Corrections

Allocation of the Reactive Power Support Requirements in Multitransaction Networks (Republished)

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***Abstract*—This paper presents a new physical-flow-based mechanism for allocating the reactive power support requirements provided by the generators in multitransaction networks. The allocatable reactive support requirements are defined with respect to the support required for the network with no transactions in place. The requirements in the presence of the proposed transactions are formulated as the sum of two specific components—the voltage magnitude variation component and the voltage angle variation component. The formulation utilizes the multitransaction framework used for the allocation of losses. The formulation leads to a natural allocation as a function of the amount of each transaction. The physical interpretation of each allocation as a sensitivity of the reactive output of a generator is discussed. The extensive testing indicates that the allocation scheme approximates with good fidelity the actual net VAr outflow from each generator and is able to deal effectively with the nonlinearities due to the generator reactive power limits. The numerical results also indicate that the proposed scheme behaves in a physically reasonable and intuitive way.**

***Index Terms*—Allocation, ancillary services, electricity markets, multitransaction networks, reactive support, transmission services, unbundling, voltage control, voltage profiles.**

I. Introduction

THE RAPIDLY spreading open access regime is leading to the disintegration of the vertically integrated utility structure. The dramatic increase in the number of new players and the growing pressures of the competitive marketplace are resulting in the proliferation in the number and volume of power transactions. The introduction of open access transmission is often accompanied by the establishment of an independent grid operator (IGO) and the unbundling of electricity services. The IGO, in its various implementations around the world such as independent system operators (ISOs) [1] or regional transmission organizations [2], has, typically, exclusive authority for maintaining the security/reliability of the system and is responsible for provision/acquisition of the required ancillary services. Among the ancillary services, the reactive power support is essential for the system operator to maintain an acceptable system voltage profile by setting the voltage magnitudes at the controllable buses so as to keep the voltage magnitudes at the other buses within specified ranges. The reactive support provided by generators is one of the six ancillary services specified in the FERC Order no. 888 [1]. The focus of this paper is the allocation of the voltage support requirements among the various transactions on the system.

The characterization and evaluation of the reactive power support requirement allocations present a very challenging task. This is particularly true in the new unbundled regime with the many transactions in place at any point in time. Typically, in the commercial marketplace the transactions specify the real power traded and no mention is made of reactive power. Yet, reactive power support is a fundamental need for a network to provide transmission service. In fact, the reactive power support requirements of a network in the absence of any transactions is considered to be an intrinsic part of the transmission service, since in its absence the required voltage profile could not be maintained. As such, we consider this support as an integral part of the transmission service and consequently a quantity that cannot be allocated to the transactions. We use the intrinsic voltage support requirements as the reference level from which the additional requirements due to the presence of transactions are evaluated. Some of the complications in this evaluation are due to the local nature of reactive power. As is well known, reactive power cannot be delivered from a source to an electrically remote sink, due to the reactive nature of the transmission grid. This local nature of the reactive power also implies that a generator may provide the reactive power support for a number of transactions in which the particular generator is not involved. In addition, whether a generator generates or absorbs reactive power depends on the state of the system. Consequently, the allocated contributions of the individual generator's reactive output to a particular transaction may be negative or nonnegative. A negative sign indicates that the reactive support required by the transaction is absorption by the

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particular generator; while, a nonnegative sign indicates that the VAr support from the generator is injection or additional reactive power. Physically, the algebraic sum of the allocations for all the transactions to a specific generator is meaningful since it is the net support provided by that generator. On the other hand, the algebraic sum of the allocations provided by the various generators is a nonhomogeneous term and, as such, the total reactive support required by each transaction is a physically less meaningful term.

The reactive power generation/absorption at a generator is easily computed given the system state. The reactive output at each power generator is measurable, but there is no physical way of verifying its allocation among the transactions present on the system. For commercial reasons, there is a need for allocating the reactive power support to the transactions on an equitable and transparent basis and to make such information available on an *a priori* basis. Thus the allocation of VAr support requirements is not a simple matter and involves a good amount of arbitrariness. Since there is no physically meaningful measurement scheme nor a theoretically-based evaluation methodology to determine the VAr requirements caused by each transaction, a new scheme must be developed to make available the required information.

A number of different allocation schemes has been proposed based on various assumptions and approximations. In [3], real and reactive power flows are considered separately and the reactive power generated or absorbed by the line is taken into account by using fictitious nodes in a system model specifically developed for the analysis of the reactive power flows. A decomposition of the power injections based on real and imaginary currents has been proposed in [4], where the results depend on the location of the system angle reference. The loss allocation scheme proposed in [5] uses an operating point defined in terms of bilateral transactions. The allocation mechanism in [6] uses the power flow without transactions and requires two additional power flows for each transaction. The Aumann–Shapley methodology [7] was also applied to determine the allocation of reactive support requirements [8]. This application did not fully exploit the physical characteristics of the reactive support nor the nature of the allocation problem.

The objective of this paper is to allocate reactive power support requirements in a multitransaction system taking into account the generation VAr limits and the interaction with the network. Thus, by design, the proposed allocation does not incorporate economic efficiency goals. We make use of the mathematical multitransaction framework developed in [9] and extend it to incorporate a detailed description of the reactive power flows. We formulate the VAr support requirements in terms of the change from the no transaction reference case corresponding to the intrinsic reactive support required by the transmission system for the specified set of control variables. Using the voltage magnitude specifications used in the no transaction case, we compute the power flow solution with all the specified transactions to determine the allocatable reactive support requirements. We decompose the requirements into two specific components—the voltage magnitude variation component and the voltage angle variation component—and allocate these components to each transaction. Note that the power flow solution with all the transactions in place is the IGO-imposed transaction schedule after all the necessary actions for congestion management have been undertaken. The proposed scheme gives physically meaningful results. Moreover, a key attribute is its ability to handle the nonlinearities arising from the generators' reactive power limits. The physically intuitive behavior of the allocation scheme under a variety of conditions for different test systems is an attractive feature of the proposed mechanism. We give a sample of representative results to illustrate the scheme's application.

The paper has four additional sections. We review briefly the multitransaction framework in Section II and discuss the characterization of reactive power support requirements in Section III. We then construct the proposed VAr allocation scheme in Section IV. We present a representative sample of numerical results indicating the strong performance of the proposed scheme in Section V.

II. Transaction Framework

We use the multitransactions framework developed in [9] to consider a system $N+1$ buses and M transactions. For $m = 1, 2, \ldots M$, the transaction $\boldsymbol{T}^{(m)}$ involving the set of selling entities $\boldsymbol{S}^{(m)}$, the set of buying entities $\boldsymbol{B}^{(m)}$ and the MW amount $t^{(m)}$ is defined by the triplet

$$\boldsymbol{T}^{(m)} = \left\{ t^{(m)},\ \boldsymbol{S}^{(m)},\ \boldsymbol{B}^{(m)} \right\} \tag{1}$$

where

$$\boldsymbol{S}^{(m)} = \left\{ \left(s_i^{(m)},\ \sigma_i^{(m)} \right),\ i = 1, 2, \ldots, N_s^{(m)} \right\} \tag{2}$$

$$\boldsymbol{B}^{(m)} = \left\{ \left(b_j^{(m)},\ \beta_j^{(m)} \right),\ j = 1, 2, \ldots, N_b^{(m)} \right\}. \tag{3}$$

For each transaction m, the selling bus $s_i^{(m)}$ provides the fraction $\sigma_i^{(m)}$ of the total MW amount $t^{(m)}$ with $i = 1, 2, \ldots, N_s^{(m)}$. Similarly, the buying bus $b_j^{(m)}$ receives the fraction $\beta_j^{(m)}$ of the total MW amount $t^{(m)}$ with $j = 1, 2, \ldots, N_b^{(m)}$. The fractions $\sigma_i^{(m)}$ and $\beta_j^{(m)}$ must satisfy the convexity conditions

$$\sum_{i=1}^{N_s^{(m)}} \sigma_i^{(m)} = 1 \text{ and } \sigma_i^{(m)} \in [0,1] \text{ for } i = 1, 2, \ldots, N_s^{(m)} \tag{4}$$

$$\sum_{j=1}^{N_b^{(m)}} \beta_j^{(m)} = 1 \text{ and } \beta_j^{(m)} \in [0,1] \text{ for } j = 1, 2, \ldots, N_b^{(m)}. \tag{5}$$

We formulate the expressions for the power flows in the network explicitly in terms of the M transactions. We assume all the real power losses to be compensated by the slack bus at node 0. In this framework, the amount of the power injection at any bus $h = 0, 1, \ldots, N$ due to the transaction m is expressed as

$$P_h^{(m)} = \delta_h^{(m)} t^{(m)} \tag{6}$$

where the components of the vector $\underline{\delta}^{(m)}$ are

$$\delta_h^{(m)} = \begin{cases} \sigma_i^{(m)}, & \text{if } h = s_i^{(m)}, i = 1, 2, \ldots, N_s^{(m)} \\ -\beta_j^{(m)}, & \text{if } h = b_j^{(m)}, i = 1, 2, \ldots, N_b^{(m)} \\ \sigma_i^{(m)} - \beta_j^{(m)}, & \text{if } h = s_i^{(m)} = b_j^{(m)}, i = 1, 2, \ldots, N_s^{(m)} \\ & \qquad j = 1, 2, \ldots, N_b^{(m)} \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Additional discussion of this formulation may be found in [9]. We complete the formulation by defining the following sets:

$\boldsymbol{Q} \triangleq$ the set of the generator buses, with cardinality $|\boldsymbol{Q}|$

$\boldsymbol{Q}^c \triangleq$ the set of the nongenerator buses, with cardinality $|\boldsymbol{Q}^c|$.

Note that $|\boldsymbol{Q}| + |\boldsymbol{Q}^c| = N+1$. We associate with each bus $i = 0, 1, 2, \ldots, N$ the set $\boldsymbol{H}_i$ of buses connected to bus i, the voltage magnitude V_i, the voltage angle θ_i, and the real power load P_i^d and the reactive power load Q_i^d. Without any loss of generality, we set the voltage angle $\theta_0 = 0$ at the designated slack bus 0. The voltage magnitude at the generator buses $k \in \boldsymbol{Q}$ is specified as $V_k = V_k^s$.

For the transmission line, we consider the π-model. For $k \in \boldsymbol{Q}$ and $i \in \boldsymbol{H}_k$, we characterize the line connecting bus k to bus i with the series resistance r_{ki}, the series reactance x_{ki}, and the modulus of the series impedance z_{ki}. We denote by b_{ki} the total shunt susceptance. The k, ith element of the bus admittance matrix is $G_{ki} + jB_{ki}$.

The real power flow equation at bus h is

$$\begin{aligned}\sum_{m=1}^{M} \delta_h^{(m)} t^{(m)} =& G_{hh} V_h^2 + V_h \sum_{i \in \boldsymbol{H}_h} V_i \\ &\cdot \left[G_{hi}\cos(\theta_h - \theta_i) + B_{hi}\sin(\theta_h - \theta_i)\right] \\ h =& 1, 2, \ldots, N. \end{aligned} \tag{8}$$

We complete the formulation of the framework by stating the reactive power balance equations from which we derive the expression for the reactive power support requirements of the transmission system. The reactive power equation at bus $j \in \boldsymbol{Q}^c$ is

$$\begin{aligned}-Q_j^d = -B_{jj}V_j^2 + V_j \sum_{i\in \boldsymbol{H}_j} V_i \Big[&G_{ji}\sin(\theta_j - \theta_i) \\ &- B_{ji}\cos(\theta_j - \theta_i)\Big],\ j \in \boldsymbol{Q}^c. \end{aligned} \tag{9}$$

This support is considered to be provided entirely by the generator buses. At each generator bus $k \in \boldsymbol{Q}$, the net reactive power outflow is the sum of the reactive powers Q_{ki} injected into the line connecting bus k to bus $i \in \boldsymbol{H}_k$

$$Q_k^g - Q_k^d = \sum_{i \in \boldsymbol{H}_k} Q_{ki} \tag{10}$$

where

$$\begin{aligned}Q_{ki} = -\frac{b_{ki}}{2}V_k^2 - \frac{r_{ki}}{z_{ki}^2} V_k V_i \sin(\theta_k - \theta_i) \\ + \frac{x_{ki}}{z_{ki}^2}\left[V_k^2 - V_k V_i \cos(\theta_k - \theta_i)\right]. \end{aligned} \tag{11}$$

We use the framework of this section for the formulation of the reactive support requirements of a multitransaction network.

III. Reactive Power Support Requirements

In this section, we evaluate the VAr support requirements under certain simplifying assumptions. The focus is on the system-wide requirements and so we assume that any local reactive load is locally met at the bus. In other words, we assume that at each load bus the power factor is 1.0. We use the assumption that $(\theta_k - \theta_i)$, for $k \in \boldsymbol{Q}$ and $i \in \boldsymbol{H}_k$, is small. This assumption has the following implications:

$$\begin{aligned}\sin(\theta_k - \theta_i) \approx& \theta_k - \theta_i \\ \cos(\theta_k - \theta_i) \approx& 1 - \frac{(\theta_k - \theta_i)^2}{2}. \end{aligned} \tag{12}$$

These conditions are weaker than the dc power flow assumptions (see, for example, [9]). We first compute the reactive power injected in the line connecting bus $k \in \boldsymbol{Q}$ to bus $i \in \boldsymbol{H}_k$ for the case of no transactions on the system and we use the superscript 0 to denote this case. Since for $k \in \boldsymbol{Q}$, the voltage magnitude $V_k^0 = V_k^s$ and (11) becomes[1]

$$\begin{aligned}Q_{ki}^0 = -\frac{b_{ki}}{2}V_k^2 - \frac{r_{ki}}{z_{ki}^2} V_k V_i^0 \left(\theta_k^0 - \theta_i^0\right) \\ \frac{+x_{ki}}{z_{ki}^2}\left[V_k^2 - V_k V_i^0 \left(1 - \frac{\left(\theta_k^0 - \theta_i^0\right)^2}{2}\right)\right]. \end{aligned} \tag{13}$$

Then

$$Q_k^{g,0} = \sum_{i \in \boldsymbol{H}_k} Q_{ki}^0 \tag{14}$$

is the no-transaction generation at bus $k \in \boldsymbol{Q}$. Equation (14) expresses the *intrinsic* reactive power support provided by the generators to maintain the specified voltage profile in the absence of any transactions. This quantity is an intrinsic part of the provision of transmission services and as such is not allocated to any of the transactions that may be undertaken.

We next consider the network with the proposed M transactions. The presence of transactions changes the voltages at the buses that are not voltage-controlled and we write these voltage magnitudes as

$$V_i = V_i^0 + \Delta V_i \text{ for } i \in \boldsymbol{H}_K. \tag{15}$$

Note that $V_k = V_k^s$ for $k \in \boldsymbol{Q}$. For any generator reaching its reactive power limit, the power flow imposes the reactive power generation at the active limit and changes the voltage from V_k^s to a new value V_k^q. We set $V_k = V_k^q$ in such a case. We rewrite (11) by using (12), (15), and (16)

$$\begin{aligned}Q_{ki} =& -\frac{b_{ki}}{2}V_k^2 - \frac{r_{ki}}{z_{ki}^2} V_k V_i^0 (\theta_k - \theta_i) \\ &+ \frac{x_{ki}}{z_{ki}^2} V_k \left[V_k - V_i^0\left(1 - \frac{(\theta_k - \theta_i)^2}{2}\right)\right] \\ &- \frac{V_k}{z_{ki}^2}\left[r_{ki}(\theta_k - \theta_i) + x_{ki}\left(1 - \frac{(\theta_k - \theta_i)^2}{2}\right)\right]\Delta V_i \end{aligned} \tag{16}$$

We further substitute for the voltage angles

$$\theta_i = \theta_i^0 + \Delta\theta_i \text{ for } i \in \boldsymbol{H}_k \tag{17}$$

$$\theta_k = \theta_k^0 + \Delta\theta_k \text{ for} k \in \boldsymbol{Q} \text{ and } k \neq 0 \tag{18}$$

[1]If $r_{ij} \ll x_{ij}$ for all the lines of the network, the no-transaction network is essentially purely reactive. In such case, the angles $\theta_i^0 - \theta_j^0 \approx 0$ for $i, j = 0, 1, 2, \ldots, N$ and the reactive power flowing into the line connecting bus i and bus j becomes $Q_{ki}^0 = -b_{ki}/2V_k^2 + x_{ki}/z_{ki}^2\,[V_k^2 - V_i^0 V_k]$.

into the second and third term of (16). Furthermore, let

$$Q_{ki}^{\Delta\theta} = \frac{V_k V_i^0}{z_{ki}^2}\left[-r_{ki}\left(\Delta\theta_k - \Delta\theta_i\right) + x_{ki}\frac{\left(\Delta\theta_k - \Delta\theta_i\right)^2}{2} + x_{ki}\left(\theta_k^0 - \theta_i^0\right)\left(\Delta\theta_k - \Delta\theta_i\right)\right] \quad (19)$$

and

$$Q_{ki}^{\Delta V} = -\frac{V_k}{z_{ki}^2}\left[r_{ki}\left(\theta_k - \theta_i\right) + x_{ki}\left(1 - \frac{\left(\theta_k - \theta_i\right)^2}{2}\right)\right]\Delta V_i \quad (20)$$

and recall the definition of Q_{ki}^0 from (13). We may thus rewrite (16) as

$$Q_{ki} = Q_{ki}^0 + Q_{ki}^{\Delta V} + Q_{ki}^{\Delta\theta}. \quad (21)$$

We call $Q_{ki}^{\Delta V}$ the *voltage magnitude variation* component and $Q_{ki}^{\Delta\theta}$ the *voltage angle variation* component of the reactive power injected into the line connecting a generator bus $k \in \boldsymbol{Q}$ to a bus $i \in \boldsymbol{H}_k$. These components are in addition to the intrinsic VAr support requirement Q_{ki}^0 of the network. The presence of the two components $Q_{ki}^{\Delta V}$ and $Q_{ki}^{\Delta\theta}$ is attributed entirely to the transactions. These additional reactive support requirements expressed by these two components result in added VAr support from the generators. In (16)–(20), V_k is either V_k^s or, in case the reactive limit is reached, V_k^q. We next develop the physical flow-based allocation of these additional requirements.

IV. Proposed Allocation Approach

In analogy to the split in (21), the net reactive power outflow from a generation bus $k \in \boldsymbol{Q}$ may be written as

$$Q_k^{g,net} = Q_k^g - Q_k^d = Q_k^{g,0} + Q_k^{g,\Delta V} + Q_k^{g,\Delta\theta} \quad (22)$$

where $Q_k^{g,0}$ is given by (14) and

$$Q_k^{g,\Delta V} = \sum_{i\in\boldsymbol{H}_k} Q_{ki}^{\Delta V}$$
$$Q_k^{g,\Delta\theta} = \sum_{i\in\boldsymbol{H}_k} Q_{ki}^{\Delta\theta}. \quad (23)$$

The allocation of the *net* VAr outflow from bus $k \in \boldsymbol{Q}$ is accomplished by allocating the components $Q_{ki}^{\Delta V}$ and $Q_{ki}^{\Delta\theta}$ among the M transactions. The reactive load Q_k^d indirectly contributes toward operating the generator closer to its reactive limits.

In line with our assumption that the component $Q_{ki}^{\Delta V}$ is entirely due to the transactions, we attribute the voltage variation ΔV_i at each bus $i \in \boldsymbol{Q}^c$ to the presence of the transactions in the system. Using (8), we may write the *net* real power load P_h^d at any bus, $h = 0, 1, 2, \ldots, N$ in terms of the M transactions as

$$P_h^d = -\sum_{m=1}^{M} \min\left(0; \delta_h^{(m)}\right) t^{(m)}. \quad (24)$$

Let us multiply and divide ΔV_i by the total weighted net load $\sum_{h=0}^{N} P_h^d w_{ih}$

$$\Delta V_i = \frac{\Delta V_i}{\sum_{h=0}^{N} P_h^d w_{ih}} \sum_{h=0}^{N} P_h^d w_{ih} = \sum_{m=1}^{M} \frac{\Delta V_i}{\sum_{h=0}^{N} P_h^d w_{ih}} \sum_{h=0}^{N}\left[-\min\left(0; \delta_h^{(m)}\right) w_{ih} t^{(m)}\right] \quad (25)$$

where the weighting factor w_{ih} is used to reflect the local nature of the reactive support. Due to the inability of reactive power to *travel*, the w_{ih} of the buses h in the vicinity of the bus i have an impact. As such, we use the device of a simple decay function for attenuating the impact of the loads located at far buses on the voltage magnitude variation component. A natural candidate is an exponential function of the *concentric* distance of the load bus from node i. We set $w_{ih} = 1$ if $h = i$ or $h \in \boldsymbol{H}_i$, $w_{ih} = e^{-1}$ if the concentric distance is two buses, $w_{ih} = e^{-2}$ if the concentric distance is three buses and so on. It follows then from (23) that

$$Q_k^{g,\Delta V} = \sum_{i\in\boldsymbol{H}_k} Q_{ki}^{\Delta V} = \sum_{m=1}^{M} \nu_k^{(m)} t^{(m)} \; k \in \boldsymbol{Q} \quad (26)$$

where

$$\nu_k^{(m)} = \sum_{i\in\boldsymbol{H}_k}\left\{-\frac{V_k}{z_{ki}^2}\left[r_{ki}\left(\theta_k - \theta_i\right) + x_{ki}\left(1 - \frac{\left(\theta_k - \theta_i\right)^2}{2}\right)\right] \cdot \frac{\sum_{h=0}^{N}\left[-\min\left(0; \delta_h^{(m)}\right) w_{ih}\right]}{\sum_{h=0}^{N} P_h^d w_{ih}}\right\}\Delta V_i$$
$$k \in \boldsymbol{Q}. \quad (27)$$

We adopt the approach used in [9] for the allocation of the real power line losses to allocate the component $Q_{ki}^{\Delta\theta}$. We next impose the assumption that $r_{ij} \ll x_{ij}$. Then, the voltage angle changes with respect to the reference case are approximated by using the dc power flow results. Let $\underline{\hat{\theta}} = \left[\hat{\theta}_1, \hat{\theta}_2 .., \hat{\theta}_N\right]^T$ be the vector of the bus voltage angles computed by the dc power flow using the $N \times N$ submatrix $\underline{\mathbf{B}}'$ obtained from the $(N+1)$-bus network susceptance matrix $\underline{\mathbf{B}}$ by removing the slack bus. Let $\underline{\mathbf{D}} = [d_{ij}] = (\underline{\mathbf{B}}')^{-1}$. It is then possible to write the dc power flow equations in the form

$$\underline{\mathbf{B}}'\underline{\hat{\theta}} = -\sum_{m=1}^{M} \underline{\delta}^{(m)} t^{(m)}. \quad (28)$$

The allocation of the voltage angle component is based on rewriting the voltage angle variation component in terms of the approximation

$$\tilde{Q}_{ki}^{\Delta\theta} = \frac{V_k V_i^0}{z_{ki}^2}\left(-r_{ki} + \frac{x_{ki}}{2}\left(\theta_k - \theta_i\right) + \frac{x_{ki}}{2}\left(\theta_k^0 - \theta_i^0\right)\right)\left(\hat{\theta}_k - \hat{\theta}_i\right). \quad (29)$$

The term $(\hat{\theta}_k - \hat{\theta}_i)$ is then expressed in terms of the transactions using the same approach as in [9]. Thus

$$\hat{\theta}_k - \hat{\theta}_i = \sum_{m=1}^{M} \pi_{ki}^{(m)} t^{(m)} \quad k \in \boldsymbol{Q}, i \in \boldsymbol{H}_k \tag{30}$$

where

$$\pi_{ki}^{(m)} = \mu_k^{(m)} - \mu_i^{(m)} \tag{31}$$

with $\mu_0^{(m)} = 0$ for $m = 1, 2, \ldots, M$, $d_{h0} = 0$ for $h = 1, 2, \ldots, N$, and

$$\mu_h^{(m)} = -\sum_{\nu=1}^{N} d_{h\nu}\delta_\nu^{(m)} = \sum_{j=1}^{N_b^{(m)}} d_{hb_j^{(m)}}\beta_j^{(m)} - \sum_{i=1}^{N_s^{(m)}} d_{hs_i^{(m)}}\sigma_i^{(m)}. \tag{32}$$

We use this expression for each line connecting the generator bus $k \in \boldsymbol{Q}$, obtaining the voltage angle variation component for each generator

$$\tilde{Q}_k^{g,\Delta\theta} = \sum_{i \in \boldsymbol{H}_k} \tilde{Q}_{ki}^{\Delta\theta} = \sum_{m=1}^{M} \varsigma_k^{(m)} t^{(m)} \tag{33}$$

where

$$\varsigma_k^{(m)} = \sum_{i \in \boldsymbol{\mathcal{H}}_k} \frac{V_k V_i^0}{z_{ki}^2} \left(-r_{ki} + \frac{x_{ki}}{2}(\theta_k - \theta_i) + \frac{x_{ki}}{2}\left(\boldsymbol{\theta}_k^0 - \boldsymbol{\theta}_i^0\right) \right) \pi_{ki}^{(m)}. \tag{34}$$

Note that in (34) $V_k = V_k^s$ or, in case the reactive limit is reached, $V_k = V_k^q$. The sum of the entities in (27) and (34) gives the total amount of VArs allocated to each transaction for each generator

$$Q_{k,a}^{(m)} = \eta_{k,a}^{(m)} t^{(m)} \quad m = 1, 2, \ldots, M \tag{35}$$

where

$$\eta_{k,a}^{(m)} = \nu_k^{(m)} + \varsigma_k^{(m)} \quad m = 1, 2, \ldots, M. \tag{36}$$

Since the sum of the allocated amounts will be an approximation of the voltage magnitude and voltage angle variation components, we have that may be approximated by

$$\tilde{Q}_k^{g,net} = \tilde{Q}_k^g - Q_k^d = Q_k^{g,0} + Q_{k,a}^g \tag{37}$$

where

$$Q_{k,a}^g = \sum_{m=1}^{M} Q_{k,a}^{(m)}.$$

We next examine the application of the proposed allocation scheme to test systems.

V. Numerical Results

We have tested the proposed scheme on a number of different test systems. We present a representative sampling of the results on two test systems: *Test System A* and *Test System B*. *Test System A* is derived from the IEEE 30-bus system by eliminating the shunt capacitors at buses 10 and 24 and by setting all the transformer ratios to 1 p.u. *Test System B* is constructed using the IEEE 57-bus system by eliminating the shunt capacitors at buses 18, 25, and 53. In *Test System B*, the synchronous condensers at buses 2, 3, 6, and 9 are considered as generators and consequently are included in the provision of reactive power support. The slack bus of each network has been renumbered to be bus 0. We use the values of the voltage setting points V_k^s, $k \in \boldsymbol{Q}$ and the reactive power generation limits as specified for the IEEE test systems. The voltage profiles required at buses $j \in \boldsymbol{Q}^c$ are within the range [0.9, 1.05] p.u.

TABLE I
TRANSACTIONS IN THE *TEST SYSTEM A*

m	$t^{(m)}$ [MW]	$S^{(m)}$	$\mathcal{B}^{(m)}$
1	35.0	{(8,100%)}	{(7,47%),(10,14%),(21,23%),(23,16%)}
2	32.4	{(11,93%), (13,7%)}	{(3,7%),(8,93%)}
3	56.1	{(0,72%), (13,28%)}	{(5,78%),(23,6%),(24,16%)}
4	50.0	{(5,100%)}	{(5,100%)}
5	33.2	{(2,100%)}	{(14,19%),(19,29%),(21,52%)}
6	21.7	{(13,100%)}	{(2,100%)}
7	55.0	{(0,15%), (2,85%)}	{(1,19%),(4,14%),(7,41%),(10,11%), (15,15%)}

TABLE II
TRANSACTIONS IN THE *TEST SYSTEM B*

m	$t^{(m)}$ [MW]	$S^{(m)}$	$\mathcal{B}^{(m)}$
1	255.0	{(0,100%)}	{(0,22%), (12,78%)}
2	227.0	{(12,100%)}	{(6,22%), (12,78%)}
3	161.0	{(0,75%), (3,25%)}	{(5,8%),(9,75%),(10,3%), (15,14%)}
4	322.0	{(8,100%)}	{(2,1%), (3,13%), (6,8%), (8,47%), (13,5%), (16,13%), (17,13%)}
5	122.5	{(0,61%), (12,39%)}	{(1,5%),(14,9%),(18,23%), (19,3%), (20,2%),(23,5%),(25,5%),(32,2%), (33,3%),(41,5%),(47,24%),(54,3%), (55,5%), (56,6%)}
6	163.3	{(8,88%), (12,22%)}	{(27,6%), (28,3%), (29,10%), (30,2%), (31,4%), (38,9%), (42,4%), (43,1%), (44,7%), (49,11%), (50,13%), (51,11%), (52,3%), (53,12%)}

We defined transactions for each of the two systems to supply the loads in the network. The sets of transactions are specified in Tables I and II for the *Test Systems A* and *B*, respectively. We assume that the reactive power allocation is to be performed on a set of feasible transactions with no system congestion.

We discuss our investigations of the overall ability of the proposed scheme to approximate the VAr outflows from the generation buses when compared to those computed by an ac power flow. $\tilde{Q}_k^{g,net}$ computed using (37) is compared to the power flow solution of (10) and (11) for each generator bus $k \in \boldsymbol{Q}$. We tested the robustness of this approximation by uniformly scaling all the transaction amounts $t^{(m)}$, $m = 1, 2, \ldots, M$. The results indicate agreement over a broad range of scaling factors. Fig. 1 shows a representative comparison of *Test System A* for scaling factors over the range [0, 2]. The unity scaling factor corresponds to the base case. These results are representative of the good tracking of the ac results by the approximation in (37) for each of the generating units of the network over this wide range.

We study the behavior of the allocation scheme in response to the variation in the scaling factor. We found that the allocation responds in a physically reasonable way for the various systems tested. For example, Fig. 2 shows the impacts of varying the transaction amounts on the reactive support requirement allocation $Q_{11,a}^{(m)}$ for the generator at bus 13, for each transaction. The

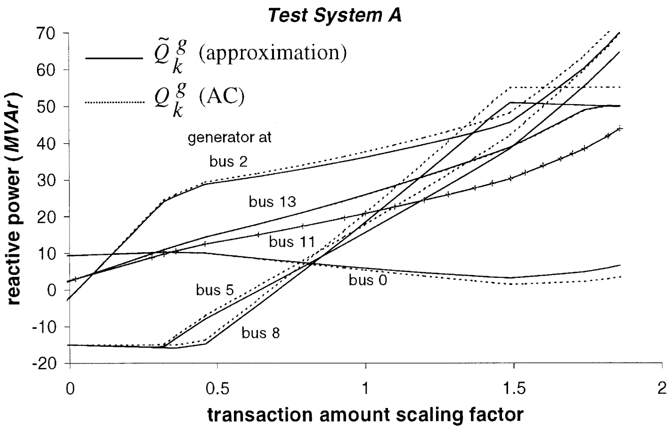

Fig. 1. Comparison of the performance of the approximation of the proposed allocation scheme with the ac power flow results.

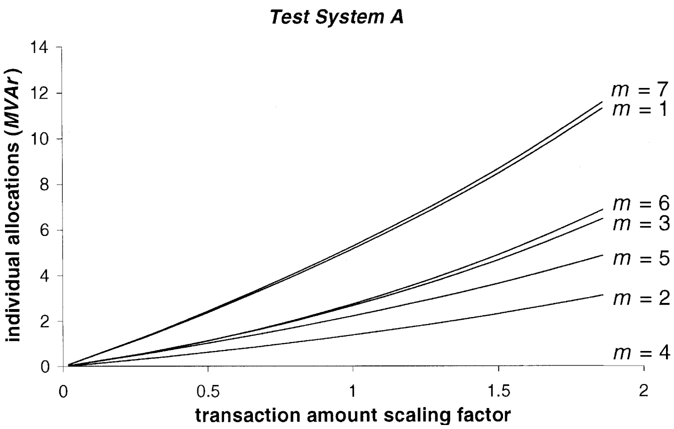

Fig. 2. Response of the allocation scheme for the generator at bus 13 to the uniform variation in the transaction amounts.

TABLE III
GENERATOR BUS INDIVIDUAL ALLOCATIONS IN MVARS FOR EACH TRANSACTION FOR THE BASE CASE OF *TEST SYSTEM B*

m \ k	0	2	3	6	8	9	12
	$Q_{0,a}^{(m)}$	$Q_{2,a}^{(m)}$	$Q_{3,a}^{(m)}$	$Q_{6,a}^{(m)}$	$Q_{8,a}^{(m)}$	$Q_{9,a}^{(m)}$	$Q_{12,a}^{(m)}$
1	0.07	19.76	0.19	-5.71	0.67	-0.33	2.77
2	0.08	1.20	0.31	0.03	0.22	0.23	-5.19
3	0.65	0.22	-1.53	-12.83	3.93	10.29	10.18
4	0.12	3.95	1.47	0.58	0.56	-11.40	-3.40
5	0.76	33.22	40.96	-0.19	79.14	98.53	-29.78
6	0.09	2.09	2.55	-6.38	7.24	-4.17	-5.16
$Q_{k,a}^g$	1.77	60.44	43.95	-24.51	91.75	93.15	-30.59
$Q_k^{g,0}$	-89.25	-78.53	-42.07	71.37	-118.79	4.63	145.99
$\tilde{Q}_k^{g,net}$	-87.48	-18.09	1.88	46.87	-27.04	97.77	115.41

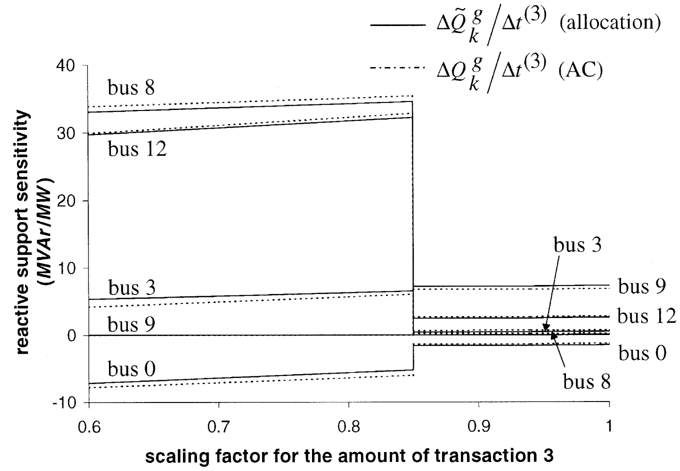

Fig. 3. Comparison of the allocation scheme response to that based on the ac power flow calculation to the variation of the amount of transaction 3 with all other transactions remaining fixed.

scaling factor is uniformly varied over the range [0, 2]. Note that for transaction 4, defined with both the seller and the buyer at bus 5, no reactive power support is allocated at any generator bus. This is correct since, in effect, this transaction does not *use the transmission system directly*.

We consider in more detail the allocated support requirements to the generators for each transaction for the base case condition. The results are shown in Table III for *Test System B*. The behavior of the allocation scheme is examined with respect to the intrinsic reactive support displayed in the penultimate row. The final row provides the net reactive support of each generator. The individual support requirement allocations $Q_{k,a}^{(m)}$ at bus k for transaction m may be positive or negative. An interpretation of $Q_{k,a}^{(m)}$ from (35) is obtained by considering the impacts on the allocation of the variation of the particular transaction amount. We consider all transactions with fixed amounts, save for transaction m for which we vary the amount $t^{(m)}$. At each generator, the resulting change in gives a satisfactory approximation of the change in the generator reactive power output. We illustrate this approximation using the *Test System B*. We hold all transactions constant and vary the amount $t^{(3)}$ with a scaling factor over the range [0.6, 1]. The scaling factor 1 corresponds to the base case $t^{(3)} = 161$ MW. Using 1 MW steps we compute the *sensitivities* $\Delta\tilde{Q}_k^g/\Delta t^{(3)}$ and $\Delta\tilde{Q}_k^g/\Delta t^{(3)}$ at each value of $t^{(3)}$ over the range of the scaling factor. We plot the results in Fig. 3 for some of the generators. For a scaling factor in the range [0.6, 0.85] the generator at bus 9 violates its lower limit. For scaling factor in the range (0.85, 1] no violation occurs, the generator at bus 9 provides additional reactive support and causes a reduction in the sensitivities for all the generators. In all cases, the allocation results track very closely the ac power flow results.

VI. CONCLUSION

A new physical-flow-based scheme for allocating the reactive power support provided by the generators to the transactions in a multitransaction framework has been presented. The proposed scheme allocates the net reactive power outflow from the generator buses in a physically meaningful way, by explicitly taking into account the interactions with the network and the nonlinearities due to the generator reactive power limits. The logical next step is to marry the reactive power allocation with reactive power pricing [10]–[12], so as to incorporate the economic efficiency goals. This is a highly challenging undertaking since the pricing of reactive power is considerably more complex than that of real power due to generator capability curve impacts

[12]. The presence of positive and negative terms in the reactive power allocation of each generator presents a further complication. While a generator needs to be compensated for its net reactive power output for each transaction, the fair compensation mechanism must explicitly ensure that no double-charging for the positive and negative allocations occurs. These issues are being addressed in our current research and the results of our work will be reported in a future paper.

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