraints. Those physical and operational constraints need to be ensured to guarantee an operating state feasible and when those constraints binding the congested system show remarkable economic impacts. Strategic interactions among market participants with the objective of maximising their surplus must be taken into account when modelling competitive electricity markets. The network constraints, specific of the electricity markets, provide opportunity of exercising strategic behaviour of the market participants. Game theory provides a tool to model such a context. This study provides a comparative analysis of the application of game theory models to network constrained electricity markets with the focus on the strategic behaviour of the electricity producers. Different models such as supply function equilibrium, Cournot, Stackelberg and conjecture supply function are considered and their appropriateness to model the electricity markets is discussed. Under network constraints with reference to the IEEE 30- and IEEE 57-bus test systems, various models are compared in quantitative way to provide analysis of the market performance under different representation of the oligopoly competition in the electricity markets.

Nomenclature

Set

$\boldsymbol{G}$	index set of the electricity producers, $\boldsymbol{G} = [1, 2, \ldots, g-1, g, g+1, \ldots, N_G]$
$\boldsymbol{D}$	index set of the electricity consumers, $\boldsymbol{D} = [1, 2, \ldots, d-1, d, d+1, \ldots, N_D]$

Vector

$\boldsymbol{a}^m/\boldsymbol{b}^m$	intercept (\$/MW) and slope (\$/MW^2) values of the marginal costs of the electricity producers, $\dim(\boldsymbol{a}^m) = \dim(\boldsymbol{b}^m) = N_G$
$\boldsymbol{e}/\boldsymbol{h}$	intercept (\$/MW) and slope (\$/MW^2) values of the demand of the electricity consumers, $\dim(\boldsymbol{e}) = \dim(\boldsymbol{e}) = N_D$

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$\boldsymbol{p}/\boldsymbol{q}$	power production and demand vector, MW, $\dim(\boldsymbol{p}) = N_G$, $\dim(\boldsymbol{q}) = N_D$
$\boldsymbol{i}_G/\boldsymbol{i}_D$	all one vector for producers/consumers, $\dim(\boldsymbol{i}_G) = N_G$, $\dim(\boldsymbol{i}_D) = N_D$
$\boldsymbol{P}^+/\boldsymbol{P}^-$	upper and lower production limits of the producers, MW, $\dim(\boldsymbol{P}^+) = \dim(\boldsymbol{P}^-) = N_G$
$\boldsymbol{\omega}^+/\boldsymbol{\omega}^-$	Lagrange multipliers of the upper and lower production limits, \$/MW, $\dim(\boldsymbol{\omega}^+) = \dim(\boldsymbol{\omega}^-) = N_G$
$\boldsymbol{F}$	flow limits of the considered transmission lines, MW
$\boldsymbol{\mu}^+/\boldsymbol{\mu}^-$	Lagrange multipliers of the line flow limits, \$/MW, $\dim(\boldsymbol{\mu}^+) = \dim(\boldsymbol{\mu}^-) = \dim(\boldsymbol{F})$
$\boldsymbol{\lambda_G}/\boldsymbol{\lambda_D}$	nodal prices at the generator and load buses, \$/MW, $\dim(\boldsymbol{\lambda_G}) = N_G$, $\dim(\boldsymbol{\lambda_D}) = N_D$. For example, λ_g, $g \in \boldsymbol{G}$, is the nodal price at the bus with which the producer g is connected

Matrix

$\boldsymbol{H}$	diagonal matrix formulated by the vector of $\boldsymbol{h}$
$\boldsymbol{B}^m$	diagonal matrix formulated by the vector of $\boldsymbol{b}^m$
$\boldsymbol{J}$	matrix of power transfer distribution factors
$\boldsymbol{J}_G^{\mathrm{T}}, \boldsymbol{J}_D^{\mathrm{T}}$	generator and load buses rows of the transpose of $\boldsymbol{J}$ matrix, respectively

Scalars

λ_N	the price at the reference bus \$/MW, that is, market clearing price without network constraints
$\bar{\lambda}$	average price weighted by the quantity, \$/MW, that is, market clearing price with the consideration of the network constraints that is expressed by

$$\bar{\lambda} = (\boldsymbol{\lambda}_G^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{\lambda}_D^{\mathrm{T}}\boldsymbol{q})/(\boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{q})$$

S^{S}, S^{M}	social surplus and merchandise surplus, \$, respectively
S_g^G, S_d^D	surplus of producer g and consumer d, \$, respectively

The above surplus values are defined in [4, 10].

1 Introduction

Nowadays, the restructuring of the power industry has been implemented in many countries. The introduction of the deregulation has not always proved to be as efficient as expected. In California [1, 2], the market experienced huge problems. From May 2000 to May 2001, the price hit frequently the price cap. The average price of December 2000 was 317 \$/MWh, almost ten times higher than usual. In June 1998, wholesale electricity price in Midwest US market reached 7000 \$/MWh [3]. Starting from the regulated monopoly, the competition introduced to the electricity markets was aimed to improve market efficiency towards the theoretical reference model of perfect competition. Actually, due to the structural characteristics, the electricity markets are oligopoly in which the market performance is in-between perfect competition and monopoly [4]. In this context, the task of the regulators is to force the market performance towards perfect competition while monitoring continuously the distance from such a condition.

In the electricity markets, as well as in other markets, strategic behaviour of the market participants may arise, striving for larger amount of profits or surpluses with high prices and production withdrawals [4]. Game theory [5, 6] can capture the strategic interactions among market participants who are aware that their results depend on other competitors' decisions. This paper is aimed at discussing the application of game theory models to physical constrained electricity markets with the goal of providing tools for simulating and assessing the market performance.

In the electricity markets, network constraints may induce possibilities of market power under strategic bidding behaviour, very specific of this context. The power systems that accommodate the economic transactions in the market need to be operated under strict physical and operational constraints to assure its security. If those constraints are binding, the system is said to be congested and proper countermeasures need to be undertaken [7]. To our concern, another goal of this paper is to pinpoint the network constraints impacts on the market performance under different game theory models.

This paper consists of five additional sections. In Section 2, the market clearing model under network constraints is introduced. In Section 3, the market clearing game models are formulated whereas in Section 4, the market power assessing indices are indicated. In Section 5, numerical comparisons of the outcomes of various models are presented with examples using IEEE 30- and IEEE 57-bus systems. Some conclusive remarks are outlined in Section 6.

2 Market clearing model

A firm that is unable to exercise market power is known as price taker [8]. According to the classic micro-economic theory, a price-taking producer that wishes to maximise the profits would bid his/her production at the marginal cost [8, 9]. The market is characterised as perfect competition where all the market participants are price-takers. Although the perfect competition is completely unrealistic, it can serve as a reference case to identify market power behaviour

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in an actual market, based on the fact of that perfect competition would lead to the most efficient market performance [4, 5, 10].

In the pool-operated electricity markets, the independent system operator (ISO) is responsible to coordinate the aggregate offers from the supply side and the aggregate demands from the demand side for a specified time interval, usually 1 h. Owing to the peculiarities of the electricity transmission, the transactions must be settled according to the physical conditions of the electricity network and different nodal prices may arise when the flow limits are binding. Considering the network constraints, the market clearing can be modelled with an optimisation problem subject to the electricity network constraints modelled by the DC power flow. The market clearing is formulated as

$$\max \ S^S = \frac{1}{2}\boldsymbol{q}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{q} + \boldsymbol{q}^{\mathrm{T}}\boldsymbol{e} - \frac{1}{2}\boldsymbol{p}^{\mathrm{T}}\boldsymbol{B}^m\boldsymbol{p} - \boldsymbol{p}^{\mathrm{T}}\boldsymbol{a}^m \tag{1}$$

$$\text{s.t. } \boldsymbol{i}_G^T\boldsymbol{p} - \boldsymbol{i}_D^T\boldsymbol{q} = 0 \tag{2}$$

$$-\boldsymbol{F} \le \boldsymbol{J}(\boldsymbol{p} - \boldsymbol{q}) \le \boldsymbol{F} \tag{3}$$

$$\boldsymbol{P}^- \le \boldsymbol{p} \le \boldsymbol{P}^+ \tag{4}$$

The equality constraint (2) is for the balance of the power production and consumption. The inequality constraints (3) and (4) represent the line flow limits and the power production limits, respectively. Note that the inequality and equality symbols in (3) and (4) represent the component-wise inequality and equality between two vectors.

The solution of the above optimisation problem provides the nodal prices as

$$\begin{cases} \boldsymbol{\lambda}_G = \lambda_N \boldsymbol{i}_G - \boldsymbol{J}_G^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) \\ \boldsymbol{\lambda}_D = \lambda_N \boldsymbol{i}_D - \boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) \end{cases} \tag{5}$$

When network constraints are not considered, that means $\boldsymbol{\mu}^+ = \boldsymbol{\mu}^- = 0$, the prices at all buses are equal to λ_N that is usually called market clearing price. In another words, the geographic property of electricity network is degenerated to one location with an identity price of the whole market.

3 Oligopoly competition models: game theory applications

The present electricity markets may be better described in terms of oligopoly than of perfect competition from which they may be rather far. In the last 50 years, game theory has become a useful analytic tool for the assessment of strategic behaviour of the oligopoly market players. In an oligopoly electricity market, the producer is a market player submitting offers higher than the marginal cost and aiming to the surplus maximisation. The objective function of the market clearing is altered by replacing the marginal costs in (3) with the strategic offers from the producers.

In general, game solution is done through an iterative process (multi moves) in which each producer in turn will determine the best strategy given the strategies of his competitors, to maximise its payoff, that is, the producer surplus in this paper, by solving the model of (6) [4]. The Nash equilibrium (NE) is found when no player can improve his payoff by changing his strategy if his competitors do not

$$\max \ S_g^G = \lambda_g p_g - a_g^m p_g - \frac{1}{2} b_g^m p_g^2 \quad \forall g \in \boldsymbol{G} \tag{6}$$

s.t. KKT conditions of the model (1)–(4). The KKT conditions are expanded in the table. Referring to (5), the nodal price, λ_g, in (6) is a function of the λ_N and $\boldsymbol{\mu}^+$ and $\boldsymbol{\mu}^-$ depending on the strategy variables with respect to the different game models. The detail of the producer surplus maximisation problem is summarised in Table 1.

However, the unique/existence of the NE is a general concern in many related references [4, 10–22]. So far the unique NE can be guaranteed under simple test system (for instance in [10], the test system is modelled with three-bus network and no capacity constraints of the producers and single line power flow limit) by using the best response functions in analytical way [10, 20]. As for the large test system used in this paper, the analytical way cannot be readily used because of the consideration of capacity constraints of the producers and multiple lines flow limits. The existence of equilibrium cannot be guaranteed analytically and ex-post check is needed [4, 10].

According to the classification of the strategic variables, there are three types of game models which are price bidding, quantity bidding and supply function bidding models. The price bidding models include Bertrand and Forchheimer models [11, 12]. However, taking into account network constraints for the analysis of hourly electricity markets, so far there is no literature using price bidding game models as an efficient tool. Another reason for the Bertrand model has not been the focus in the literature would be that Bertrand model might correspond to perfect competition case. The quantity bidding game models include Cournot [14, 15], Stackelberg [16, 17] and conjectural supply function (CSF) [18] models. An essential assumption of the former two models is that the individual player's own output decision will not have an effect on the decisions of its competitors. As for the CSF model, the basic assumption is that the output of the other competitors can be estimated to change in an expected way with respect to the output decision of the considered player. Since the strategic variable is the quantity of the electricity transacted, those game models do not give meaningful equilibrium when price elasticity of the demand curve is low (the demand quantity is fixed with the zero value of the price elasticity). The supply function bidding models, named after supply function equilibrium [13], choose a strategic supply function different with the marginal cost curve with the aim of maximising individual

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producer surplus. Different with the quantity bidding game models, for the optimal strategy formulation of one producer, the given strategies are the supply functions of other competitors but not the production quantities. The dispatched quantities of the competitors, (11) in Appendix, are determined by the supply function of the considered producer through current decision-making process.

4 Indices for assessing the market performances

Owing to the strategic behaviour of the market participants, the oligopoly market equilibrium is deviated from the perfect competition equilibrium that has the most efficient market performance. The Lerner index and the market inefficiency index, columns 2 and 3 of Table 2, are popularly used to assess the two main effects of the strategic behaviour: higher market clearing price and lower social surplus with reference to the perfect competition equilibrium. In addition, the network constraints play a major role in determining the oligopoly equilibrium. In this respect, each index has tow corresponding values. The first one, with the subscript 'u', is used to assess the strategic behaviour without the consideration of the network constraints whereas the second one is computed with the introduction of the network constraints in order to capture the peculiarities of the electricity markets. Furthermore, to differentiate the market clearing under different test cases, perfect competition case is indicated by the superscript 'p' whereas the oligopoly competition equilibrium is indicated by the superscript 'E'. For instance, S_u^{SP} and S^{SE}, respectively, denote the social surplus at perfect competition equilibrium without network constraints and social surplus at oligopoly equilibrium with network constraints.

Table 1 Detailed description of the producer surplus maximisation

Model	Strategy variable	Price at the reference bus and production of the players	KKT conditions
Cournot	p_g	$$\lambda_N = \frac{p_g + \boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{p}' + \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]}{\boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D}$$ where $\boldsymbol{p}' = [p_1', \ldots, p_{g-1}', 0, p_{g+1}', \ldots, p_{N_G}]^{\mathrm{T}}$, p_i' ($i \in \boldsymbol{G}$, $i \neq g$) is the optimal offered quantity derived from the last move of the producer i, p_g is the decision variable in the optimal problem	$P_g^- \leq p_g \leq P_g^+$ $-\boldsymbol{F} \leq \boldsymbol{J}\cdot(\boldsymbol{p}-\boldsymbol{q}) \leq \boldsymbol{F}\boldsymbol{\mu}^+\cdot$ $[\boldsymbol{J}(\boldsymbol{p}-\boldsymbol{q})-\boldsymbol{F}] = \boldsymbol{0}$ $\boldsymbol{\mu}^-\cdot[\boldsymbol{J}(\boldsymbol{p}-\boldsymbol{q})+\boldsymbol{F}] = \boldsymbol{0}$
Stackelberg	p_g^l	$$\lambda_N = \frac{\boldsymbol{i}_F^{\mathrm{T}}\boldsymbol{p}^F + p_g + \boldsymbol{i}_L^{\mathrm{T}}\boldsymbol{p}'^L + \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]}{\boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D}$$ where $\boldsymbol{p}'^L = [\ldots, p_{g-1}'^L, 0, p_{g+1}'^L, \ldots]^{\mathrm{T}}$, $p_i'^L$ ($i \in \boldsymbol{L} \subset \boldsymbol{G}, i \neq g$, $\boldsymbol{L}$ is the set of the leaders) is the optimal offered quantity of the leader i, $\boldsymbol{p}^F$ is the vector of the optimal production quantities of the followers	
conjecture supply function (CSF)	p_g	$$\lambda_N = \frac{p_g + \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}] - \boldsymbol{r}_g^{\mathrm{T}}\boldsymbol{J}_G^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{p}^{g-1} - \boldsymbol{w}_g^{g-1})}{\boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D - \boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{r}_g}$$ where $\boldsymbol{r}_g = [r_{1,g}, r_{2,g}, \ldots, r_{g-1,g}, 0, r_{g+1,g}, r_{NG,\,g}]^{\mathrm{T}}$ $$\boldsymbol{p}^{g-1} = [p_1^{g-1}, p_2^{g-1}, \ldots, p_{g-1}^{g-1}, 0, p_{g+1}^{g-1}, \ldots, p_{\mathrm{NG}}^{g-1}]^{\mathrm{T}}$$ $$\boldsymbol{w}_g^{g-1} = [r_{g,1}\Lambda_1^{g-1}, \ldots, r_{g,g-1}\Lambda_{g-1}^{g-1}, 0, r_{g,g+1}\Lambda_{g+1}^{g-1}, \ldots, r_{g,NG}\Lambda_{\mathrm{NG}}^{g-1}]^{\mathrm{T}}$$ p_i^{g-1} and Λ_i^{g-1} are the dispatched quantity and the nodal price of producer i derived from the last move of the producer $g-1$, respectively. $r_{g,i}$, $\forall g$, $i \in \boldsymbol{G}$, $i \neq g$, represents the assumed rate of change in competitor supply per unit price. The *CSF* function is $$p_i = p_i^{g-1} + r_{g,i}\,(\lambda_i - \Lambda_i^{g-1})\ \forall i \in \boldsymbol{G},\ i \neq g$$	

Continued

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Table 1 *Continued*

Model	Strategy variable	Price at the reference bus and production of the players	KKT conditions
supply function equilibrium (SFE)	a_g SFE-intercept, the offer function is $f(p_g) = a_g + b_g^m p_g$	$$\lambda_N = \frac{\boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}^m)^{-1}[\boldsymbol{J}_G^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) + \boldsymbol{a}'] - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]}{\boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}^m)^{-1}\boldsymbol{i}_G - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D}$$ $$p_g = \frac{\lambda_N - \boldsymbol{J}_g^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) - (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) - a_g}{b_g^m}$$ where $\boldsymbol{a}' = [a'_1, a'_2, \ldots, a'_{g-1}, a'_g, a'_{g+1}, \ldots, a'_N]^{\mathrm{T}}$; a'_i, $i \neq g$, is the determined value derived from the last move of producer i	$\boldsymbol{P}^- \leq \boldsymbol{p} \leq \boldsymbol{P}^+$ $-\boldsymbol{F} \leq \boldsymbol{J} \bullet (\boldsymbol{p} - \boldsymbol{q}) \leq \boldsymbol{F}$ $\boldsymbol{\mu}^+ \bullet [\boldsymbol{J}(\boldsymbol{p} - \boldsymbol{q}) - \boldsymbol{F}] = \boldsymbol{0}$ $\boldsymbol{\mu}^- \bullet [\boldsymbol{J}(\boldsymbol{p} - \boldsymbol{q}) + \boldsymbol{F}] = \boldsymbol{0}$ $\boldsymbol{\omega}^+ \bullet (\boldsymbol{p} - \boldsymbol{P}^+) = \boldsymbol{0}$ $\boldsymbol{\omega}^- \bullet (\boldsymbol{p} + \boldsymbol{P}^-) = \boldsymbol{0}$
	b_g SFE-slope, the offer function is $f(p_g) = a_g^m + b_g p_g$	$$\lambda_N = \frac{\boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}')^{-1}[\boldsymbol{J}_G^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) + \boldsymbol{a}^m] - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]}{\boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}')^{-1}\boldsymbol{i}_G - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D}$$ $$p_g = \frac{\lambda_N - \boldsymbol{J}_g^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) - (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) - a_g^m}{b_g}$$ where $\boldsymbol{B}'$ is the diagonal matrix formulated with the vector $[b_1', \ldots, b_{g-1}', b_g, b_{g+1}', \ldots, b_{NG}']^{\mathrm{T}}$, b_i', $i \neq g$, is the known value derived from the last move of producer i	
	k_g SFE-k multiplier, the offer function is $f(p_g) = k_g(a_g^m + b_g^m p_g)$	$$\lambda_N = \frac{\boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{K}[\boldsymbol{J}_G^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-)] + \boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}^m)^{-1}\boldsymbol{a}^m + \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]}{\boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{K}\boldsymbol{i}_G - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D}$$ $$p_g = \frac{\lambda_N - \boldsymbol{J}_g^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) - (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) - k_g a_g^m}{k_g b_g^m}$$ where $\boldsymbol{K}$ is a diagonal matrix formulated by the vector $[1/(k_1' b_1^m), \ldots, 1/(k'_{g-1} b_{g-1}^m), 1/(k_g b_g^m), 1/(k'_{g+1} b_{g+1}^m), \ldots, 1/(k'_{N_G} b_{N_G}^m)]^{\mathrm{T}}$, k_i', $i \neq g$, is the determined value derived from the last move of producer i	

The higher level of the strategic bidding behaviour can also be outlined by the higher total producer surplus at the decrement of the total consumer surplus, with reference to the perfect competition. However, the benefit of the strategic behaviour to the whole supply side may not apply to the individual producers. With the producer surplus deviation index, column 4 of Table 2, we can check the individual gain and loss under oligopoly games. In order to extract the 'just network impacts' from the strategic behaviour, the network impacts index, column 5 of

Table 2 Indices for assessing the market performance

Scenarios	Lerner indices	Market inefficiency indices	Producer surplus deviation index	Network impacts index
with network constraints	$\sigma = (\bar{\lambda}^E - \lambda_N^p)/\bar{\lambda}^E$	$\xi = 100^*(S^{\mathrm{SE}} - S_u^{\mathrm{SP}})/S_u^{\mathrm{SP}}$	$s_g = (S_g^{\mathrm{GE}} - S_{gu}^{\mathrm{GP}})/S_{gu}^{\mathrm{GP}}\ \forall g \in \boldsymbol{G}$	$\tau = 100^*(S^{\mathrm{SE}} - S_u^{\mathrm{SE}})/S_u^{\mathrm{SE}}$
without network constraints	$\sigma_u = (\lambda_N^E - \lambda_N^p)/\lambda_N^E$	$\xi_u = 100^*(S_u^{\mathrm{SE}} - S_u^{\mathrm{SP}})/S_u^{\mathrm{SP}}$	$s_{gu} = ((S_{gu}^{\mathrm{GE}} - S_{gu}^{\mathrm{GP}})/S_{gu}^{\mathrm{GP}}\ \forall g \in \boldsymbol{G}$	

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Table 2, is proposed to assess the social surplus under a given game model. That is expressed by referring to the unconstrained oligopoly equilibrium as the benchmark.

5 Numerical studies

IEEE 30-bus test system is composed with six producers (generators) and 20 consumers (loads), as shown in Fig. 1 in Appendix. The parameters of the producers and the consumers are shown in Appendix Table 12. The lines selected, as an example, to consider the network constraints are shown in Appendix Table 13, other lines are assumed to have infinitive line flow limits.

Market clearings under different game models are shown in Tables 3 and 4. Among those models, the CSF-1 model is characterised with zero conjectural parameters ($r_{g,i} = 0$, $\forall i \in \boldsymbol{G}$, $i \neq g$) to show its equivalence to the Cournot model as we discussed in Section 3. The CSF-2 model is used to model the increased competition level in the oligopoly market with unit conjectural parameters ($r_{g,i} = 1$, $\forall i \in \boldsymbol{G}$, $i \neq g$). As for the Stackelberg model, we choose producers 1, 5 and 6 as the leader producers.

Furthermore, we tested different game models based on the IEEE 57-bus transmission network, as shown in Appendix Fig. 2, to generalise some common conclusions on different but larger system. There are ten producers and 44 consumers whose parameters are shown in Appendix Table 14. The lines selected to consider the network constraints are shown in Appendix Table 15. The simulation results are shown in Tables 5 and 6. For the Stackelberg model, we choose producers 1, 8, 9 and 10 as the leaders.

5.1 Perfect competition and monopoly

Perfect competition and monopoly models represent the two extreme market structures. In the monopoly market, all the producers are assumed as one firm to realise the maximum total producer surplus. Monopoly has the highest level of market power behaviour with the highest market clearing price and the lowest social surplus both under constrained

Table 3 Market clearing without network constraints IEEE 30-bus test system

Models	$\bar{\lambda}$, \$/MW	$\sum_g P_g$, MW	$\sum_g S_g^G$, \$	$\sum_d S_d^C$, \$	S^M, \$	S^S, \$
monopoly	71.7	201.2	9640.7	5618.9	0	15 259.6
Cournot	44.5	326.4	5934.8	12 805.3	0	18 740.1
CSF-1	44.5	326.4	5936.5	12 803.3	0	18 739.7
Stackelberg	41.8	338.5	5155.5	13 675.3	0	18 830.7
CSF-2	38.3	354.9	4028.8	14 912.2	0	18 941
SFE-slope	37.8	357.1	3906.8	15 085.9	0	18 992.7
SFE-*k* parameter	37.5	358.4	3808.4	15 190	0	18 998.3
SFE-intercept	37.2	360	3694.2	15 310	0	19 004
perfect competition	34.5	372.2	2741.8	16 285.6	0	19 027.5

Table 4 Market clearing with network constraints IEEE 30-bus test system

Models	$\bar{\lambda}$, \$/MW	$\sum_g P_g$, MW	$\sum_g S_g^G$, \$	$\sum_d S_d^C$, \$	S^M, \$	S^S, \$
monopoly	72.5	197.5	9603	5458.1	0	15 061.1
Cournot	51	285.2	6827.5	10 117.2	911	17 855.6
CSF-1	51.1	285.2	6827.8	10 116.9	911	17 855.7
Stackelberg	44.5	309.6	5091.7	11 825.8	1260.3	18 177.8
CSF-2	43.7	314.9	4864.1	12 216.1	1114.9	18 195.1
SFE-slope	43.4	312.7	4756.6	12 074.5	1359.4	18 190.5
SFE-*k* parameter	42.4	316.2	4443.7	12 344.9	1422.1	18 210.2
SFE-intercept	42	317.6	4303.2	12 452	1454	18 209.2
perfect competition	37.4	336.9	2853.2	13 927.4	1550	18 330.4

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Table 5 Market clearing without network constraints IEEE 57-bus test system

Models	$\bar{\lambda}$, \$/MW	$\sum_g P_g$, MW	$\sum_g S_g^G$, \$	$\sum_d S_d^C$, \$	S^M, \$	S^S, \$
Monopoly	71.7	436.1	20 820	12 062	0	32 882
Cournot	42.3	735.2	11 095	29 253	0	40 348
Stackelberg	40.6	753.2	9843	30 566	0	40 409
SFE-slope	37.7	782.6	7788	32 782	0	40 570
SFE-*k* parameter	37.6	783.1	7747	32 823	0	40 571
SFE-intercept	37.6	783.5	7717	32 854	0	40 571
perfect competition	36.4	796	6770	33 812	0	40 582

Table 6 Market clearing with network constraints IEEE 57-bus test system

Models	$\bar{\lambda}$, \$/MW	$\sum_g P_g$, MW	$\sum_g S_g^G$, \$	$\sum_d S_d^C$, \$	S^M, \$	S^S, \$
monopoly	72.1	432.4	20 766	11 902	0	32 668
Cournot	50.1	614.6	14 575	22 397	784	37 739
SFE-slope	43.3	674	10 934	26 594	938	38 466
SFE-*k* parameter	42.4	696	10 569	27 690	859	39 118
SFE-intercept	42.5	678.6	10 405	27 059	997.6	38 462
Stackelberg	41.7	714	10 211	28 650	761	39 623
perfect competition	37.4	762	6814	32 225	781	39 820

network and unconstrained network. In fact, the unconstrained network provides more favourable opportunity for the monopolist to realise maximum total surplus than the constrained network does. Although not evident, the total producer surplus are \$ 9641 (or \$ 32 882) and \$ 9603 (or \$ 32 668) for the constrained and unconstrained network based on IEEE 30-bus test system (or IEEE 57-bus test system), respectively. Owing to the assumption of the infinitive flow limits of other lines, the monopolist still has enough line resources to circumvent the constrained paths to deliver the power since all the generators are under his control. This explains the market clearing values are very similar both under the unconstrained and constrained network.

5.2 Market clearing under oligopoly model

A more common case is the oligopoly of which the equilibrium is in-between the two preceding cases. The models in Tables 3 and 4 are ranked based on the fact that how much does the oligopoly equilibrium deviate from the perfect competition equilibrium. Cournot model and CSF-1 model (they are equivalent to each other) have the highest oligopoly level both under constrained and unconstrained network. Other oligopoly models take the successive positions. Higher level of oligopoly behaviour from the supply side produces higher market clearing prices and lower social surpluses, leading to higher Lerner indices and lower market inefficiency indices, as shown in Tables 7 and 8.

Without network constraints, the market clearing under CSF-2, SFE-slope, SFE-*k* parameter and SFE-intercept models are very close to the perfect competition equilibrium. Those game models result in very low values of the σ_u indices (lower than 0.1) and ξ_u indices (in absolute value lower than 1%) both for IEEE 30-bus and IEEE 57-bus test systems. Owing to the abundant production ability in both two test systems, the total production capacity are 480 MW (or 1270 MW) and the total maximum demand are 530 MW (or 1166 MW) for IEEE 30-bus test system (or IEEE 57-bus test system), the ISO can always find enough cheap power to meet the market demand without network constraints. That makes the market players compete for supply in a mild level, pushing the oligopoly equilibrium close to the perfect competition.

5.3 Network constraints impacts

The network constraints contribute to higher level of the market power behaviour with higher Lerner indices (between 0.32 and 0.18 for IEEE 30-bus system, Table 7, whereas between 0.38 and 0.14 for IEEE 57-bus system, Table 8) and lower market inefficiency indices (between −6.16% and −4.3% for IEEE 30-bus system, Table 7, whereas between −7% and −2.4% for the IEEE 57-bus system, Table 8). Under

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Table 7 Market performance indices, IEEE 30-bus test system

Index		Models 1	2	3	4	5	6	7	8
Lerner (p.u.)	σ	0.52	0.32	0.32	0.23	0.21	0.21	0.19	0.18
	σ_u	0.52	0.22	0.22	0.18	0.1	0.09	0.08	0.07
market inefficiency (%)	ξ	−20	−6.16	−6.16	−4.47	−4.37	−4.4	−4.3	−4.3
	ξ_u	−20	−1.51	−1.51	−1.03	−0.45	−0.18	−0.15	−0.12
network impacts (%)	τ	−1.3	−4.95	−4.95	−3.59	−4.1	−4.41	−4.33	−4.37

Models: 1. monopoly, 2. Cournot, 3. CSF-1, 4. Stackelberg, 5. CSF-2, 6. SFE-slope, 7. SFE-k parameter, 8. SFE-intercept.

Table 8 Market performance indices, IEEE 57-bus test system

Index		Models 1	2	3	4	5	6
Lerner (p.u.)	σ	0.49	0.38	0.14	0.19	0.17	0.17
	σ_u	0.49	0.16	0.11	0.04	0.03	0.03
market inefficiency (%)	ξ	−19.5	−7	−2.4	−5.2	−3.6	−5.2
	ξ_u	−19	−0.6	−0.4	$\simeq$0	$\simeq$0	$\simeq$0
network impacts (%)	τ	$\simeq$0	−6.5	−2	−5.2	−3.6	−5.2

Models: 1. monopoly, 2. Cournot, 3. Stackelberg, 4. SFE-slope 5. SFE-*k* parameter, 6. SFE-intercept.

CSF-2, SFE-slope, SFE-*k* parameter and SFE-intercept models, the electricity producers that have no obvious strategic behaviour under unconstrained network will find the opportunities to exert strategic behaviour under constrained network. Since the producers bear the notion of that the ISO cannot arrange the transactions just according to the merit order rule, they can use the opportunity of the network constraints to isolate other competitors delivering power to the specific consumers. In this way, the producers can sell profitable but expensive energy to obtain higher surplus. With the introduction of the network constraints for the IEEE 30-bus test system, Table 7, the σ indices of the aforementioned models are, respectively, 0.21, 0.206, 0.187 and 0.178, which are remarkably higher than corresponding values of σ_u index. Similar numerical trend is also presented in the simulation case with the IEEE 57-bus test system, Table 8.

Furthermore, within a given oligopoly model, the introduction of network constraints will decrease the social surplus. That effect can be assessed by resorting to the τ index, column 5 of Table 2, used to just account for the network impacts, since the social surplus comparison is done between two oligopoly equilibriums (with and without network constraints). The values range between −5% and −3.6% among the different oligopoly models for IEEE 30-bus system, Table 7, and between −6.5% and −2% for the IEEE 57-bus test system, Table 8. The incurred network constraints dissipates the social surplus obtained at the oligopoly equilibrium, showing that the strategic behaviour exacerbate the inefficient market performance under the constrained network than the case of unconstrained network.

Another important point is that, with the introduction of network constraints, the social surplus at the perfect competition equilibrium (\$ 18 330.4 or \$ 39 820, the last row of Table 4 or 6) is even smaller than the social surplus at the Cournot equilibrium (\$ 18 740.1 or \$ 40 348, row 3 of Table 3 or 5) under unconstrained network. This fact can somewhat justify that reinforcing the transmission lines and letting them not to be congested is a positive approach for the market regulator to improve the market efficiency, from the social surplus point of view.

5.4 Producer and consumer surplus under oligopoly models

The goals of the producers are to maximise their individual surplus. With reference to the perfect competition, the total producer surplus ($\sum_g S_g^G$) and the total consumer surplus ($\sum_d S_d^C$) are higher and lower under both unconstrained and constrained network for different oligopoly models, respectively, as shown in columns 4 and 5 in Tables 3 and 4 or Tables 5 and 6. As an effect of strategic behaviour of the producers, social surplus passes from the consumer side to the producer side. As a result, the social surplus may not change significantly, the market inefficiency indices range

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between the −6.2% and −4.3% under constrained network (ξ) whereas in-between −1.5% and −0.12% (ξ_u) under unconstrained network for IEEE 30-bus test system, Table 7. For example by referring to perfect competition in Table 3, the SFE-slope model under IEEE 30-bus test system, row 7 of Table 4, the extra total surplus that the producers acquire because of the strategic behaviour ($ 2014.8 = $ 4756.6 − $ 2741.8) and the arising of merchandise surplus ($S^M =$ $ 1359.4) because of the network constraints is balanced in large part by the decreased total consumer surplus ($ 4211.1 = $ 16 285.6 − $ 12 074.5). That results in the social surplus ($ 18 190.5) close to the value of perfect competition equilibrium ($ 19 027.5) and the small value of market inefficiency index (in absolute value, 4.4% in Table 7). In this respect, the main effects of the strategic behaviour from the supply side are more remarkably presented in striving to obtain extra surplus from the consumer side with higher market clearing prices and production withdrawn.

Differently from the monopoly, the constrained network will impact the market clearing under oligopoly markets where there are a bunch of independent market players. The market clearing results deviate more from the perfect competition equilibrium with the introduction of the network constraints. Within a given oligopoly model, the total producer surplus ($\sum_g S_g^G$) and the total consumer surplus ($\sum_d S_d^C$) are higher and lower under constrained network than those values under unconstrained network, respectively, except for the Stackelberg model, as shown in columns 4 and 5 in Tables 3 and 4 for IEEE 30-bus system or Tables 5 and 6 for IEEE 57-bus system. For example under Cournot model, the total producer surplus is increased from $ 5934.8, Table 3 (or $ 11 095, Table 5) to $ 6827.5, Table 4 (or $ 14 575, Table 6) with the increased market clearing price, from 44.5 $/MW, Table 3 (or 42.3 $/MW, Table 5) to 51 $/MW, Table 4 (or 50.1 $/MW, Table 6) because of the network constraints under IEEE 30-bus test system (or IEEE 57-bus test system). However, the benefits of the network constraints to the supply side are along with the negative impact on the consumer side, the total consumer surplus is decreased from $ 12 805 (or $ 29 253) to $ 10 117 (or $ 22 397) under IEEE 30-bus test system (or IEEE 57-bus test system).

Although the constrained network will globally benefit the supply side, that effect does not apply to the individual producers. Tables 9 and 10 are shown for the individual producer surplus deviation indices, ς_g and ς_{gu}, under Cournot and SFE-slope models, respectively. All the producers obtain higher surplus both under constrained and unconstrained network with reference to the perfect competition equilibrium. However, the advantage of obtaining higher surplus under the case of constrained network over the case of unconstrained network can only benefit some producers. For example by referring to the IEEE 30-bus test system, both the Cournot and SFE-slope models, the producers at bus 13, 23 and 27 obtain higher surplus under constrained network than they do under unconstrained network, the ς_g index values are larger than the ς_{gu} index values. The producers at bus 2 and 22 will loss their surplus with smaller ς_g index values than the ς_{gu} indices because of the network constraints. As a matter of fact, with the introduction of the network constraints, the higher surpluses obtained by the beneficial producers outweigh the less surpluses of the loss producers results in the higher total producer surplus under most oligopoly models. Similar conclusions can be drawn on the IEEE 57-bus test system.

5.5 Comparison of the SFE models

With respect to the market performance, the three SFE models are very close to each other in terms of the market clearing prices and social surpluses, as shown in rows 7–9 of Tables 3 and 4 (or rows 5–7 of Table 5 and rows 4–6 of Table 6). However, there are differences for the individual surplus among the electricity producers. Under constrained network, the SFE-slope model yields the highest surpluses

Table 9 Producer surplus deviation index, Cournot model

IEEE 30-bus test system							
producers		1	2	3	4	5	6
buses		1	2	13	22	23	27
index	ς_g	1.03	1.05	2.51	0.97	2.49	1.61
	ς_{gu}	1.02	1.08	2.06	1.36	1.33	0.86

IEEE 57-bus test system											
producers		1	2	3	4	5	6	7	8	9	10
buses		1	3	8	12	23	28	39	46	50	53
index	ς_g	0.62	0.09	1.49	1.37	1.03	1.21	4.08	0.67	0.86	0.74
	ς_{gu}	0.61	0.67	1.07	0.78	0.78	0.66	0.66	0.49	0.56	0.54

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Table 10 Producer surplus deviation index, SFE-slope model

IEEE 30-bus test system							
producers		1	2	3	4	5	6
buses		1	2	13	22	23	27
index	ς_g	0.34	0.36	1.16	0.06	1.75	1.02
	ς_{gu}	0.38	0.41	0.73	0.49	0.44	0.33

IEEE 57 bus test system											
producers		1	2	3	4	5	6	7	8	9	10
buses		1	3	8	12	23	28	39	46	50	53
index	ς_g	0.04	−0.37	0.39	0.42	0.2	0.61	4.31	0.23	0.22	0.25
	ς_{gu}	0.12	0.13	0.19	0.15	0.15	0.13	0.13	0.11	0.12	0.11

Table 11 Individual producer surplus under SFE models constrained network

IEEE 30-bus test system						
producers	1	2	3	4	5	6
buses	1	2	13	22	23	27
SFE-slope	730	714	486	412	1030	1384
SFE-k parameter	667	645	450	369	993	1320
SFE-intercept	644	614	430	345	978	1293

IEEE 57-bus test system										
producers	1	2	3	4	5	6	7	8	9	10
buses	1	3	8	12	23	28	39	46	50	53
SFE-slope	705	422	452	735	620	1080	3568	1276	1034	1040
SFE-k parameter	677	384	390	708	660	1071	3356	1305	1035	982
SFE-intercept	648	402	406	666	574	1021	3551	1217	966	952

for all of the producers, as shown in Table 11. Therefore compared with other two SFE oligopoly models, the SFE-slope model can achieve a better equilibrium, from the supply side point of view, while does not bring obviously negative impacts on the market performances. The market inefficiency index has no significant difference between the three models because of the fact of the social surpluses are almost the same, as shown in column 7 of Tables 3 and 4 (or Tables 5 and 6) for the SFE models.

5.6 Discussion of the CSF model

The characteristic of the CSF model is that it can represent the Cournot model and the models with higher competition level by adjusting the conjectural parameters. Note that row 4 of Tables 3 and 4, the CSF-1 model is almost the same as the Cournot model both under constrained and unconstrained network. With the increase of the conjectural parameters, CSF-2 model provides market clearing that is closer to the perfect competition, showing that the competition level is increased, referring to row 6 of Tables 3 and 4. However, different assumptions of the distributions and values of the conjectural parameters among the market players, in this paper, we use the uniform parameters among different players for the IEEE 30-bus test case, will affect the oligopoly equilibrium. In addition, there is no solid ground to make estimation on the conjectural parameters or justify the rationality of those values, which are the application disadvantages of the CSF in modelling the strategic behaviour among the electricity market participants.

6 Conclusions

In the electricity markets, the power transactions are undertaken on a grid that needs to be operated under strict physical and operational constraints. For this reason, specific

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occasions of strategic behaviour related to network congestion may arise, giving a further source of market inefficiency.

Game theory models used to represent the competitive electricity markets have been analysed and tested by using the IEEE 30- and IEEE 57-bus systems. All simulations show a worsening of the market performance, as measured by the market inefficiency index and the Lerner index, when compared to the ideal model of perfect competition. Cournot model shows the worst behaviour, both under constrained and unconstrained network. CSF model with increased conjecture parameters provides the oligopoly market equilibrium closer to the perfect competition equilibrium. The three SFE models cause no obvious differences in terms of the Lerner indices and the market inefficiency indices. From the supply side point of view, if each producer behaves the way the SFE-slope model describes, their surpluses are more than they can attain in other SFE models.

Furthermore, social surplus experiences transition from the consumer side to the supply side along with the strategic behaviour of the producers. Actually, due to value transition, the social surplus at the oligopoly equilibrium may not be changed so much with reference to the perfect competition equilibrium. The main effects of the strategic behaviour from the supply side are more remarkably presented in obtaining extra surplus from the consumer side with higher market clearing prices and production withdrawn.

Owing to the network constraints, the transmission network plays a major role in determining the market equilibrium. Under constrained network, from the simulation results of each oligopoly model, the market clearing price is higher and the cleared demand are lower than the corresponding values under unconstrained network. As for the producer surpluses, the network constraints provide some producers with opportunities to obtain higher producer surplus at the expense of the total consumer surplus, leading to a higher level of market inefficiency compared with the unconstrained electricity market. In this respect, improving the electricity transmission network contributes to mitigate the market power in present with the strategic bidding behaviour of the electricity producers.

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8 Appendix

8.1 Derivation of the price at the reference bus

For the supply function equilibrium game models, SFE-intercept model for example, apply the KKT conditions to the optimisation problem (1)–(4) we have

$$\boldsymbol{a} + \boldsymbol{B}^m\boldsymbol{p} + \boldsymbol{\omega}^+ - \boldsymbol{\omega}^- = \lambda_N \boldsymbol{i_G} - \boldsymbol{J}_G^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) \tag{7}$$

$$\boldsymbol{Hq} + \boldsymbol{e} = \lambda_N \boldsymbol{i_D} - \boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) \tag{8}$$

$$\boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{p} - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{q} = 0 \tag{9}$$

From the above three equations, we can have

$$\lambda_N = \frac{\begin{array}{c}\boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}^m)^{-1}[\boldsymbol{J}_G^T(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) + \boldsymbol{a}^m] \\ -\boldsymbol{i}_D^T\boldsymbol{H}^{-1}[\boldsymbol{J}_D^T(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]\end{array}}{\boldsymbol{i}_G^{\mathrm{T}}(\boldsymbol{B}^m)^{-1}\boldsymbol{i}_G - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D} \tag{10}$$

From 7, the dispatched quantity of the current producer g, is

$$p_g = \frac{\lambda_N - \boldsymbol{J}_g^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) - (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) - a_g}{b_g^m} \tag{11}$$

whereas the dispatched quantities of the other competitor is

$$p_i = \frac{\lambda_N - \boldsymbol{J}_i^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) - (\boldsymbol{\omega}^+ - \boldsymbol{\omega}^-) - a_g'}{b_g^m}, \; i \in \boldsymbol{G}, \; i \neq g \tag{12}$$

where a_i' is the determined parameter derived from last iteration move of producer i.

As for the quantity bidding models, for example the Cournot model, we have

$$\boldsymbol{Hq} + \boldsymbol{e} = \lambda_N \boldsymbol{i_D} - \boldsymbol{J}_D^{\mathrm{T}}(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) \tag{13}$$

$$p_g + \boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{p}' - \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{q} = 0 \tag{14}$$

where $\boldsymbol{p}' = [p_1', \ldots, p_{g-1}', 0, p_{g+1}', \ldots, p_{N_G}]^{\mathrm{T}}$, p_i' $(i \in \boldsymbol{G}, i \neq g)$ is the determined optimal quantity derived from the last move of the producer i, p_g is the decision variable in the optimal problem (6).

The price at the reference bus is

$$\lambda_N = \frac{p_g + \boldsymbol{i}_G^{\mathrm{T}}\boldsymbol{p}' + \boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}[\boldsymbol{J}_D^T(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-) + \boldsymbol{e}]}{\boldsymbol{i}_D^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{i}_D} \tag{15}$$

8.2 Tables and Figures for the IEEE-30 bus and IEEE-57 bus test systems

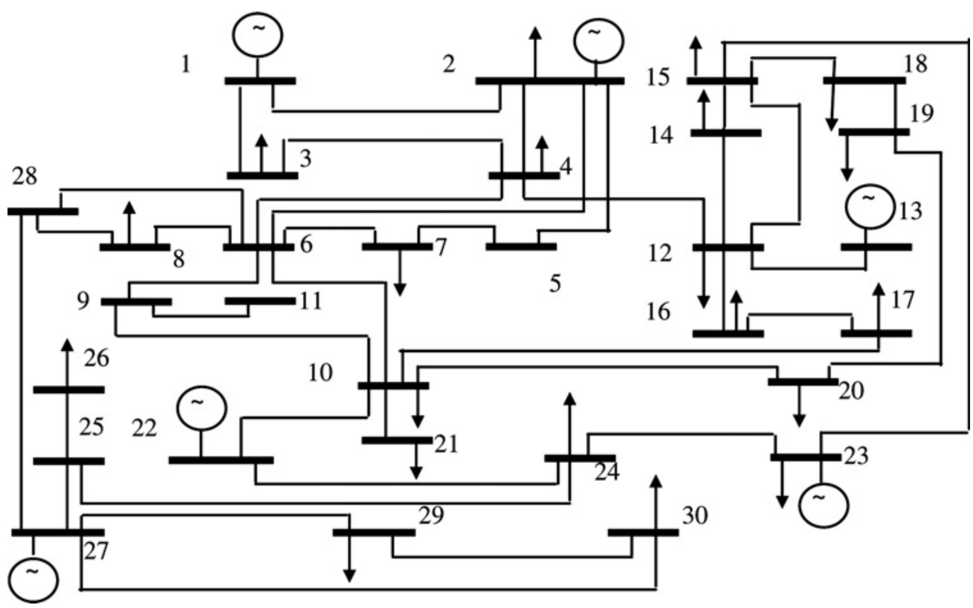

Figure 1 *IEEE 30-bus transmission network*

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Table 12 Parameters of the producers for IEEE 30-bus test system

Bus	g	a_g^m, \$/MW	b_g^m, \$/MW^2	P_g^-, MW	P_g^+, MW	Bus	g	a_g^m, \$/MW	b_g^m, \$/MW^2	P_g^-, MW	P_g^+, MW
1	1	18	0.25	5	100	22	4	22	0.2	5	80
2	2	20	0.2	5	80	23	5	22	0.2	5	50
13	3	25	0.2	5	50	27	6	16	0.25	5	120

Table 13 Parameters of the consumers for IEEE 30-bus test system

Bus	d	e_d, \$/MW	f_d, \$/MW^2	Bus	d	e_d, \$/MW	f_d, \$/MW^2	Bus	d	e_d, \$/MW	f_d, \$/MW^2	Bus	d	e_d, \$/MW	f_d, \$/MW^2
2	1	120	−5	10	6	95	−3	17	11	90	−3.5	23	16	120	−5
3	2	130	−5.5	12	7	150	−5.5	18	12	95	−3.5	24	17	150	−6
4	3	120	−4.5	14	8	125	−4	19	13	90	−3.5	26	18	100	−4.5
7	4	135	−5	15	9	100	−4.5	20	14	90	−3.5	29	19	95	−3.5
8	5	150	−5	16	10	150	−5	21	15	160	−6	30	20	125	−4.5

Table 14 Considered lines of the constrained network for IEEE 30-bus test system

Lines l	From bus	To bus	Flow limits MW
10	6	8	10
17	12	14	8
26	10	17	10

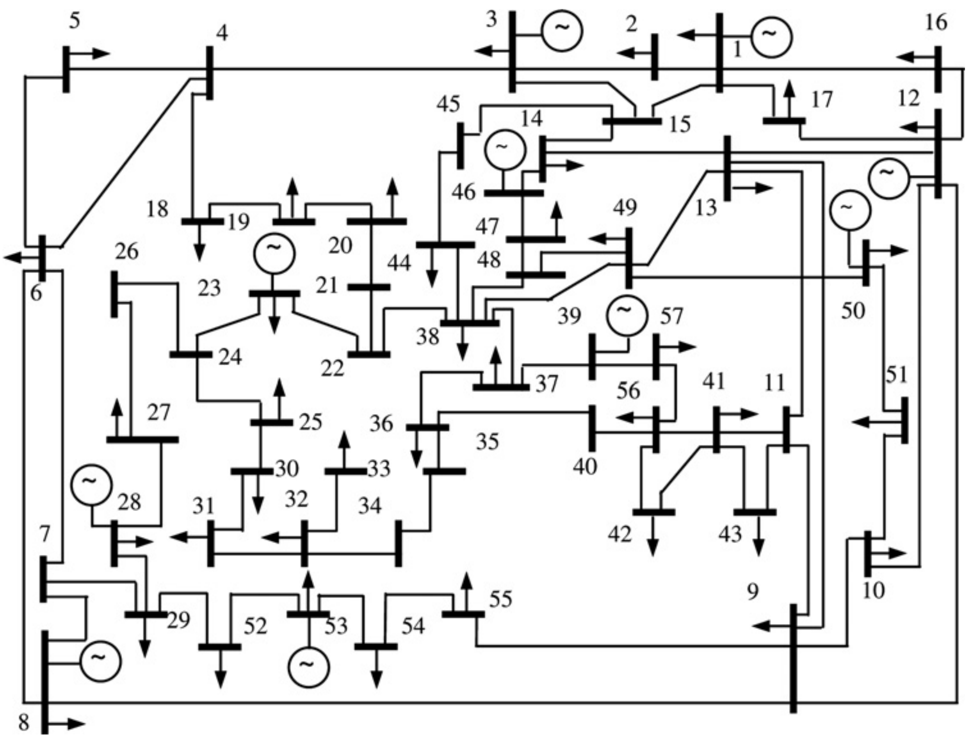

Figure 2 *IEEE 57-bus transmission network*

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Table 15 Parameters of the producers and consumers for IEEE 57-bus test system

Bus	g	a_g^m, \$/MW	b_g^m, \$/MW^2	P_g^-, MW	P_g^+, MW	Bus	g	a_g^m, \$/MW	b_g^m, \$/MW^2	P_g^-, MW	P_g^+, MW
1	1	18	0.25	5	150	28	6	20	0.2	5	100
3	2	20	0.2	5	110	39	7	20	0.2	5	110
8	3	25	0.2	5	100	46	8	16	0.2	5	150
12	4	22	0.2	5	110	50	9	18	0.2	5	170
23	5	22	0.2	5	100	53	10	16	0.25	5	170

Table 16 Parameters of the consumers for IEEE 57 -bus test system

Bus	D	e_d \$/MW	f_d \$/MW^2	Bus	d	e_d \$/MW	f_d \$/MW^2	Bus	d	e_d \$/MW	f_d \$/MW^2	Bus	d	e_d \$/MW	f_d \$/MW^2
1	1	120	−5	15	12	95	−3.5	30	23	135	−5	44	34	90	−3.5
2	2	130	−5.5	16	13	90	−3.5	31	24	150	−5	47	35	160	−6
3	3	120	−4.5	17	14	90	−3.5	32	25	95	−3	49	36	120	−5
5	4	135	−5	18	15	160	−6	33	26	150	−5.5	50	37	150	−6
6	5	150	−5	19	16	150	19	35	27	125	−4	51	38	100	−4.5
8	6	95	−3	20	17	100	20	36	28	100	−4.5	52	39	95	−3.5
9	7	150	−5.5	23	18	95	23	37	29	150	−5	53	40	125	−4.5
10	8	125	−4	25	19	125	25	38	30	100	−4.5	54	41	100	−4.5
12	9	100	−4.5	27	20	120	27	41	31	90	−3.5	55	42	150	−5
13	10	150	−5	28	21	130	28	42	32	95	−3.5	56	43	90	−3.5
14	11	90	−3.5	29	22	120	29	43	33	90	−3.5	57	44	95	−3.5

Table 17 Considered lines of the constrained network for IEEE 57-bus test system

Lines l	From bus	To bus	Flow limits MW
4	4	5	5
10	9	11	5
49	36	37	10

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