APPLICATION OF SUBSTITUTABILITY IN CONGESTION RELIEF

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Abstract – The principal cause of transmission congestion is the lack of adequate transfer capabilities in the network. A modification of the network resulting in the increase of the constrained transfer capability(ies) is a viable scheme for congestion relief. An alternative approach is the addition of supply sources at nodes at which additional injection can be accommodated without violating the network constraints. In this sense, we may view the generation resource addition (*GRA*) and the transmission transfer capability enhancement (*TCE*) as *substitutable* congestion relief schemes. In this paper, we define and quantify the notion of *substitutability* in terms of the impacts of the *GRA* and those of the *TCE* on the electricity market outcomes. We use the social welfare as the metric to quantify the impacts of the modification of the resource mix or of the network. This measure allows the quantification to be performed on a consistent basis. We derive criteria under which the *GRA* and the *TCE* are substitutable for congestion relief. Simulation results on various test systems indicate the comparative ability of attaining congestion relief through a *GRA* or a *TCE*. Illustrative results on the IEEE 57-bus network are provided.

***Index Terms*— congestion relief, generation resource addition, transfer capability enhancement, locational marginal price, social welfare.**

1 INTRODUCTION

The advent of open access transmission [1], [2] and the wider implementation of competitive electricity markets [3] have resulted in the growing prominence of transmission congestion. Congestion occurs whenever the preferred generation/demand pattern of the various market players requires the provision of transmission services beyond the capability of the transmission system to provide. Under the ideal conditions of the transmission-unconstrained markets, the various buyers and sellers are able to consummate desired deals as long as their respective surpluses are nonnegative [4], [5]. When the constraints on the transmission network are taken into account, the constrained transfer capabilities of the network may be unable to accommodate the preferred unconstrained market schedule without violating one or more constraints. The presence of congestion introduces unavoidable losses in market efficiency so that the benefits foreseen through restructuring may not be fully realized [4], [5]. There is a growing realization that congestion is a major obstacle to vibrant competitive electricity markets [6]. Therefore, effective management and relief of congestion is a critically important contributor to the establishment of vibrant competitive electricity markets.

Various approaches from the *command-and-control* transmission loading relief (*TLR*) scheme used by *NERC* security coordinators [7] to the market-based methods using locational marginal prices (*LMPs*) [8] and other economic signals [9]-[13] have been proposed to manage and relieve transmission congestion. The essence of these methods is to modify the injection and withdrawal pattern so that the transmission network can accommodate them without violating the constraints. The congestion relief problem may also be addressed in alternative ways. Congestion results from insufficient transfer capabilities to simultaneously transfer electricity between the various selling and buying entities [13]. It follows that a modification of the network that leads to an increase of the constrained transfer capability(ies) is a viable scheme for congestion relief. Typical modifications include addition of a new transformer/transmission line, reconductoring of existing transmission line(s) and installation of electronic devices such as wavetraps. An alternative approach is

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the addition of supply sources at nodes at which additional injection can be accommodated without violating the network transfer capabilities. Therefore, either a generation resource addition (*GRA*) or a transmission transfer capability enhancement (*TCE*), when introduced appropriately, is a viable approach to relieve the transmission congestion.

In this paper, we analyze and compare the impacts of the *GRA* and the *TCE* modifications on the outcomes of the electricity markets. We consider a centralized day-ahead market operated by the independent grid operator (*IGO*) [14] with pool sellers and buyers operating along with bilateral transactions. While the comprehensive valuation of such new assets requires the consideration of various asset attributes, including construction costs, environmental impacts and economic benefits during the asset life span, our focus is on their impacts on the transmission congestion for a single hour *h* so as to evaluate their operational impacts. Our analysis shows that the *GRA* and *TCE* modifications may be viewed as *substitutable* congestion relief schemes in the sense that a *GRA* and a *TCE* provide interchangeable schemes to relieve transmission congestion. We define and quantify this *substitutability* notion by formulating a single appropriate metric for evaluating the impacts of the *GRA* and *TCE* modifications. We study the relationship between the location of a modification and its impacts on the market outcomes and derive conditions for the substitutability. These results are discussed in detail and illustrated with appropriately selected numerical results.

In some cases, the system may benefit more with both the *GRA* and the *TCE* than with just either of them. In this sense, these two modifications have complementarity [15]. While the substitutability term in economic theory expressly excludes complementarity [15], in this paper, we slightly abuse the term and allow complementarity in our discussions of substitutability.

This paper contains five additional sections. We start in section 2 with the formulation of the appropriate analytical basis for the substitutability study. In section 3, we provide analytical results to allow the quantification of the impacts of a *GRA* and a *TCE* on a consistent basis. This analysis provides a practical and meaningful performance metric for substitutability. We present in section 4 representative results of the various simulations performed on several test systems and conclude with a summary in section 5.

2 SUBSTITUTABILITY STUDY PLATFORM

To evaluate the substitutability notion, we need to quantify the effects of the *GRA* and *TCE* modification on the market outcomes on a consistent basis so as to allow effective comparative analysis. To make the quantification possible, we evaluate the effects of a *GRA* with $\overline{p}_i^{\mathcal{G}}$ *MW* capacity at node *i* and also the effects of a *TCE* that results in increasing the transfer capability from node *m* to node *n* by $\overline{p}_{m,n}^{\mathcal{T}}$ *MW*. We neglect the maintenance costs for the *TCE*. The variable cost of the new generation is a_i *$/MWh*. While several metrics are commonly used in the literature to measure the effectiveness of a congestion relief scheme [13], we use the *social welfare* [4], [5] as the metric to quantify the impacts of the two modifications since it provides a comprehensive measure on both the quantity and the economic benefits attained by the modifications. We analyze the substitutability in terms of this metric and term it as the $\mathscr{S}_h$*–substitutability* where *h* refers to the specified hour.

In this section, we establish a platform for this purpose. The basic idea is to determine the changes in the market outcomes with a *GRA* or with a *TCE* from those in the reference case of the “as is” system.

We consider a network with N+1 buses and L lines. We denote by $\mathcal{N} \triangleq \{0,1,2,\cdots,N\}$ the set of buses, with bus 0 being the slack bus, and by $\mathcal{L} \triangleq \{\ell_1,\ell_2,\cdots,\ell_L\}$ the set of transmission lines and transformers that connect the buses in the set $\mathcal{N}$. We associate with each element $\ell \in \mathcal{L}$ the *ordered* pair $\ell = (i, j)$ to denote the line ℓ with series admittance $g_\ell - jb_\ell$ and real power flow limit f_ℓ^{max}. We adopt the convention that the direction of the flow in line ℓ is *from* node *i* *to* node *j* so that $f_\ell \geq 0$, where f_ℓ is the real power flow in line ℓ. Let $\underline{\boldsymbol{f}} \triangleq [f_1, f_2,\cdots, f_L]^T$ and $\underline{\boldsymbol{f}}^{max} \triangleq [f_1^{max}, f_2^{max},\cdots, f_L^{max}]^T$. We denote by $\underline{\boldsymbol{B}}_{\boldsymbol{d}} \triangleq diag\{b_1,b_2,\cdots, b_L\}$ the diagonal $L \times L$ *branch susceptance matrix* and by $\underline{\boldsymbol{A}}$ the *reduced incidence matrix* [16]. $\underline{\boldsymbol{B}} \triangleq \underline{\boldsymbol{A}}^T \underline{\boldsymbol{B}}_{\boldsymbol{d}} \underline{\boldsymbol{A}}$ is then the *reduced nodal susceptance matrix*. $\underline{\boldsymbol{b}}_0$ is the column of the nodal susceptance matrix [16] corresponding to the slack bus.

We consider the centralized day-ahead market with both pool players and bilateral transactions for the specified hour *h*. Without loss of generality, we assume there is a single pool seller and a single pool buyer[1] at each network node $n \in \mathcal{N}$. Each seller submits a sealed offer to the *IGO* specifying the *MWh/h* amount of electricity and the associated price it is willing to sell. We denote the integral of the seller's marginal offer price by $\beta_n^s(p_n^s)$, where $p_n^s \in \left[0, \overline{p}_n^s\right]$ is the amount of energy offered to the *IGO* by the seller *n*. We assume $\beta_n^s(\cdot)$ to be nonnegative, differentiable and convex. Also, each pool buyer submits a bid with the desired purchase amount and the associated maximum price willing to pay. The integral of the marginal bid price is denoted by $\beta_n^b(p_n^b)$ with $p_n^b \in \left[0, \overline{p}_n^b\right]$. We assume $\beta_n^b(\cdot)$ to be nonnegative, differentiable and concave. Let $\underline{\boldsymbol{p}}^s \triangleq [p_1^s, p_2^s,\cdots, p_N^s]^T$, $\underline{\overline{\boldsymbol{p}}}^s \triangleq [\overline{p}_1^s, \overline{p}_2^s,\cdots, \overline{p}_N^s]^T$, $\underline{\boldsymbol{p}}^b \triangleq [p_1^b, p_2^b,\cdots, p_N^b]^T$ and $\underline{\overline{\boldsymbol{p}}}^b \triangleq [\overline{p}_1^b, \overline{p}_2^b,\cdots, \overline{p}_N^b]^T$. The bilateral transactions are represented by $\mathcal{W} \triangleq \{\overline{\omega}^1, \overline{\omega}^2, \cdots, \overline{\omega}^W\}$ where each element $\overline{\omega}^w$ is an *ordered* triplet $\overline{\omega}^w \triangleq \{m^w, n^w, \overline{t}^w\}$ representing a transaction with receipt point (*from* node) m^w, delivery point (*to* node) n^w and the *desired* transaction amount $\overline{t}^w$ *MW*. Let $\underline{\overline{\boldsymbol{t}}} \triangleq [\overline{t}^1, \overline{t}^2,\cdots, \overline{t}^W]^T$. The integration of the bilateral transactions, with the submission of their willingness to pay for congestion charges explicitly considered, together with the pool players developed in [17], leads to the more efficient market operations. We adopt this scheme in the construction of the approach proposed in this paper. We assume each bilateral transaction $\overline{\omega}^w$ provides a function $\alpha^w(t^w)$ in its transmission request to indicate the maximum charges willing to be borne [17] as a function of the delivered transaction amount $t^w \in [0, \overline{t}^w]$. We assume $\alpha^w(\cdot)$ to be nonnegative, concave and differentiable.

The *IGO* collects the bids/offers and transmission requests and determines simultaneously the market outcomes and the transmission schedules based on a security constrained optimal power flow (*SCOPF*) formulation. In a nonlinear AC *OPF* representation, full consideration of the impacts of reactive power and voltage limits provides a detailed representation of the transmission grid. However, the tractability of the computations and the speed requirements associated with market applications make the simplified, fast solution speed DC *OPF* more practical in the various *IGO* operations and planning applications, including the day-ahead market clearing computations. The impacts of the reactive powers and the voltage limits may be approximated by appropriately modifying the real power flow limits of the lines/interfaces. Therefore, we follow the practice of actual market operators and adopt a DC *OPF* formulation in the *SCOPF*. We refer to our formulation as the generalized transmission scheduling problem ($GTSP_h$) [14].

1 The offers of multiple sellers at a node may be combined into a composite offer function submitted by a single seller; similarly, the bids of multiple buyers at a node may be combined into a composite bid function submitted by a single buyer.

$$
\begin{cases}
max \;\; \mathscr{S}_h\left(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}\right) = \sum_{w=1}^{W} \alpha^w\left(t^w\right) + \\
\qquad \sum_{n=0}^{N}\left[\beta_n^b\left(p_n^b\right) - \beta_n^s\left(p_n^s\right)\right] \\
s.t. \qquad p_0^s - p_0^b + p_0^{\mathcal{W}} = \underline{\boldsymbol{b}}_0^T \underline{\theta} \;\; \leftrightarrow \;\; \mu_0 \\
\qquad \underline{\boldsymbol{p}}^s - \underline{\boldsymbol{p}}^b + \underline{\boldsymbol{p}}^{\mathcal{W}} = \underline{\boldsymbol{B}}\underline{\theta} \;\; \leftrightarrow \;\; \underline{\boldsymbol{\mu}} \\
\qquad \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}}\underline{\theta} \le \underline{\boldsymbol{f}}^{max} \\
\qquad 0 \le p_0^s \le \overline{p}_0^s \\
\qquad 0 \le p_0^b \le \overline{p}_0^b \\
\qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{p}}^s \le \underline{\overline{\boldsymbol{p}}}^s \\
\qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{p}}^b \le \underline{\overline{\boldsymbol{p}}}^b \\
\qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{t}} \le \underline{\overline{\boldsymbol{t}}}
\end{cases}
\tag{1}
$$

with

$$
\underline{\boldsymbol{p}}^{\mathcal{W}} \triangleq \left[p_1^{\mathcal{W}}, p_2^{\mathcal{W}}, \cdots, p_N^{\mathcal{W}}\right]^T \;, \tag{2}
$$

where

$$
p_n^{\mathcal{W}} \triangleq \sum_{w \in \mathcal{W},\, m^w = n} t^w - \sum_{w \in \mathcal{W},\, n^w = n} t^w \;, \quad n = 0, 1, \cdots, N \tag{3}
$$

is the net nodal injection corresponding to the set $\mathcal{W}$ of bilateral transactions undertaken. The optimal solution ($p_0^{s*}, p_0^{b*}, \underline{\boldsymbol{p}}^{s*}, \underline{\boldsymbol{p}}^{b*}, \underline{\boldsymbol{t}}^*$) determines the market outcomes and the transmission schedules for the hour *h*. The dual variables ($\mu_0^*, \underline{\boldsymbol{\mu}}^*$) determine the corresponding *LMP*s. The optimal value of the objective function is

$$
\mathscr{S}_h^* = \mathscr{S}_h\left(p_0^{s*}, p_0^{b*}, \underline{\boldsymbol{p}}^{s*}, \underline{\boldsymbol{p}}^{b*}, \underline{\boldsymbol{t}}^*\right) \;, \tag{4}
$$

the value of the maximum social welfare attained for the reference system.

We next restate the scheduling problem for the same hour *h* for the modified system with a *GRA*. We consider the *GRA* of $\overline{p}_i^{\mathcal{G}}$ *MW* at node *i* with the variable production cost of a_i *$/MWh*. Such an additional resource may be modeled as an additional seller[2] at node *i* with the offer function $\tilde{\beta}_i^s\left(p_i^{\mathcal{G}}\right) = a_i\, p_i^{\mathcal{G}}$, $p_i^{\mathcal{G}} \in [\,0, \overline{p}_i^{\mathcal{G}}\,]$. The resulting scheduling problem $GTSP_h^{\mathcal{G}}$ for this *GRA* is

$$
\begin{cases}
max \;\; \mathscr{S}_h^{\mathcal{G}}\left(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}, p_i^{\mathcal{G}}\right) = \sum_{w=1}^{W} \alpha^w\left(t^w\right) + \\
\qquad \sum_{n=0}^{N}\left[\beta_n^b\left(p_n^b\right) - \beta_n^s\left(p_n^s\right)\right] - a_i\, p_i^{\mathcal{G}} \\
s.t. \qquad p_0^s - p_0^b + p_0^{\mathcal{W}} = \underline{\boldsymbol{b}}_0^T \underline{\theta} \;\; \leftrightarrow \;\; \mu_0^{\mathcal{G}} \\
\qquad \underline{\boldsymbol{p}}^s - \underline{\boldsymbol{p}}^b + \underline{\boldsymbol{p}}^{\mathcal{W}} + p_i^{\mathcal{G}} \underline{\boldsymbol{e}}_i = \underline{\boldsymbol{B}}\,\underline{\theta} \;\; \leftrightarrow \;\; \underline{\boldsymbol{\mu}}^{\mathcal{G}} \\
\qquad \underline{\boldsymbol{B}}_d \underline{\boldsymbol{A}}\underline{\theta} \le \underline{\boldsymbol{f}}^{max} \\
\qquad 0 \le p_0^s \le \overline{p}_0^s \\
\qquad 0 \le p_0^b \le \overline{p}_0^b \\
\qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{p}}^s \le \underline{\overline{\boldsymbol{p}}}^s \\
\qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{p}}^b \le \underline{\overline{\boldsymbol{p}}}^b \\
\qquad \boldsymbol{0} \le \underline{\boldsymbol{t}} \le \underline{\overline{\boldsymbol{t}}} \\
\qquad p_i^{\mathcal{G}} \le \overline{p}_i^{\mathcal{G}}
\end{cases}
\tag{5}
$$

2 To be consistent with the assumption of a single seller and a single buyer at each node, we need to represent the *GRA* by modifying the offer function of the seller at node *i*. However, to allow us to evaluate the effect of the resource addition, we relax the single seller assumption and represent the *GRA* as an additional seller.

where,

$$\underline{\boldsymbol{e}}_i \triangleq [0\cdots0\overset{i}{1}0\cdots0]^T \in \Re^N . \tag{6}$$

All the market outcomes and the corresponding *LMP*s with the *GRA* are evaluated using $GTSP_h^{\mathcal{G}}$. The maximum social welfare for the *GRA* modified system is

$$\mathscr{S}_h^{\mathcal{G}*} = \mathscr{S}_h^{\mathcal{G}}\left(p_0^{s*}, p_0^{b*}, \underline{\boldsymbol{p}}^{s*}, \underline{\boldsymbol{p}}^{b*}, \underline{\boldsymbol{t}}^*, p_i^{\mathcal{G}*}\right) . \tag{7}$$

We can analogously perform the evaluation of the impacts of the *TCE* that increases the transfer capability from bus *m* to bus *n* by $\overline{p}_{m,n}^{\mathcal{T}}$ *MW*. Conceptually, we may view such an enhancement to be equivalent to adding a $\overline{p}_{m,n}^{\mathcal{T}}$ *MW* counter flow from the node *n* to the node *m*. Mathematically, we represent this counter flow by a new transaction $\overline{\omega}^{\mathcal{T}} = \{n, m, \overline{p}_{m,n}^{\mathcal{T}}\}$ with a zero willingness to pay. We modify the scheduling problem for this *TCE* and refer to it as $GTSP_h^{\mathcal{T}}$:

$$\begin{cases} max \quad \mathscr{S}_h^{\mathcal{T}}\left(p_0^s, p_0^b, \underline{\boldsymbol{p}}^s, \underline{\boldsymbol{p}}^b, \underline{\boldsymbol{t}}, p_{m,n}^{\mathcal{T}}\right) = \\ \qquad \sum_{w=1}^{W} \alpha^w\left(t^w\right) + \sum_{n=0}^{N}\left[\beta_n^b\left(p_n^b\right) - \beta_n^s\left(p_n^s\right)\right] \\ s.t. \qquad p_0^s - p_0^b + p_0^{\mathcal{W}} = \underline{\boldsymbol{b}}_0^T\underline{\theta} \leftrightarrow \mu_0^{\mathcal{T}} \\ \underline{\boldsymbol{p}}^s - \underline{\boldsymbol{p}}^b + \underline{\boldsymbol{p}}^{\mathcal{W}} + p_{m,n}^{\mathcal{T}}\left(\underline{\boldsymbol{e}}_n - \underline{\boldsymbol{e}}_m\right) = \underline{\boldsymbol{B}}\underline{\theta} \leftrightarrow \underline{\boldsymbol{\mu}}^{\mathcal{T}} \\ \qquad \underline{\boldsymbol{B}}_d\,\underline{\boldsymbol{A}}\,\underline{\theta} \le \underline{\boldsymbol{f}}^{max} \\ \qquad 0 \le p_0^s \le \overline{p}_0^s \\ \qquad 0 \le p_0^b \le \overline{p}_0^b \\ \qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{p}}^s \le \overline{\underline{\boldsymbol{p}}}^s \\ \qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{p}}^b \le \overline{\underline{\boldsymbol{p}}}^b \\ \qquad \underline{\boldsymbol{0}} \le \underline{\boldsymbol{t}} \le \overline{\underline{\boldsymbol{t}}} \\ \qquad p_{m,n}^{\mathcal{T}} \le \overline{p}_{m,n}^{\mathcal{T}} \end{cases} \tag{8}$$

Solution of the $GTSP_h^{\mathcal{G}}$ determines the market outcomes and the *LMP*s for the *TCE* case. We denote by

$$\mathscr{S}_h^{\mathcal{T}*} = \mathscr{S}_h^{\mathcal{T}}\left(p_0^{s*}, p_0^{b*}, \underline{\boldsymbol{p}}^{s*}, \underline{\boldsymbol{p}}^{b*}, \underline{\boldsymbol{t}}^*\right) \tag{9}$$

the maximum social welfare for the *TCE* system.

The three different *GTSP* statements, $GTSP_h$ of the reference system in (1)-(3), $GTSP_h^{\mathcal{G}}$ of the system with the *GRA* in (5)-(6) and $GTSP_h^{\mathcal{T}}$ of the system with the *TCE* in (8), constitute the key building blocks of the platform for the $\mathscr{S}_h$*–substitutability* analysis. The solutions of these three problems provide the information used to quantify and compare the impacts of the two modifications. Due to the larger feasible regions produced by each modification than that of the reference problem, it follows that

$$\mathscr{S}_h^{\mathcal{G}*} \ge \mathscr{S}_h^* \quad \text{and} \quad \mathscr{S}_h^{\mathcal{T}*} \ge \mathscr{S}_h^* . \tag{10}$$

Let

$$\mathscr{V}_h^{\mathcal{G}} \triangleq \mathscr{S}_h^{\mathcal{G}*} - \mathscr{S}_h^* \ge 0 \tag{11}$$

represent the increase in the social welfare due to the bus *i GRA*. If $\mathscr{V}_h^{\mathcal{G}} > 0$, then, the *GRA* $\overline{p}_i^{\mathcal{G}}$ is said to be effective for congestion relief in hour *h*. $\mathscr{V}_h^{\mathcal{G}}$ is termed the *value* of the $\overline{p}_i^{\mathcal{G}}$ *MW GRA* at node *i* for the specified hour *h*. Analogously, we denote by

$$\mathscr{V}_h^{\mathcal{T}} \equiv \mathscr{S}_h^{\mathcal{T}*} - \mathscr{S}_h^{*} \geq 0 \tag{12}$$

the *value* of the $\overline{p}_{m,n}^{\mathcal{T}}$ *MW TCE* from node *m* to node *n* in hour *h*. We consider the *TCE* to be an effective congestion relief tool for hour *h* whenever $\mathscr{V}_h^{\mathcal{T}} > 0$.

The notion of the $\mathscr{S}_h$*–substitutability* for hour *h* is made more precise with the quantification of $\mathscr{V}_h^{\mathcal{G}}$ and $\mathscr{V}_h^{\mathcal{T}}$. If $\mathscr{V}_h^{\mathcal{G}} > 0$ and $\mathscr{V}_h^{\mathcal{T}} > 0$, we consider the $\overline{p}_i^{\mathcal{G}}$ *MW GRA* and the $\overline{p}_{m,n}^{\mathcal{T}}$ *MW TCE* to be $\mathscr{S}_h$*–substitutable* for the specified hour *h*. The substitutability notion may be extended to other metrics, but we limit our investigation to $\mathscr{S}_h$*–substitutability* in this paper.

3 ANALYSIS OF $\mathscr{S}_h$ –*SUBSTITUTABILITY*

With the quantification of $\mathscr{S}_h$*–substitutability*, we next embark on the study of the impacts on the social welfare of a *GRA* and those of a *TCE*. We focus first on the analysis of $GTSP_h^{\mathcal{G}}$ as stated in (5). Now, the optimal solution of (5) is clearly a function of the parameter $\overline{p}_i^{\mathcal{G}}$, the *GRA* under consideration. Our notation of $\mathscr{S}_h^{\mathcal{G}*}$ and $\mu_n^{\mathcal{G}*}$, $n \in \mathcal{N}$, for the optimal social welfare and *LMP*s, respectively, clearly fails to explicitly represent this dependence. We indicate such dependence on $\overline{p}_i^{\mathcal{G}}$ by explicitly expressing the optimal solution of each decision and dual variable as a function of $\overline{p}_i^{\mathcal{G}}$:

$$\begin{aligned}\mathscr{S}_h^{\mathcal{G}*} = \mathscr{S}_h^{\mathcal{G}}\Big[& p_0^{s*}\left(\overline{p}_i^{\mathcal{G}}\right), p_0^{b*}\left(\overline{p}_i^{\mathcal{G}}\right), \underline{\boldsymbol{p}}^{s*}\left(\overline{p}_i^{\mathcal{G}}\right), \underline{\boldsymbol{p}}^{b*}\left(\overline{p}_i^{\mathcal{G}}\right), \\ & \underline{\boldsymbol{t}}^{*}\left(\overline{p}_i^{\mathcal{G}}\right), p_i^{\mathcal{G}*}\left(\overline{p}_i^{\mathcal{G}}\right)\Big] \equiv \hat{\mathscr{S}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right)\end{aligned} \tag{13}$$

and

$$\mu_n^{\mathcal{G}*} = \hat{\mu}_n^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right), \quad n = 0,1,\cdots,N \ . \tag{14}$$

$\hat{\mathscr{S}}_h^{\mathcal{G}}(\cdot)$ is a non-decreasing function of $\overline{p}_i^{\mathcal{G}}$ since, as $\overline{p}_i^{\mathcal{G}}$ increases, there is increased choice of resources for meeting the loads and possibly replacing the supply of higher priced resources, i.e., an increase in the value of $\overline{p}_i^{\mathcal{G}}$ corresponds to a larger decision space. Note that, for $\overline{p}_i^{\mathcal{G}} = 0$, the $GTSP_h^{\mathcal{G}}$ simply reduces to the $GTSP_h$ of the reference network. Therefore, $\hat{\mathscr{S}}_h^{\mathcal{G}}(0) = \mathscr{S}_h^{*}$ and $\hat{\mu}_n^{\mathcal{G}}(0) = \mu_n^{*}$. The necessary conditions of optimality for (5) implies that the social welfare $\hat{\mathscr{S}}_h^{\mathcal{G}}(\cdot)$ satisfies

$$\partial \hat{\mathscr{S}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) \Big/ \partial \overline{p}_i^{\mathcal{G}} = max\left\{ 0, \ \hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) - a_i \right\} \ . \tag{15}$$

Equation (11) implies that $\mathscr{V}_h^{\mathcal{G}}$ is also a function of $\overline{p}_i^{\mathcal{G}}$ and we denote this functional relationship by

$$\hat{\mathscr{V}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) = \hat{\mathscr{S}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) - \mathscr{S}_h^{*} \ . \tag{16}$$

Then, $\hat{\mathscr{V}}_h^{\mathcal{G}}(0) = 0$. It follows from (15) that

$$\left.\frac{\partial \hat{\mathscr{V}}_h^{\mathcal{G}}}{\partial \overline{p}_i^{\mathcal{G}}}\right|_{\overline{p}_i^{\mathcal{G}}} = \left.\frac{\partial \hat{\mathscr{S}}_h^{\mathcal{G}}}{\partial \overline{p}_i^{\mathcal{G}}}\right|_{\overline{p}_i^{\mathcal{G}}} = max\left\{ 0, \hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) - a_i \right\} \ . \tag{17}$$

Integration of both sides of (17) yields

$$\hat{\mathscr{V}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) = \int_0^{\overline{p}_i^{\mathcal{G}}} \max\left\{ 0, \hat{\mu}_i^{\mathcal{G}}(\zeta) - a_i \right\} d\zeta \ . \tag{18}$$

Equation (18) implies that the value of the *GRA* may be evaluated in terms of the *LMP* function $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ at node *i*. Also, for $\mu_i^* = \hat{\mu}_i^{\mathcal{G}}(0) \le a_i$, $\hat{\mathscr{V}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) = 0$ as either none of the $\overline{p}_i^{\mathcal{G}}$ *MW GRA* is utilized or its utilization does not produce an increase in the social welfare, in the following discussion. Therefore, we assume $\mu_i^* > a_i$ holds. Using the duality theorem and the Kuhn-Tucker conditions [18] of the *GTSP*, we can show that

- $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ is a non-increasing function of $\overline{p}_i^{\mathcal{G}}$;
- $\hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) \ge a_i$, $\forall\ \overline{p}_i^{\mathcal{G}} \ge 0$; and,
- there exists some value $p > 0$ such that, $\hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) = a_i$, $\forall\ \overline{p}_i^{\mathcal{G}} \ge p$.

The proof is given in [14]. It is useful to interpret these results in economic terms. The *LMP* at node *i* indicates the marginal costs to supply an additional unit of demand at node *i*. Clearly, the *GRA* at node *i* does not increase the marginal costs. In fact, if the *LMP* μ_i^* at node *i* of the reference system is greater than a_i and if $\overline{p}_i^{\mathcal{G}}$ is sufficiently large, the *GRA* reduces the *LMP* from μ_i^* to $\hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) = a_i$. However, the *GRA* cannot reduce the *LMP* at node *i* below a_i since, once $\overline{p}_i^{\mathcal{G}}$ reaches the value p such that $\hat{\mu}_i^{\mathcal{G}}(p) = a_i$, increasing $\overline{p}_i^{\mathcal{G}}$ to a value larger than p does not change the *LMP* at node *i*. Considering the relationship in (18), the implication is that a *GRA* with capacity $\overline{p}_i^{\mathcal{G}} > p$ cannot offer more increase to the social welfare than that with capacity p.

These results imply that $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ is a non-decreasing concave function of $\overline{p}_i^{\mathcal{G}}$. We derive an upper bound and a lower bound for this function. Assuming $\mu_i^* = \hat{\mu}_i^{\mathcal{G}}(0) > a_i$, then, we can show that the function $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ of $\overline{p}_i^{\mathcal{G}}$ satisfies

$$0 \le \overline{p}_i^{\mathcal{G}}\left[\hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) - a_i\right] \le \hat{\mathscr{V}}_h^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right) \le \overline{p}_i^{\mathcal{G}}\left[\mu_i^* - a_i\right]. \tag{19}$$

Equation (19) and the preceding result lead to the conclusion that, for a given value $\tilde{p}$ of $\overline{p}_i^{\mathcal{G}}$, $\hat{\mathscr{V}}_h^{\mathcal{G}}(\tilde{p}) > 0$ if and only if there exists $\zeta \in (0, \tilde{p}]$ such that

$$\hat{\mu}_i^{\mathcal{G}}(\zeta) > a_i\ . \tag{20}$$

The proof is given in [14]. This result provides the necessary and sufficient condition for the *GRA* to provide congestion relief.

Determination of $\hat{\mu}_i^{\mathcal{G}}(\zeta)$ requires the solution of the $GTSP_h^{\mathcal{G}}$ with $\overline{p}_i^{\mathcal{G}} = \zeta$. Considering the number of candidate nodes for the *GRA*, such a solution is impractical. Fortunately, since the bids/offers and the willingness-to-pay functions are assumed to be continuous, $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ is a continuous function of $\overline{p}_i^{\mathcal{G}}$. Therefore, $\mu_i^* > a_i$ implies $\hat{\mu}_i^{\mathcal{G}}(\zeta) > a_i$ for some small ζ. Consequently, we may relax the range of ζ in (20) to include 0. In other words, $\mu_i^* = \hat{\mu}_i^{\mathcal{G}}(0) > a_i$ is also a sufficient condition for $\hat{\mathscr{V}}_h^{\mathcal{G}}(\tilde{p}) > 0$ in this case. Note that, μ_i^* is the *LMP* at node *i* determined in the $GTSP_h$ of the reference system. Consequently, the usefulness of a *GRA* at a particular node may be judged by examining the reference system *LMP* only. Such a simplified conclusion is consistent with the intuitive understanding of the way the *LMP*s behave. A generation asset addition is valuable if the *LMP* at the bus in the reference system is greater than the marginal cost of the asset. The application of our framework to the *GRA* valuation therefore leads to the same result with the introduction of the simplifications we used.

We consider next the contributions of the $\overline{p}_{m,n}^{\mathcal{T}}$ *MW TCE* from node *m* to node *n* to the social welfare. We use an analogous development to that used for a *GRA* to study the properties of the $GTSP_h^{\mathcal{T}}$ solutions and to represent them as functions of the parameter $\overline{p}_{m,n}^{\mathcal{T}}$. In particular, we denote by $\hat{\mathscr{S}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \square \mathscr{S}_h^{\mathcal{T}*}$ and by $\hat{\mu}_n^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \square \mu_n^{\mathcal{T}*}$, $\forall\, n \in \mathcal{N}$. Now, $\hat{\mathscr{S}}_h^{\mathcal{T}}(\cdot)$ is a non-decreasing function of $\overline{p}_{m,n}^{\mathcal{T}}$ since an increase in the value of $\overline{p}_{m,n}^{\mathcal{T}}$ corresponds to a larger decision space for the $GTSP_h^{\mathcal{T}}$. Also, for $\overline{p}_{m,n}^{\mathcal{T}} = 0$, the $GTSP_h^{\mathcal{T}}$ reduces simply to the $GTSP_h$ so that $\hat{\mathscr{S}}_h^{\mathcal{T}}(0) = \mathscr{S}_h^{*}$ and $\hat{\mu}_k^{\mathcal{T}}(0) = \mu_k^{*}$, $\forall\ k \in \mathcal{N}$. From (12), $\mathscr{V}_h^{\mathcal{T}}$ of the *TCE* is also a function of $\overline{p}_{m,n}^{\mathcal{T}}$ and we denote this functional relationship by

$$\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) = \hat{\mathscr{S}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) - \mathscr{S}_h^{*} \quad . \tag{21}$$

In addition, we can show

$$\begin{aligned} \partial \hat{\mathscr{V}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \Big/ \partial\, \overline{p}_{m,n}^{\mathcal{T}} &= \partial \hat{\mathscr{S}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \Big/ \partial\, \overline{p}_{m,n}^{\mathcal{T}} \\ &= max\left\{\, 0\,,\ \hat{\mu}_n^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) - \hat{\mu}_m^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \right\} \end{aligned} \tag{22}$$

and

$$\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) = \int_0^{\overline{p}_{m,n}^{\mathcal{T}}} max\left\{0,\, \hat{\mu}_n^{\mathcal{T}}(\zeta) - \hat{\mu}_m^{\mathcal{T}}(\zeta)\right\} d\zeta \ . \tag{23}$$

Equation (23) indicates that the impact of the *TCE* on the social welfare may be evaluated by studying the *LMP* difference between node *n* and node *m* only. Also, if $\mu_n^{*} = \hat{\mu}_n^{\mathcal{T}}(0) \le \hat{\mu}_m^{\mathcal{T}}(0) = \mu_m^{*}$, then $\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) = 0$ since either none of the $\overline{p}_{m,n}^{\mathcal{T}}$ transfer capability is utilized or its utilization does not produce an increase in the social welfare. Therefore, we assume henceforth that $\mu_n^{*} > \mu_m^{*}$ holds. Using the duality theorem and the Kuhn-Tucker conditions [18] of the $GTSP_h^{\mathcal{T}}$, we can show that

- $\hat{\mu}_n^{\mathcal{T}}(\cdot) - \hat{\mu}_m^{\mathcal{T}}(\cdot)$ is a non-increasing function of $\overline{p}_{m,n}^{\mathcal{T}}$;
- $\hat{\mu}_n^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) - \hat{\mu}_m^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \ge 0$, $\forall\ \overline{p}_{m,n}^{\mathcal{T}} \ge 0$; and,
- there exists some value $p > 0$ such that $\hat{\mu}_n^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) - \hat{\mu}_m^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) = 0$, $\forall\ \overline{p}_{m,n}^{\mathcal{T}} \ge p$.

We interpret these results in economic terms. As $\overline{p}_{m,n}^{\mathcal{T}}$ increases, more choice on the use of supplies to meet the demand at node *n* exists. Such an increase cannot increase the *LMP* at node *n* which is the marginal costs to supply an additional unit of demand at node *n*. On the other hand, the *TCE* provides a larger set of demands that may be supplied by the seller at node *m* and, consequently, may lead to a higher *LMP* at node *m*. The *LMP* difference [$\hat{\mu}_n^{\mathcal{T}}(\cdot) - \hat{\mu}_m^{\mathcal{T}}(\cdot)$], therefore, is a non-increasing function of $\overline{p}_{m,n}^{\mathcal{T}}$. In fact, if the *LMP* μ_n^{*} at node *n* of the reference system is greater than μ_m^{*} at node *m*, we can always make the *LMP*s at these two nodes equal by a sufficiently large *TCE* from node *m* to node *n*. However, such enhancement cannot result in $\hat{\mu}_n^{\mathcal{T}}(\cdot) < \hat{\mu}_m^{\mathcal{T}}(\cdot)$ since, once $\hat{\mu}_n^{\mathcal{T}}(\cdot) = \hat{\mu}_m^{\mathcal{T}}(\cdot)$, any additional *TCE* from node *m* to node *n* has no impact on the *LMP* difference between these two nodes. The relationship in (23) implies that a *TCE* with $\overline{p}_{m,n}^{\mathcal{T}} > p$ cannot offer more social welfare increase than that in the amount p.

These results, together with (23), imply that $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ is a non-decreasing concave function of $\overline{p}_{m,n}^{\mathcal{T}}$. We may derive an upper bound and a lower bound for $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$. We can show that, if $\mu_n^{*} > \mu_m^{*}$, then,

$$\overline{p}_{m,n}^{\mathcal{T}}\left[\hat{\mu}_n^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right)-\hat{\mu}_m^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right)\right] \le \mathscr{V}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right) \le \overline{p}_{m,n}^{\mathcal{T}}\left[\mu_n^*-\mu_m^*\right] . \tag{24}$$

Equation (24) and the preceding analysis yield that, for a given value $\tilde{p}$ of $\overline{p}_{m,n}^{\mathcal{T}}$, $\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\tilde{p}\right) > 0$ if and only if there exists $\zeta \in \left(0, \tilde{p}\right]$ such that

$$\hat{\mu}_n^{\mathcal{T}}\left(\zeta\right) - \hat{\mu}_m^{\mathcal{T}}\left(\zeta\right) > 0 . \tag{25}$$

This conclusion gives the necessary and sufficient condition for the *TCE* to provide congestion relief.

Determination of $\hat{\mu}_n^{\mathcal{T}}\left(\zeta\right) - \hat{\mu}_m^{\mathcal{T}}\left(\zeta\right)$ requires the solution of the $GTSP_h^{\mathcal{T}}$ with $\overline{p}_{m,n}^{\mathcal{T}} = \zeta$. Considering the large possible number of candidate node pairs for a *TCE*, such a solution is impractical. Using the similar argument in the discussion for the *GRA*, we may relax the range of ζ in (25) to include 0 since we have assumed the bids/offers and the willingness-to-pay functions to be continuous. In other words, $\mu_n^* - \mu_m^* > 0$ is also a sufficient condition for $\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\tilde{p}\right) > 0$. Note that, μ_m^* and μ_n^* are determined in the $GTSP_h$ of the reference system. Consequently, the assessment of the usefulness of a *TCE* at particular node pairs is made simply by examining the reference system *LMP* differences. This conclusion obtained with the introduction of the simplifications also agrees with the intuitive notions we have of the *LMP* differences. A nonzero difference between the *LMP*s at two buses implies that the limit of the transfer capability between the two buses is reached and constrains the ability of the system to be dispatched in the most economic manner so as not to violate the limit. Clearly, an increase in the transfer capability between the two buses helps reduce the congestion and increases the social welfare in the particular hour. This conclusion obtained from the application of our framework to the *TCE* valuation under the simplifications introduced shows the insights attainable from the proposed approach.

The analytical results presented in this section provide sufficient and necessary conditions of the $\mathscr{S}_h$ – *substitutability*. A key application of the $\mathscr{S}_h$ –*substitutability* notion is in the comparison of alternative investment plans, be they of a *GRA* nature or a *TCE* type. The analysis presented in this section provides the basis for developing a tool for comparative purposes in terms of the contribution of a proposed investment to the social welfare increase. The consideration from the extension of the hourly information to the much longer-term durations associated with the horizons of planning studies and incorporation of additional factors, such as construction costs and environmental impacts, is beyond the scope of this paper. Results of our work on that front will be discussed in a future publication.

4 SIMULATION RESULTS

To investigate the $\mathscr{S}_h$ –*substitutability* results, we have carried out an extensive set of simulations on a number of test systems to study and compare various cases of *GRA* and *TCE*. The test systems include the IEEE 7-bus, 14-bus, 57-bus and 118-bus systems. For each case of these test systems, we examine the impacts of a modification, be it a *GRA* or a *TCE*, on the outcomes of the hour *h* market. The study of the relationship of the reference system *LMP*s to the increase in the social welfare attained by each modification is the focus of the investigation. The data used in these simulations are available in [14]. We next summarize a set of representative results from the various simulations.

We begin by considering the impacts of a *GRA* in the various test systems. For each system, we consider a *GRA* at a specified node and examine sensitivity cases by varying the *MW* output of this additional generation capacity. We repeat such investigations at different nodes of each test system. We present representative plots of the behavior of the *LMP*

functions $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ and the value function $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ for the IEEE 7-bus system in Figures 1 and 2. For these plots, the offer price of the *GRA* is held constant at $a_i = a = 13$ *$/MWh* for each node *i* of the network. The plots in Figure 1 depict the behavior of the *LMP* functions $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ as a function of the *GRA* $\overline{p}_i^{\mathcal{G}}$ for each node *i* of the network. The horizontal axis is the capacity of the new generation asset $\overline{p}_i^{\mathcal{G}}$ and the vertical axis is the *LMP* at bus *i* with the generation addition. Clearly, the intersection of each plot and at the 0 capacity value corresponds to $\hat{\mu}_i^{\mathcal{G}}(0) = \mu_i^*$, the *LMP* at bus *i* of the reference system without the addition. These plots exhibit two types of variations depending on the relationship of the *LMP* μ_i^* of the reference system to the *GRA* offer price a. Whenever $\mu_i^* > 13$, $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ has a monotonically non-increasing behavior. For $\overline{p}_i^{\mathcal{G}}$ above a certain value, $\hat{\mu}_i^{\mathcal{G}}\left(\overline{p}_i^{\mathcal{G}}\right)$ reduces to a and remains unchanged. On the other hand, if $\mu_i^* \leq 13$ as is the case at the node *i* = 0, the *LMP* function $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ remains constant as $\overline{p}_i^{\mathcal{G}}$ varies. This observed behavior verifies the analytical results presented in Section 4.

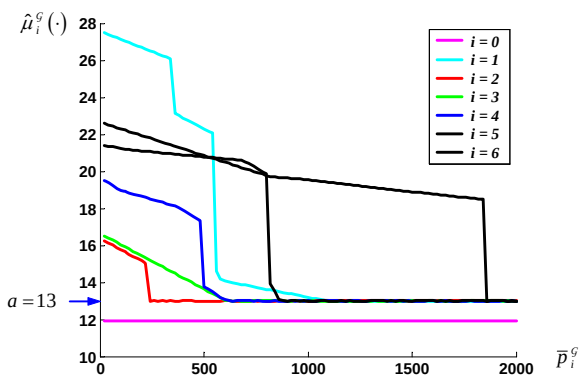

Figure 1: The behavior for the IEEE-7 bus system of the *LMP* $\hat{\mu}_i^{\mathcal{G}}(\cdot)$ at bus *i* with the generation addition as a function of the output from the new generation capacity $\overline{p}_i^{\mathcal{G}}$

Figure 2 shows the typical behavior of the value function $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ as the new capacity $\overline{p}_i^{\mathcal{G}}$ at bus *i* varies in the IEEE 7-bus network with *i* = 1. $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ is a non-decreasing function of $\overline{p}_1^{\mathcal{G}}$ with a slope that decreases as $\overline{p}_1^{\mathcal{G}}$ increases. The value function reaches its maximum when $\overline{p}_1^{\mathcal{G}} = 1000$ *MW* and remains unchanged thereafter. Such a monotonic behavior in the $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ functions is exhibited in all the cases studied.

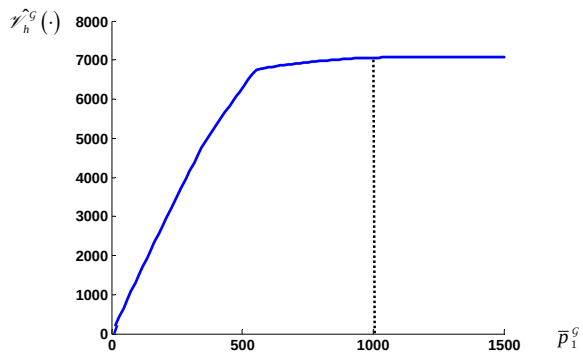

Figure 2: The behavior of the value function $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ for the IEEE-7 bus system as a function of the output of the generation capacity addition $\overline{p}_1^{\mathcal{G}}$

A logical question arises as to the selection of the node at which a *GRA* is installed. We study the relationship between the value of the *GRA* and the reference system *LMP* of the node at which the new resource is located. We consider a *GRA* with capacity $\overline{p}_i^{\mathcal{G}} = 500\ MW$ and the offer price of $a_i = a = 20$ *$/MWh*. We compare its value for the specified hour *h* when installed at various nodes *i*. Figure 3 shows a representative scatter plot obtained for the IEEE 57-bus system to determine a relationship of value $\mathscr{V}_h^{\mathcal{G}}$ with respect to the difference between the reference system *LMP* μ_i^* at the node *i* and the constant offer price $a = 20$. The horizontal axis in the plot is the difference $\mu_i^* - a$ and the vertical axis is the value $\mathscr{V}_h^{\mathcal{G}}$ of the 500 *MW GRA*. The scatter plot clearly shows that the higher the difference, the larger the value $\mathscr{V}_h^{\mathcal{G}}$ and, therefore, the larger the contribution of the *GRA* to the social welfare. Such a pattern is observed in all the test cases we have studied on the various systems. We also observe in Figure 3 an upper bound of $\mathscr{V}_h^{\mathcal{G}}$ that equals $\overline{p}_i^{\mathcal{G}}\left(\mu_i^* - a\right)$ as indicated by the bold line. Such a study provides a good illustration of the strength of the analytical results presented in Section 4.

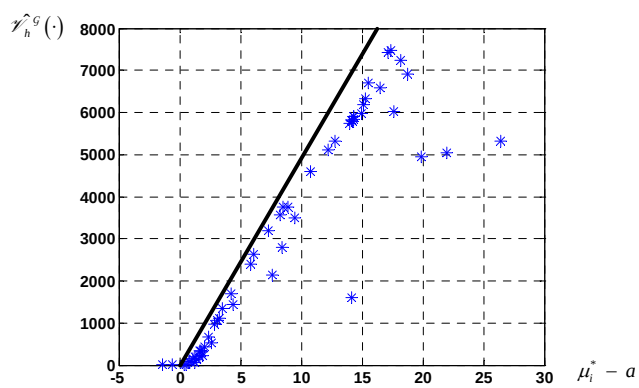

Figure 3: The relationship between the value $\hat{\mathscr{V}}_h^{\mathcal{G}}(\cdot)$ of the *GRA* for the IEEE-57 bus system and the difference $\mu_i^* - a$ between the reference case *LMP* at the bus and the offer price of the generation resource

We next discuss the simulation results for the impacts of a *TCE* on the *LMP*s and on the social welfare. For each test system, we consider the value of a *TCE* for a specified *from* and *to* node pair and examine sensitivity cases by varying the *MW* value of the enhancement. We repeat such simulations for different *from* and *to* node pairs in the test systems considered. The focus is on the specified hour *h*. We present in Figures 4 and 5 representative plots for the IEEE 7-bus system of the behavior of the *LMP* differences a function of the *MW*s of the *TCE* enhancement, with the *TCE* added from the node $m = 0$ to the node $n = 6$. The plot in Figure 4 shows the behavior of the *LMP* difference $[\hat{\mu}_n^{\mathcal{T}}(\cdot) - \hat{\mu}_m^{\mathcal{T}}(\cdot)]$ with the *TCE* as a function of the *TCE MW* $\overline{p}_{m,n}^{\mathcal{T}}$. The horizontal axis is the *TCE MW* and the vertical axis is the *LMP* difference with the *TCE*. The plot exhibits the expected monotonically non-increasing behavior of the function $[\hat{\mu}_n^{\mathcal{T}}(\cdot) - \hat{\mu}_m^{\mathcal{T}}(\cdot)]$. Above the value $\overline{p}_{m,n}^{\mathcal{T}} = 370$ *MW*, this difference vanishes and the congestion between *m* and *n* is completely removed. Such behavior is typical in all the cases we studied.

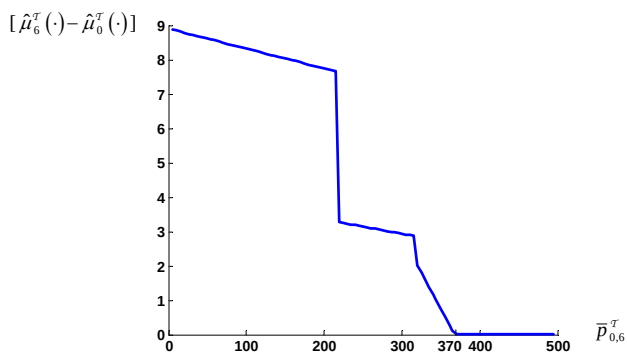

Figure 4: The behavior for the IEEE-7 bus system of the *LMP* difference $[\hat{\mu}_6^{\mathcal{T}}(\cdot) - \hat{\mu}_0^{\mathcal{T}}(\cdot)]$ as a function of the *TCE MW* $\overline{p}_{0,6}^{\mathcal{T}}$

We present in Figure 5 a representative plot of the value $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ of the *TCE* as a function of the *MW* enhancement $\overline{p}_{m,n}^{\mathcal{T}}$. $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ is a non-decreasing function of $\overline{p}_{m,n}^{\mathcal{T}}$ with a slope that decreases as $\overline{p}_{m,n}^{\mathcal{T}}$ increases. The value function reaches its maximum when $\overline{p}_{m,n}^{\mathcal{T}} = 370$ *MW* and remains unchanged thereafter. Such monotonic behavior is exhibited in all the cases investigated and is consistent to the analytical results presented in Section 4.

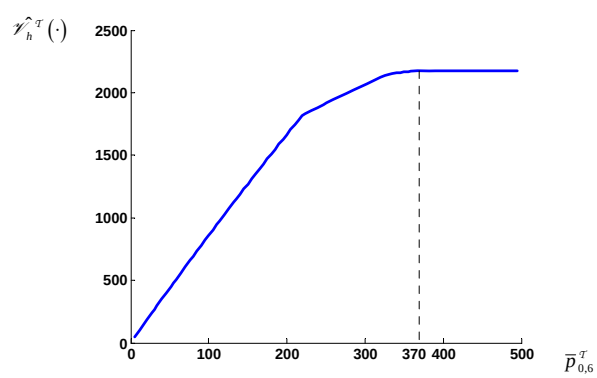

Figure 5: The behavior for IEEE-7 bus system of the *TCE* value $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ from bus 0 to bus 6 as a function of the *TCE MW* $\overline{p}_{0,6}^{\mathcal{T}}$

In addition, we investigate the relationship of the value function $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ of the *TCE* to the reference system *LMP* difference $\mu_n^* - \mu_m^*$ between the *to* and *from* node pair of the *TCE*. We set the *TCE MW* at $\overline{p}_{m,n}^{\mathcal{T}} = 230$ *MW* and com-

pare the changes in the social welfare when such an enhancement is undertaken between different node pairs *m* and *n*. Figure 6 shows a representative scatter plot for the IEEE 57-bus system of the relationship between the value $\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right)$ of the *TCE* and the *LMP* difference $\mu_n^* - \mu_m^*$. We note a clear pattern in this figure that a *TCE* at a node pair with higher *LMP* difference results in a larger increase in the social welfare and, therefore, a higher value of $\hat{\mathscr{V}}_h^{\mathcal{T}}\left(\overline{p}_{m,n}^{\mathcal{T}}\right)$. Such a pattern is observed in every case and test system we studied. We also observe in Figure 6 an upper limit for $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ that equals $\overline{p}_{m,n}^{\mathcal{T}}\left(\mu_n^* - \mu_m^*\right)$ as indicated by the broken line. This observation is consistent with the analytical results presented in Section 4.

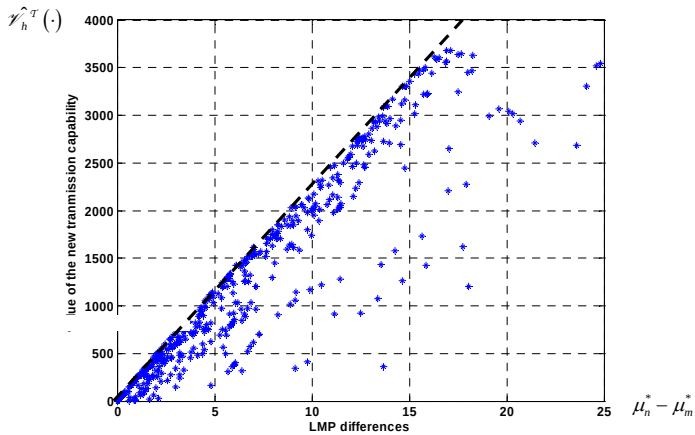

Figure 6: The relationship for the IEEE-57 bus system between the value $\hat{\mathscr{V}}_h^{\mathcal{T}}(\cdot)$ of the *TCE* and the difference $\mu_n^* - \mu_m^*$ between the reference case *LMP* at the *to* and *from* buses of the *TCE*

The comparison of Figures 2, 3 and 4 and Figures 5, 6 and 7 shows the similar impacts of a *GRA* and a *TCE* on the outcomes of the hour *h* market. Such similarities reinforce the usefulness of the notion of the substitutability for the two congestion relief schemes.

5 SUMMARY

In this paper, we defined the notion of substitutability and applied it to the study of the comparative effectiveness of congestion relief through the implementation of a *GRA* and a *TCE*. We focus our analysis on a single hour *h* and construct the formulation to evaluate the impacts of the *GRA* and a *TCE* on a centralized DA *MWh* market with both the pool players and bilateral transactions. We use the social welfare as the metric for assessing the overall impacts of such modifications and compare them on a consistent basis. We introduce the notion of substitutability and derive the conditions under which substitutability can be applied in the specified hour. Our results indicate that the *GRA* and the *TCE*, when appropriately implemented, may both provide effective transmission congestion relief. The notion of substitutability is highl useful in evaluating the effectiveness on a consistent basis.

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