Probabilistic flows for reliability evaluation of multiarea power system interconnections

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The development of a solution method and computer program for the evaluation of multiarea reliability is reported. An interconnected power system is modelled as a capacitated network with probabilistic arc capacities. The proposed solution method consists of an analytic state space decomposition phase and a Monte-Carlo simulation phase. An optimization problem is solved to minimize the total computational time for the two phases. The solution of the optimal mix problem determines the termination of the decomposition phase and the size of sample for the Monte-Carlo phase. A new reliability index, the inadequate transfer capability, is introduced. This measure indicates the relative effectiveness of either increasing existing capacities or opening new interconnections between two areas. The proposed method has been incorporated into a computationally efficient production grade software package, called Remain (Reliability Evaluation of Multiarea Interconnections). The application of Remain to a seven-area example for planning-system enhancement is given. Computational-times data is also presented.

Keywords: electric power systems, generation system reliability, planning reliability calculations

The development of an efficient computational tool for the evaluation of multiarea reliability in generation-planning studies is described. An area refers to a utility company, a geographic region within a utility, or a jointly owned generation. A single-area representation of the power system is commonly used in reliability evaluation of generation resource plans. In such representations, the transmission system is not modelled. The loss-of-load probability (LOLP) method is generally used for single-area reliability evaluation[1]. Quite often, resource planners are interested in assessing the benefits of interconnection. The objectives of reliability studies of such multiarea power systems are to evaluate the enhancement of reliability due to interconnection and to identify interconnections whose improvement is most effective in increasing the system reliability.

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Typical reliability indices for multiarea systems include the system LOLP, area LOLP and expected unserved demand. The area LOLP index depends on the interconnection policy which determines how the power is routed in the case of a loss of load. The following interconnection policies are considered:

- The load loss sharing (LLS) policy: whenever loss of load occurs in the system, areas must share the unserved demand as far as possible.
- The no load loss sharing (NLLS) policy: each area attempts to meet its own demand. If there is excess power, it is supplied to the neighbouring areas according to the order specified by a priority list.
- The isolation policy: this is considered for purposes of comparison and describes the case where no power exchange exists between any of the areas of the system.

One important objective of multiarea reliability studies is to identify weak links in the interconnection, i.e. tielines whose improvement is most effective in increasing the system reliability. Therefore, a new reliability index is introduced, the inadequate transfer capability from area i to area j. This is the probability that a lack of power transfer capability from area i to area j contributes to the loss of load. It can be evaluated for any two areas, whether or not they are directly connected. This measure indicates the relative effectiveness of either increasing existing capacities or opening new interconnections between two areas.

For multiarea reliability evaluation, the system can be modelled as a probabilistic flow-network. Doulliez and Jamoulle developed a state space decomposition method for probabilistic flow-network reliability evaluation[2] which was later used for transmission system reliability studies[3]. A decomposition approach was also used for reliability

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evaluation of composite systems including unconventional energy sources[4]. Pang and Wood[5] developed a computer program for multiarea reliability based on the inclusion and exclusion formula for evaluating the probability of the union of nondisjoint sets. In the present paper, a composite state space decomposition and Monte-Carlo method is presented for multiarea reliability calculations.

The method provides an estimate of each reliability index. Each estimate is obtained in such a way that its standard deviation is smaller than a specified quantity. This is accomplished by selecting the stopping criterion of the decomposition phase and the number of states in random samples of the Monte-Carlo phase using the solution of an optimization problem. The optimization problem is to minimize the total computation time, subject to the constraint that the standard deviations of the estimates are smaller than a specified quantity.

The combined decomposition-Monte-Carlo method has been incorporated into a computationally efficient production grade software package, called Remain (Reliability Evaluation of Multiarea Interconnections). Applications of Remain include the study of transmission bottlenecks between regions of a large utility and the investigation of the reliability of power pools. The results of a computational experience with a seven-area model of a power pool are presented. The use of the results to plan system enhancements for improving reliability is also described.

The notation used in the paper is standard. Vectors are denoted by bold type, e.g. $\mathbf{x}$. The ith component of the (row) vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is x_i. We write $\mathbf{x} \geqslant \mathbf{y}$ if and only if $x_i \geqslant y_i$ for $i = 1, 2, \ldots, n$. The notation $x \triangleq A$ means that x is defined by the expression A. The set $\{\mathbf{x}:A\}$ is the collection of vectors $\mathbf{x}$ with the property A. Random variables are indicated thus: $\underset{\sim}{c}$. The expression $P\{x:A\}$ is used for the probability of the set of events $\mathbf{x}$ characterized by property A. $P\{A \mid B\}$ is the conditional probability of event A, given that event B has occurred.

I. Flow-network model for multiarea reliability

I.1 *Multiarea power system*

In multiarea reliability evaluation, the total generation capacity in megawatts within an area is expressed as a discrete random variable. The probability distribution of the area generation capacity, which takes into account the possible forced outages of the generators, can be obtained by the convolution formula using forced outage/partial forced outage rates of individual generators. Between two areas that are directly connected, there is a maximum power, in megawatts, that can be transferred due to the limitations on the power-carrying capability of the tielines. Again, because of the possible forced outages of the tielines, the maximum power transfer capability between two areas can be represented by a discrete random variable. It is assumed that all these random variables are statistically independent. For each area, there may be a load demand in megawatts which is assumed to be a deterministic quantity.

I.2 *Probabilistic flow-network model*

Power flows from each area's generation to meet its own load and through the tielines to other areas. The power flows are limited only by the (random) generation and transfer capacities in the system. A flow-network with probabilistic arc capacities is used to model the multiarea power system. Each area is represented by a node. Two additional nodes are introduced: a generation source node s and a demand sink node t. A directed arc from the source node to each area node is introduced to represent the area generation capacity. A directed arc from each area node to the sink node is introduced to represent the area load demand. The capacity of this arc is the area load demand. Bidirectional arcs between area nodes are used to represent the tielines between areas. The capacity of each such arc is a random variable that describes the maximum power transfer capability between the two areas. The power network is assumed to be connected since the subject under study is the multiarea power system interconnections.

Let the number of arcs in the network be n. Let $\underset{\sim}{c}_i$ denote the random variable representing the capacity of arc i. It takes values

$$\underset{\sim}{c}_i = c_{ij}$$

with probability

$$p_{ij}, \quad j = 1, 2, \ldots, \ell_i$$

where ℓ_i is the number of distinct capacity levels for arc i. For each arc $\alpha(k)$ that joins area node k to the sink node, $\ell_k = 1$ so that $\underset{\sim}{c}_{\alpha(k)}$ becomes a deterministic variable. In this case, $\underset{\sim}{c}_{\alpha(k)}$ equals the area k demand D_k with probability 1. As mentioned above, the random variables $\underset{\sim}{c}_i$, $i = 1, 2, \ldots, n$, are assumed to be independent.

When each random variable $\underset{\sim}{c}_i$ takes a value, say c_{ix_i}, we have a capacitated flow-network. This corresponds to a system state which is denoted by the vector

$$\mathbf{x} = (x_1, x_2, \ldots, x_n)$$

The probability associated with each state $\mathbf{x}$ is

$$P\{\mathbf{x}\} = \prod_{i=1}^{n} p_{ix_i} \tag{1}$$

The collection of these system states forms the state space $\mathscr{X}$ of the multiarea power system model.

I.3 *Maximal flow and minimal cut*

For a particular system state $\mathbf{x}$, a maximal flow from the source to the sink can be found by the Ford-Fulkerson algorithm[6]. Let the resulting flows in the network be denoted by $\mathbf{f(x)} = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x})]$, where $f_i(\mathbf{x})$ is the flow through the ith arc. The total amount of flow from the source to the sink is called the value of the maximal flow, $V[\mathbf{f(x)}]$. There may be more than one maximal flow through the network; however, all maximal flows have the same value. Priority lists are used that order the arcs leaving each node to select a particular maximal flow. When no additional flow can be routed in the network from the source to the sink, there is always a set of arcs whose capacities limit the flow. In other words, associated with each maximal flow $\mathbf{f}$ there is a minimal cut set of arcs

$\mathscr{C}[\mathbf{f}(\mathbf{x})]$. A minimal cut partitions the nodes in the network into two disjoint sets. These two sets are denoted by:

$$\mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\} = \{\text{nodes on the source side of the cut}\} \quad (2)$$

$$\mathcal{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\} = \{\text{nodes on the sink side of the cut}\} \quad (3)$$

It can be shown (Corollary 5.4, p. 13 of Reference 6) that if $\mathcal{N}_s^1\{\mathscr{C}^1[\mathbf{f}^1(\mathbf{x})]\}$ and $\mathcal{N}_s^2\{\mathscr{C}^2[\mathbf{f}^2(\mathbf{x})]\}$ (written $\mathcal{N}_s^1$ and $\mathcal{N}_s^2$ for short) are defined by two minimal cuts $\mathscr{C}^1$ and $\mathscr{C}^2$ of the network associated with the system state $\mathbf{x}$, the arcs joining their intersection $\mathcal{N}_s^1 \cap \mathcal{N}_s^2$ and its complement $\mathcal{N}_s^1 \cap \mathcal{N}_s^2$ form a minimal cut. Similarly, the arcs joining $\mathcal{N}_t^1 \cap \mathcal{N}_t^2$ and $\mathcal{N}_t^1 \cap \mathcal{N}_t^2$ also form a minimal cut. The sets $\mathcal{N}_s^*(\mathbf{x})$ and $\mathcal{N}_t^*(\mathbf{x})$ are now defined as follows:

$$\mathcal{N}_s^*(\mathbf{x}) = \bigcap_{\text{all } \mathscr{C} \text{ of } \mathbf{x}} \mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$$

$$\mathcal{N}_t^*(\mathbf{x}) = \bigcap_{\text{all } \mathscr{C} \text{ of } \mathbf{x}} \mathcal{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$$

Note that $\mathcal{N}_s^*(\mathbf{x})$ itself is one of $\mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$, therefore $\mathcal{N}_s^*(\mathbf{x})$ is the set having the least number of nodes among all $\mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$. Similarly, $\mathcal{N}_t^*(\mathbf{x})$ is the set having the least number of nodes among all $\mathcal{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$. The application of the Ford-Fulkerson algorithm starting from node s yields $\mathcal{N}_s^*(\mathbf{x})$, whereas the application of the Ford-Fulkerson algorithm backward from node t yields $\mathcal{N}_t^*(\mathbf{x})$.

1.4 *Reliability indices*

For D_i being the local demand of area i

$$D = \Sigma D_i$$

is the total load demand of the system. A power system is said to be experiencing loss of load whenever there is load that the system fails to supply. The following reliability indices are defined for multiarea systems:

1.4.1 *System loss-of-load probability (LOLP)* For a particular system state $\mathbf{x}$, if $V[\mathbf{f}(\mathbf{x})]$ is less than the total demand $\mathscr{D}$, there is loss of load in the system. System LOLP is defined to be the probability that there is loss of load in the system.

$$\text{system LOLP} \triangleq P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\} \quad (4)$$

1.4.2 *Expected unserved demand (EUD)* EUD is the expected value of the amount of load demand that the system is unable to meet.

$$\begin{aligned}\text{EUD} &\triangleq \sum_{\mathbf{x} \in \{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\}} \{D - V[\mathbf{f}(\mathbf{x})]\} P\{\mathbf{x}\} \\ &= E\{D - V[\mathbf{f}(\mathbf{x})] \mid V[\mathbf{f}(\mathbf{x})] < D\} P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\}\end{aligned} \quad (5)$$

1.4.3 *Area LOLP* Area LOLP is the probability that the area fails to meet its load demand. The value of the area LOLP depends on the interconnection policy adopted. Under the NLLS policy, the area LOLP can be calculated as follows:

$$\text{area } i \text{ LOLP} \mid_{\text{NLLS}} \triangleq P\{\mathbf{x}: f_{\alpha(i)}(\mathbf{x}) < D_i\} \quad (6)$$

Here $\alpha(i)$ denotes the arc that connects the area node i to the sink node. Under the LLS policy, the area LOLP can be calculated as follows.

$$\text{area } i \text{ LOLP} \mid_{\text{LLS}} \triangleq P\{\mathbf{x}: \text{node } i \in \mathcal{N}_t^*(\mathbf{x})\} \quad (7)$$

1.4.4 *Inadequate transfer capability (ITC)* For a particular system state $\mathbf{x}$, suppose there is a loss of load in the system; then an increase in the transfer capability from node i to node j reduces the amount of unserved demand if and only if node i is in $\mathcal{N}_s^*(\mathbf{x})$ and node j is in $\mathcal{N}_t^*(\mathbf{x})$. This is true because of the fact that $\mathcal{N}_s^*(\mathbf{x})$ and $\mathcal{N}_t^*(\mathbf{x})$ are the sets having the least number of nodes among all $\mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$ and $\mathcal{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$, respectively. It should be noted that the foregoing statement is not true if the condition is simply that node i is in $\mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$ and node j is in $\mathcal{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$ for some minimal cut $\mathscr{C}$ of $\mathbf{x}$. In other words, when there is a loss of load in the system, node i is in $\mathcal{N}_s^*(\mathbf{x})$ and node j is in $\mathcal{N}_t^*(\mathbf{x})$ and the lack of power transfer capability from node i to node j is a contributing factor to the loss of load. If node i is the source node and node j is an area node, the situation described above indicates that the area generation is inadequate. If both node i and node j are area nodes, the situation indicates that the tieline capacity is inadequate. The inadequate transfer capability (ITC) for each ordered pair of nodes (node i, node j) is defined as the probability that increasing the capacity from node i to node j reduces the amount of unserved demand. The value of ITC_{ij} can be computed as follows:

$$\text{ITC}_{ij} = P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D, \quad \text{node } i \in \mathcal{N}_s^*(\mathbf{x}) \text{ and node } j \in \mathcal{N}_t^*(\mathbf{x})\} \quad (8)$$

The inadequate transfer capability can be interpreted as the probability that a lack of power transfer capability from node i to node j contributes to the loss of load. The inadequate transfer capability provides a measure of the inadequacy of area generation or transfer capabilities in various parts of the system. Increases in the capacity of tielines or area generation with higher ITC values are more effective in improving reliability. The inadequate transfer capability from each area i to the system can be defined as follows:

$$\text{ITC}_{i,\text{ system}} = P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D \text{ and node } i \in \mathcal{N}_s^*(\mathbf{x})\} \quad (9)$$

Attempts have been made in the past[2] to use the probability that the arc from node i to node j is in a minimal cut as a measure of inadequacy of transfer capability. Because of the fact that there may be more than one minimal cut in the network, increasing the transfer capability of an arc in the minimal cut does not necessarily imply that the amount of load lost will be reduced. The new inadequate transfer capability index that has been introduced has the desired interpretation. Moreover, since it is defined for any pair of nodes, regardless of whether they are or are not directly connected, the ITC index can be used for comparing the relative effectiveness of increasing existing tieline capacity or building new interconnections.

II. Overview of the decomposition-Monte Carlo approach

The basic idea in the evaluation of each reliability index is to identify the appropriate set of states and to compute its probability. The algorithm described here has two phases. The first phase is an analytical state space decomposition and the second phase is a Monte-Carlo simulation. The allocation of the time between the two phases is determined by the solution of an optimization problem.

The state space decomposition phase of the algorithm is an extension of the method developed by Doulliez and Jamoulle[2]. It is an iterative process to classify the states in the state space. Initially, all the states in the state space are unclassified. At each iteration, by application of the maximal-flow algorithm, the set of unclassified states is decomposed into subsets of states having the same reliability characteristics and subsets of unclassified states. Upper and lower bounds for each reliability index are computed. As the number of iterations increases, the number of states that are classified by each iteration decreases, i.e. the efficiency of the method decreases. Because of this, a Monte-Carlo method is used in the second phase of the approach. From each subset of unclassified states, a random sample of states is selected. The maximal-flow algorithm is applied to each of the sample states. Finally, an estimate of the contribution of the unclassified states to each reliability index is obtained.

The uncertainty in the estimate of each reliability index is measured in terms of its standard deviations. The standard deviation of an estimate depends on the computation time in the two phases of the algorithm. Consider the optimization problem of minimizing the total computation time, subject to the condition that the standard deviation does not exceed a specified quantity. Under some reasonable assumptions, certain relations have been described which must be satisfied by the optimal solution. From these relations, it is possible to determine the stopping criterion for the decomposition phase and the number of states in each random sample for the Monte-Carlo phase.

A detailed description of the proposed solution method is given below.

III. State-space decomposition phase

An analytic decomposition scheme is used to avoid carrying out a separate maximal-flow computation for each of the

$$\prod_{i=1}^{n} \ell_i$$

states in the state space. The state space is decomposed in such a way that, from each maximal-flow computation carried out on one state, information concerning the reliability of some other states may be derived.

The iterative decomposition scheme makes use of two basic properties of the model. These are:

(i) Let the maximal flow corresponding to state $\mathbf{x}$ be $V[\mathbf{f}(\mathbf{x})]$. For any state $\mathbf{y} \geqslant \mathbf{x}$, $V[\mathbf{f}(\mathbf{y})] \geqslant V[\mathbf{f}(\mathbf{x})]$. In other words, the system model is coherent in the sense of reliability theory (p 6 of Reference 7).

(ii) Let $\mathbf{M}$ and $\mathbf{m}$ be two states in the state space $\mathscr{X}$, $\mathbf{M} \geqslant \mathbf{m}$. For the set of states lying between $\mathbf{M}$ and $\mathbf{m}$ defined as

$$\mathscr{S} \triangleq \{\mathbf{x} : \mathbf{m} \leqslant \mathbf{x} \leqslant \mathbf{M}\} \tag{10}$$

we have

$$P\{\mathscr{S}\} = \prod_{i=1}^{n} \sum_{m_i \leqslant x_i \leqslant M_i} p_{ix_i} \tag{11}$$

$\mathbf{M}$ and $\mathbf{m}$ are referred to as the maximum and minimum states of the set $\mathscr{S}$, respectively.

The coherency property (i) enables us to derive from a single maximal-flow computation, the reliability characteristics of a subset of the states without having to carry out maximal-flow calculations for each of these states. Properties (i) and (ii) form the basis of a recursive scheme of decomposing the state space into subsets.

There are two stages in the decomposition phase: the first is for the calculation of system LOLP and the second stage is for the calculation of the area LOLP, EUD and ITC indices.

III.1 *Recursive decomposition for system LOLP*

The initial set of the decomposition phase is the state space. The state space $\mathscr{X}$ has a maximum state $\mathbf{M} = (M_1, M_2, \ldots, M_n)$ and a minimum state $\mathbf{m} = (m_1, m_2, \ldots, m_n)$ and is in the form of the set in equation (10). $\mathscr{X}$ is decomposed into subsets of three categories (see Figure 1).

III.1.1 *Subset of acceptable states $\mathscr{A}$* Suppose that the value of the maximal flow for the state $\mathbf{M}$ is $V[\mathbf{f}(\mathbf{M})] = D$. Let f_k denote the flow through arc k. For each arc k, let u_k be the index x_k corresponding to the smallest capacity level c_{kx_k} of the arc which is not less than f_k, i.e.

$$u_k = \min\{x_k : c_{kx_k} \geqslant f_k\} \tag{12}$$

By the coherency property of the flow-network model, whenever the capacity c_k of arc k takes a value between c_{ku_k} and c_{kM_k}, $k = 1, 2, \ldots, n$, then a flow D can be sent, i.e. the total load demand D is satisfied. Thus, for the set of states

$$\mathscr{A} = \{\mathbf{x} : \mathbf{u} \leqslant \mathbf{x} \leqslant \mathbf{M}\} \tag{13}$$

the demand D can be met, i.e. $V[\mathbf{f}(\mathbf{x}\ \ )] = D$. Since the set $\mathscr{A}$ is of the form in equation (10), its probability can readily be computed.

III.1.2 *Subsets of system loss of load states $\mathscr{L}_k$* To classify the collection of states $\mathbf{x}$ for which the total load demand D cannot be satisfied, i.e. $V[\mathbf{f}(\mathbf{x})] < D$, we first determine for each arc k the minimum capacity level c_{kv_k} such that a flow D can still be sent. The following procedure is used to find the value v_k (see Figure 2). Let us start with the network corresponding to state $\mathbf{M}$ with the maximal flow $\mathbf{f}(\mathbf{M}) = [f_1(\mathbf{M}), \ldots, f_n(\mathbf{M})]$. Suppose that the flow f_k in arc k is from node q to node r. We remove arc k and consider the resulting network having flows $f_\ell(\mathbf{M})$ in the remaining arcs, $\ell \neq k$. We then apply the Ford–Fulkerson algorithm to this network to determine the maximal flow from q to r that can be superimposed on the existing flow. The value e_k of this additional flow is the largest decrease in flow for arc k such that the network can still send a maximal flow D from s to t. We define v_k to be

$$v_k = \begin{cases} \min\{x_k : | c_{kx_k} \geqslant f_k - e_k\} & \text{if } f_k - e_k \geqslant 0 \\ 1 & \text{if } f_k - e_k < 0 \end{cases} \tag{14}$$

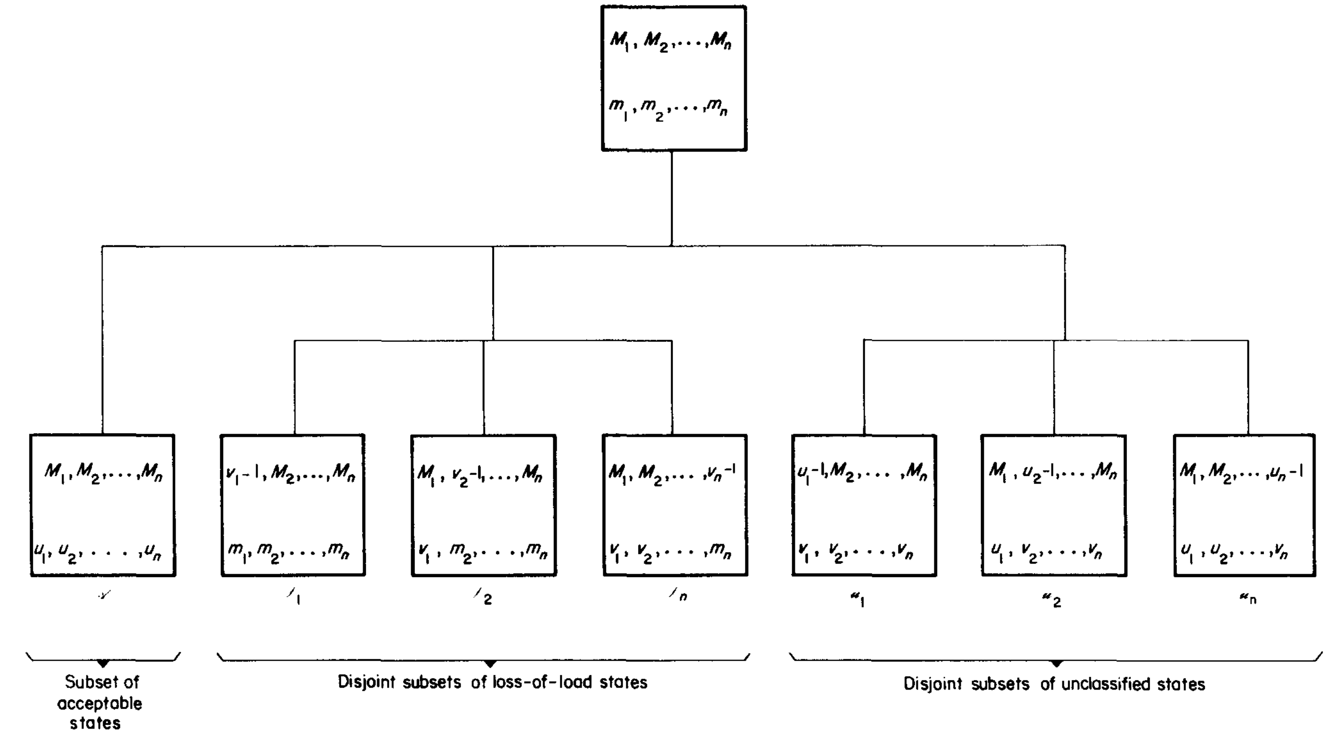

Figure 1. Recursive decomposition for the evaluation of the system LOLP index

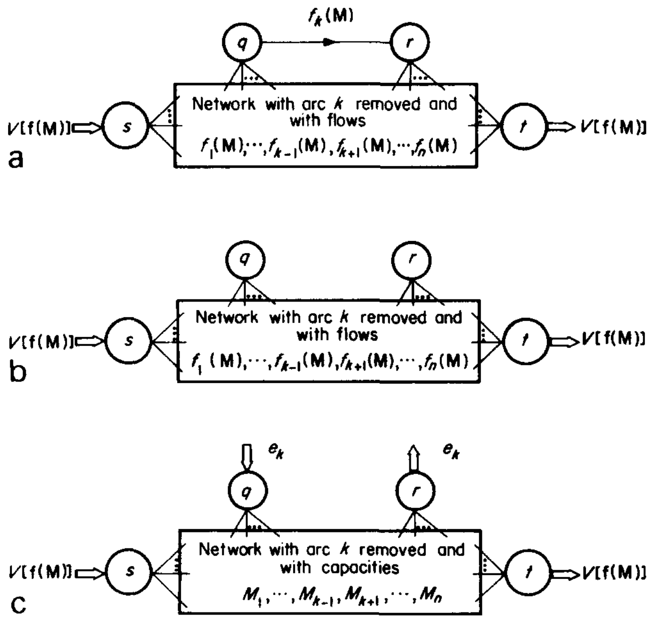

Figure 2. Procedure for determining the largest decrease in flow e_k such that a maximal flow $V[\mathbf{f}(\mathbf{x})] = D$ can still be sent from s to t; (a) network flow corresponding to state $\mathbf{M}$, (b) flow network with arc k removed and flows $f_e(M)$, $e \neq k$, unchanged, (c) determination of the maximal additional flow e_k from node q to node r

By the coherency property of the flow-network model, system loss of load occurs for any state in the set

$$\mathscr{L} = \{\mathbf{x} : \exists\, k \ni x_k < v_k\} \tag{15}$$

We may further decompose $\mathscr{L}$ into nonintersecting subsets $\mathscr{L}_1, \mathscr{L}_2, \ldots, \mathscr{L}_n$ having the form in equation (10). The states in $\mathscr{L}$ are classified into subset $\mathscr{L}_k$ if k is the first index of $\mathbf{x}$ for which $x_k < v_k$, i.e.

$$\mathscr{L}_k = \{x : (v_1, \ldots, v_{k-1}, m_k, m_{k+1}, \ldots, m_n) \leqslant \mathbf{x} \leqslant (M_1, \ldots, M_{k-1}, v_{k-1}, M_{k+1}, \ldots, M_n)\} \tag{16}$$

Note that the contribution of each subset $\mathscr{L}_k$ to the system LOLP is $P\{\mathscr{L}_k\}$, which is readily calculated. Since the subsets $\mathscr{L}_1, \mathscr{L}_2, \ldots, \mathscr{L}_n$ are disjoint, their contributions to system LOLP are additive.

III.1.3 *Subsets of unclassified states* $\mathscr{U}_k$ The remaining states in $\mathscr{X}$ are unclassified. They are characterized by

$$\mathscr{U} = \{\mathbf{x} : x_i \geqslant v_i\ \forall i \quad \text{and } \exists\, j \ni x_j < u_j\} \tag{17}$$

This set generally does not have a maximum state. In order to be able to continue the process of classification recursively, it is necessary to decompose $\mathscr{U}$ into subsets of the form in equation (10). The states in $\mathscr{U}$ are therefore classified into the subsets $\mathscr{U}_k$ if k is the first index for which $x_k < u_k$, i.e.

$$\mathscr{U}_k = \{\mathbf{x} : (u_1, \ldots, u_{k-1}, v_k, v_{k+1}, \ldots, v_n) \leqslant \mathbf{x} \leqslant (M_1, \ldots, M_{k-1}, u_{k-1}, M_{k+1}, \ldots, M_n) \tag{18}$$

The subsets $\mathscr{U}_1, \ldots, \mathscr{U}_n$ are disjoint and their union equals $\mathscr{U}$.

The recursive scheme consists of replacing $\mathscr{X}$ by $\mathscr{U}_k$ redefining the quantities $\mathbf{m}$ and $\mathbf{M}$ and repeating the decomposition into acceptable subset $\mathscr{A}$, loss-of-load subsets $\mathscr{L}_1, \ldots, \mathscr{L}_n$, and unclassified subsets $\mathscr{U}_1, \ldots, \mathscr{U}_n$. Because all the decomposed subsets are noninteresting, their probabilities can simply be added.

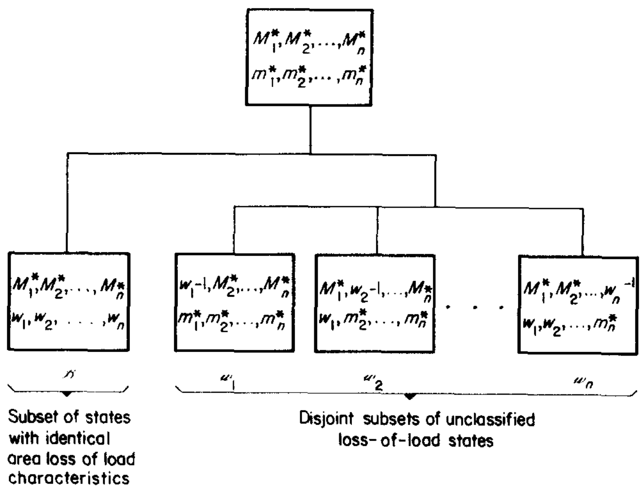

Figure 3. Recursive decomposition for evaluation of area LOLP index

III.2 *Recursive decomposition for area LOLP*
Each state in the subsets $\mathscr{L}_\ell$, $\ell = 1, 2, \ldots, n$ is a system loss-of-load state. A system loss-of-load state corresponds to the situation of one or more areas experiencing loss of load. While two states may belong to the same subset $\mathscr{L}_\ell$, their area loss-of-load characteristics may be different. In order to evaluate area LOLP, each subset $\mathscr{L}_\ell$ is recursively decomposed into subsets having identical area loss-of-load characteristics (see Figure 3).

The area LOLP depends on the interconnection policy. Let us consider first the evaluation of the area LOLP under the NLLS policy. Let m^* and M^* denote the minimum and maximum states, respectively, of $\mathscr{L}_\ell$, i.e.

$$\mathscr{L}_\ell = \{\mathbf{x} : \mathbf{m}^* \leqslant \mathbf{x} \leqslant \mathbf{M}^*\} \tag{19}$$

The Ford-Fulkerson algorithm is applied to the maximum state $\mathbf{M}^*$ to determine the set $\mathscr{N}_t^*(\mathbf{M}^*)$ and the corresponding maximal flow. One of the following two cases may occur:

- Case 1. There exists an area node $j \in \mathscr{N}_t^*(\mathbf{M}^*)$ such that the corresponding flow in the arc $\alpha(j)$ from the area node j to the sink is saturated, i.e. $f_{\alpha(j)} = D_j$.
- Case 2. For every area node $i \in \mathscr{N}_t^*(\mathbf{M}^*), f_{\alpha(i)} < D_i$, i.e. every area in $\mathscr{N}_t^*(\mathbf{M}^*)$ experiences loss of load.

Case 1 is considered first. Under the NLLS policy, no loss of load occurs for area j. On the other hand, there exists at least one area node $i \in \mathscr{N}_t^*(\mathbf{M}^*)$ for which $f_{\alpha(i)} < D_i$. Area i thus suffers loss of load. The subset of states of $\mathscr{L}_\ell$ with the same area loss-of-load characteristics, i.e. the same areas satisfy load demand and the same areas suffer loss of load, is

$$\mathscr{B} = \{\mathbf{x} : \mathbf{w} \leqslant \mathbf{x} \leqslant \mathbf{M}^*\} \tag{20}$$

where

$$w_k = \min\{x_k : c_{kx_k} \geqslant f_k(\mathbf{M}^*)\} \tag{21}$$

In Case 2 we want to find the set of states $\mathbf{x}$ for which $\mathscr{N}_t^*(\mathbf{x}) = \mathscr{N}_t^*(\mathbf{M}^*)$, i.e. the states $\mathbf{x}$ having the same area loss-of-load characteristics. Clearly, if a minimal cut $\mathscr{C}(\mathbf{x})$ of $\mathbf{x}$ is identical to the minimal cut $\mathscr{C}(\mathbf{M}^*)$ at hand, and there is no other minimal cut consisting of arcs in $\mathscr{C}(\mathbf{x})$ and arcs connecting $\mathscr{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$, than $\mathscr{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\} = \mathscr{N}_t^*(\mathbf{x})$ and is equal to $\mathscr{N}_t^*(\mathbf{M}^*)$. Let us define in this case

$$w_k = \begin{cases} \min\{x_k : c_{kx_k} \geqslant f_k\} & \text{if } k \notin \mathscr{C}(\mathbf{M}^*) \\ m_k^* & \text{if } k \in \mathscr{C}(\mathbf{M}^*) \end{cases} \tag{22}$$

Clearly, for each state $\mathbf{x}$ in the set

$$\mathscr{B} = \{\mathbf{x} : \mathbf{w} \leqslant \mathbf{x} \leqslant \mathbf{M}^*\} \tag{23}$$

$\mathscr{N}_t^*(\mathbf{x}) = \mathscr{N}_t^*(\mathbf{M}^*)$ and every area node in $\mathscr{N}_t^*(\mathbf{x})$ experiences loss of load.

Note that, in this case, under the NLLS policy, the flow along an arc connecting any two nodes in $\mathscr{N}_t^*(\mathbf{x}) - \{t\}$ is zero. Therefore, w_k in equation (22) may be alternatively determined by

$$w_k = \begin{cases} \min\{x_k : c_{kx_k} \geqslant f_k\} & \text{if both nodes connected by arc } k \text{ belong to } \mathscr{N}_s - s \\ m_k^* & \text{otherwise} \end{cases} \tag{24}$$

The remaining states in $\mathscr{L}_\ell$ can be further decomposed into disjoint subsets $\mathscr{W}_1, \mathscr{W}_2, \ldots, \mathscr{W}_n$ each of which has the form as the set in equation (10). A state $\mathbf{x}$ in $\mathscr{L}_\ell - \mathscr{B}$ is classified into the subset $\mathscr{W}_k$ if k is the first index for which $x_k < w_k$, i.e.

$$\mathscr{W}_k = \{\mathbf{x} : (w_1, \ldots, w_{k-1}, m_k^*, m_{k+1}^*, \ldots, m_n^*) \leqslant \mathbf{x} \leqslant (M_1^*, \ldots, M_{k-1}^*, w_{k-1}, M_{k+1}^*, \ldots, M_n^*) \tag{25}$$

The recursive scheme consists of replacing $\mathscr{L}_\ell$ by $\mathscr{W}_k$ and repeating the decomposition process.

The evaluation of area LOLP under the LLS policy is considered next. The following Fact shows that the set of states in $\mathscr{B}$, defined by either equations (20)-(21) or by equations (22)-(23), has the same area loss-of-load characteristics under the LLS policy.

Fact Each state $\mathbf{x}$ in the set $\mathscr{B}$, whether constructed as in Case 1 or as in Case 2, has the property that all the area nodes in $\mathscr{N}_t^*(\mathbf{x}) = \mathscr{N}_t^*(\mathbf{M}^*)$ suffer loss of load under the LLS policy.

Proof For Case 2, as under the NLLS policy, each area node in $\mathscr{N}_t(\mathbf{x})$ suffers loss of load. For Case 1, let us suppose that area node $j \in \mathscr{N}_t^*(\mathbf{x})$ and $f_{\alpha(j)} = D_j$. It can be shown that the flow in the network can be rerouted so that area j will also experience loss of load. Indeed, it is claimed that some of the flow in $f_{\alpha(j)}$ can be rerouted from node j to the sink node t through other nodes in $\mathscr{N}_t^*(\mathbf{x})$. In other words, it is claimed that there is a flow augmenting path (p 12 of Reference 6) with respect to the current flow from node j to node t through other nodes in $\mathscr{N}_t^*(\mathbf{x})$. If the claim is not true, then there must be a minimal cut consisting of arcs connecting nodes in $\mathscr{N}_t^*(\mathbf{x}) = \mathscr{N}_t^*(\mathbf{M}^*)$, which contradicts the definition of $\mathscr{N}_t^*(\mathbf{M}^*)$.

III.3 *Computation of EUD and ITC*
The expected unserved demand (EUD), which measures

the shortfall of supply for the demand D, is defined by

$$\begin{aligned}\text{EUD} &= E\{D - V[\mathbf{f}(\mathbf{x})] \mid V[\mathbf{f}(\mathbf{x})] < D\} P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\} \\ &= D \cdot P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\} - E\{V[\mathbf{f}(\mathbf{x})] \mid V[\mathbf{f}(\mathbf{x})] < D\} \\ &\quad \times P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\} \qquad (26)\end{aligned}$$

Since each $\mathscr{B}$ obtained from the decomposition process is a subset of the set $\{\mathbf{x}: V[\mathbf{f}(\mathbf{x})] < D\}$

$$\text{EUD} = D \cdot \sum_{\mathscr{B}} P\{\mathscr{B}\} - \sum_{\mathscr{B}} E\{V[\mathbf{f}(\mathbf{x})] \mid \mathbf{x} \in \mathscr{B}\} \cdot P\{\mathscr{B}\} \qquad (27)$$

where the summation is over all the subsets $\mathscr{B}$ obtained in the decomposition process. Since the set $\mathscr{B}$ has the form of $\mathscr{S}$ in equation (10), the first term in equation (27) can be readily computed

$$P\{\mathscr{B}\} = \prod_{i=1}^{n} \sum_{x_i = w_i}^{M_i^*} p_{ix_i}$$

Consider the term $E\{V[\mathbf{f}(\mathbf{x})] \mid \mathbf{x} \in \mathscr{B}\}$. For each state $\mathbf{x} \in \mathscr{B}$ constructed as in Case 1, the value of the maximal flow $V[\mathbf{f}(\mathbf{x})]$ is constant. Let $V[\mathbf{f}(\mathbf{x})] = Q$, so that

$$E\{V[\mathbf{f}(\mathbf{x})] \mid \mathbf{x} \in \mathscr{B}\} = Q \qquad (28)$$

On the other hand, in Case 2, for each state $\mathbf{x}$ in $\mathscr{B}$, the minimal cut $\mathscr{C}[\mathbf{f}(\mathbf{x})]$ has the same set of arcs. Hence

$$\begin{aligned}E\{V[\mathbf{f}(\mathbf{x})] \mid \mathbf{x} \in \mathscr{B}\} &= E\left\{\sum_{i \in \mathscr{C}} \underset{\sim}{c}_i(\mathbf{x}) \mid \mathbf{w} \leqslant \mathbf{x} \leqslant \mathbf{M}^*\right\} \\ &= \sum_{i \in \mathscr{C}} E\{\underset{\sim}{c}_i(\mathbf{x}) \mid \mathbf{w} \leqslant \mathbf{x} \leqslant \mathbf{M}^*\} \\ &= \sum_{i \in \mathscr{C}} \frac{\sum_{x_i = w_i}^{M_i} p_{ix_i} c_{ix_i}}{\sum_{x_i = w_i}^{M_i^*} p_{ix_i}} \qquad (29)\end{aligned}$$

The inadequate transfer capability between two nodes i and j is defined by

$$\begin{aligned}\text{ITC}_{ij} &= P\{\mathbf{x}: V[\mathbf{f}(\mathbf{x}) < D, \text{ node } i \in \mathscr{N}_s'^*(\mathbf{x}) \text{ and} \\ &\qquad \text{node } j \in \mathscr{N}_t^*(\mathbf{x})\} \\ &= \sum_{\mathscr{B}} P\{\text{node } i \in \mathscr{N}_s^*(\mathbf{x}) \text{ and} \\ &\qquad \text{node } j \in \mathscr{N}_t^*(\mathbf{x}) \mid \mathbf{x} \in \mathscr{B}\} P\{\mathscr{B}\} \qquad (30)\end{aligned}$$

Note that $P\{\text{node } i \in \mathscr{N}_s^*(\mathbf{x}) \text{ and node } j \in \mathscr{N}_t^*(\mathbf{x}) \mid \mathbf{x} \in \mathscr{B}\}$ is either 0 or 1. The above expression can then be readily computed.

III.4 *Bounds on reliability indices*

Each time a subset $\mathscr{L}_k$ is identified, its contribution to the system LOLP index is accumulated. Similarly, each time a subset $\mathscr{B}$ is identified, its contribution to the area LOLP, EUD and ITC indices are accumulated. When the decomposition is completed, i.e. there is no set with unclassified states, the true values of these reliability indices are obtained. However, if the decomposition is terminated earlier so that there are nonempty subsets $\mathscr{U}_k$ and $\mathscr{W}_k$, the cumulative value of each reliability index is clearly a lower bound for that index. On the other hand, if the probabilities of the remaining unclassified subsets $\mathscr{U}_k$ are added to the system LOLP, an upper bound of the system LOLP is obtained. Similarly, if the probabilities of the remaining unclassified subsets $\mathscr{U}_k$ and $\mathscr{W}_k$ are added to each of the other reliability indices, an upper bound of that index is obtained. Thus, at each stage of the recursive process, a lower bound and an upper bound for each reliability index are available.

IV. Monte-Carlo phase

The number of states in each subset that is classified in the state space decomposition process decreases rapidly as the number of iterations increases. When the efficiency of the decomposition phase decreases below the level specified by the solution of the optimal mix problem (Section V), a switch is made to Monte-Carlo simulation. The details of the Monte-Carlo phase of the proposed approach are described below.

Let r denote any reliability index defined in Section I.4 other than the EUD. At the termination of the decomposition phase, we have a lower bound r^m and an upper bound r^M for the true value r^* of the index r. The problem is to estimate the contribution to r^* of the remaining unclassified subsets $\mathscr{U}_k$ and $\mathscr{W}_k$ so as to estimate the true value of r. The Monte-Carlo phase of the algorithm consists of picking a random sample of N states $\{\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^N\}$ from the unclassified subsets $\mathscr{U}_k$ and $\mathscr{W}_k$ and estimating the contribution to r^*.

The relationship

$$p = \frac{r^* - r^m}{r^M - r^m} \qquad (31)$$

is defined as the fraction of states in the unclassified subsets that contribute to the index r. Then

$$r^* = r^m + p(r^M - r^m) \qquad (32)$$

p may be thought of as the probability that each state in the unclassified subsets makes a contribution to the index r. Thus, we can define an indicator random variable $\underset{\sim}{I}$ for each state $\mathbf{x}$ in the unclassified subsets as to its contribution to r,

$$\underset{\sim}{I}(\mathbf{x}) = \begin{cases} 1 \text{ with probability } p \\ 0 \text{ with probability } 1 - p \end{cases} \qquad (33)$$

Clearly $E\{\underset{\sim}{I}\} = p$ and $\text{var}\{\underset{\sim}{I}\} = p(1 - p)$.

Since the true value of p is not known *a priori*, the problem is to determine an estimate of p. The following unbiased estimator of p is used:

$$\underset{\sim}{\hat{p}} = \frac{1}{N} \sum_{i=1}^{N} \underset{\sim}{I}(\mathbf{x}^i) \qquad (34)$$

The variance of this estimator is (p 228 of Reference 8)

$$\text{var}\{\underset{\sim}{\hat{p}}\} = \frac{p(1 - p)}{N} \qquad (35)$$

which has an upper bound of $(1/4N)$. Based on the estimator $\underset{\sim}{\hat{p}}$ in equation (34) we obtain

$$\underset{\sim}{\hat{r}} = r^m + \underset{\sim}{\hat{p}}\,(r^M - r^m) \tag{36}$$

as an estimator of r. The variance of this estimator is

$$\mathrm{var}\{\underset{\sim}{\hat{r}}\} = (r^M - r^m)^2 \frac{p(1-p)}{N} \tag{37}$$

with an upper bound of

$$\mathrm{var}\{\underset{\sim}{\hat{r}}\} \leqslant \frac{(r^M - r^m)^2}{4N} \tag{38}$$

To estimate the value of the EUD index, the conditional expectation Q of the value of the maximal flow $V(\mathbf{x})$ is defined, given that the state $\mathbf{x}$ belongs to the family $\mathscr{R}$ of the remaining unclassified subsets $\mathscr{U}_k$ and $\mathscr{W}_k$.

$$Q = \frac{\sum_{\mathbf{x}\in\mathscr{R}} V(\mathbf{x})P(\mathbf{x})}{P\{\mathbf{x}:\mathbf{x}\in\mathscr{R}\}} \tag{39}$$

The following estimator of Q from the sample is used.

$$\hat{Q} = \frac{1}{N}\sum_{i=1}^{N} V[\mathbf{f}(\mathbf{x}^i)] \tag{40}$$

Let EUD^m be the value of the EUD index at the termination of the decomposition phase. The following estimator for EUD is used,

$$\widehat{\mathrm{EUD}} = \mathrm{EUD}^m + (D - \hat{Q})P\{\mathbf{x}:\mathbf{x}\in\mathscr{R}\} \tag{41}$$

V. The optimal mix

There are two outstanding issues that must be dealt with in the composite decomposition-Monte-Carlo method: when to terminate the decomposition phase and what the size of the sample in the Monte-Carlo phase should be.

The resolution of these questions can be expressed in terms of the following two parameters:

α = the threshold probability of an unclassified subset for decomposition, i.e. no further decomposition will be carried out for any unclassified set $\mathscr{S}$ (a $\mathscr{U}_k$ or $\mathscr{W}_k$) whenever $P\{\mathscr{S}\} < \alpha$

N = the number of states in the random sample of the Monte-Carlo phase.

The selection of the parameters α and N is based on two criteria:

- minimal total computation time,
- guaranteed accuracy of the results.

The total computation time is the sum of the computation time spent in the decomposition phase T_d and the Monte-Carlo phase T_m, of the combined method. The accuracy of the results can be measured in terms of the variance $\mathrm{var}\{\underset{\sim}{\hat{r}}\}$ of the estimator $\underset{\sim}{\hat{r}}$. Thus in a statistical sense, the accuracy of the results is guaranteed by imposing the constraint that $\mathrm{var}\{\underset{\sim}{\hat{r}}\}$ be no more than a specified value σ^2. Therefore, an optimization problem may be formulated for the selection of the parameters α and N:

$$\min_{\alpha,N}\ (T_d + T_m)$$

subject to

$$\mathrm{var}\{\underset{\sim}{\hat{r}}\} \leqslant \sigma^2 \tag{42}$$

The variance $\mathrm{var}\{\underset{\sim}{\hat{r}}\}$ is bounded by the expression in equation (38) involving $(r^M - r^m)$, the length of the uncertainty interval of the reliability index r upon the termination of the decomposition phase. Let

$$L = r^M - r^m \tag{43}$$

We may replace $\mathrm{var}\{\underset{\sim}{\hat{r}}\}$ in equation (42) by its upper bound as given in equation (38), resulting in a more conservative solution (in terms of the accuracy requirement). Thus

$$\min_{\alpha,N}\ (T_d + T_m)$$

so that

$$\frac{L^2}{4N} = \sigma^2$$

In order to solve the foregoing optimization problem, L, T_d and T_m need to be expressed in terms of the parameters α and N. L, T_d and T_m vary as a function of α and N in a complex manner. A number of factors, such as the computer hardware and software, come into play. This complexity prevents the rigorous derivation of analytic expressions for the functional relations. Experimental results are used in conjunction with intuitive reasoning to suggest analytic forms of the expressions and to determine the coefficients in these expressions by data fitting. Intuitively, it is reasonable to expect that the smaller the threshold probability α, the shorter the uncertainty interval L. Based on the experimental results shown in Figure 4A, it is assumed that

$$L = a\alpha^b \tag{44}$$

Similarly, it is expected that the smaller α is, the longer the computation time T_d is. A simple exponential expression is used to fit the experimental data shown in Figure 4B. It is assumed that

$$T_d = C\alpha^d \tag{45}$$

Furthermore, the computation time T_m should be proportional to the number of states N in the random sample. The experimental results suggest that the relation is almost linear. Therefore, it is assumed that

$$T_m = hN \tag{46}$$

As indicated in Figures 4A and 4B, it is assumed that the parameters b and d are independent of the system demand

D, whereas a and c are functions of D. While b and d are expected to be system dependent, experience indicates that the variation between systems is rather slight.

The constrained optimization problem is written here as

$$\min_{\alpha, N} (T_d + T_m)$$

subject to

$$\frac{L^2}{4N} = \sigma^2$$

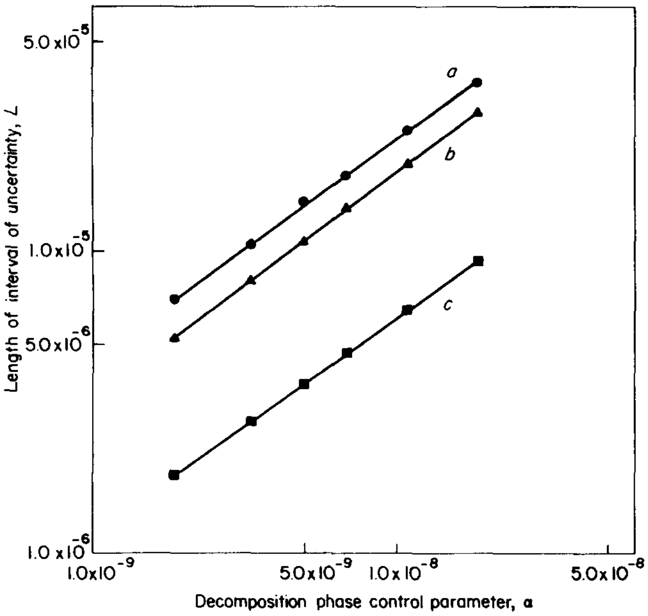

Figure 4A. Post decomposition phase interval of uncertainty L as a function of the control parameter α; (a) load scenario 1, (b) load scenario 2, (c) load scenario 3

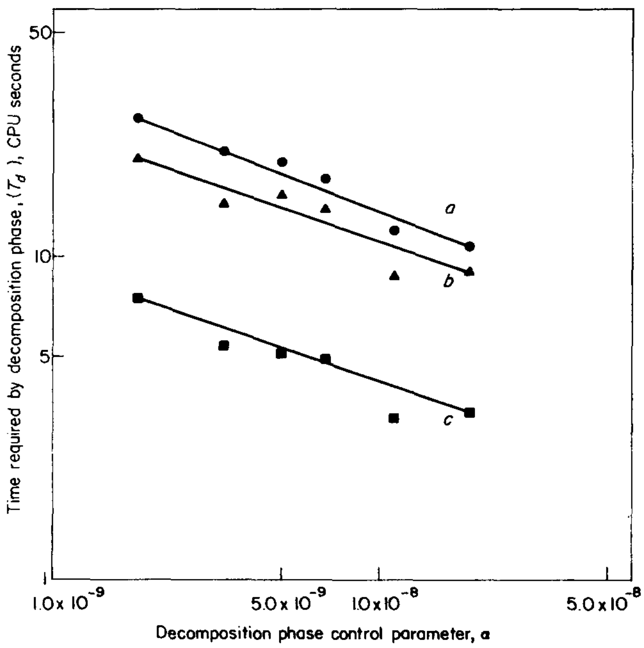

Figure 4B. Computation time of the decomposition phase as a function of the control parameter α; (a) load scenario 1, (b) load scenario 2, (c) load scenario 3

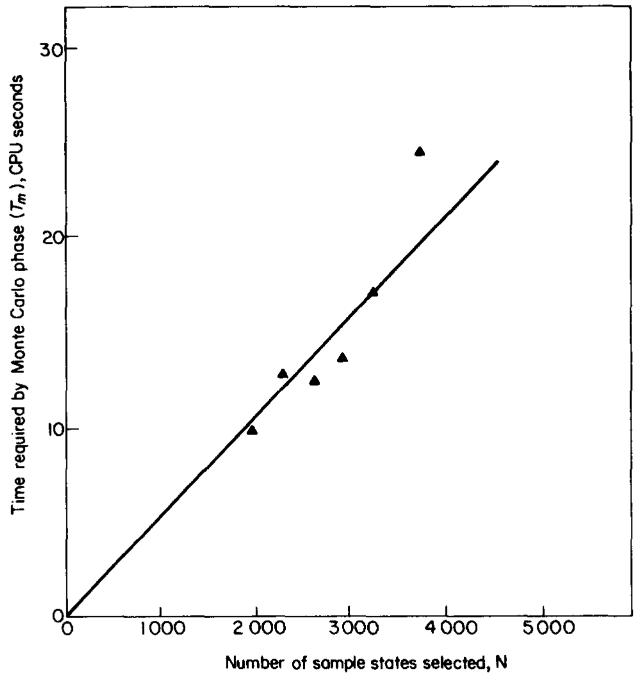

Figure 4C. Computation time of the Monte-Carlo phase as a function of the number of samples selected

where L, T_d, T_m are related to α, N through equations (43)-(45). The optimal solution of this problem is

$$\alpha = \left(-\frac{a^2 hb}{2\sigma^2 cd}\right)^{1/(d-2b)} \tag{47}$$

$$N = \frac{a^2}{4\sigma^2}\left(-\frac{a^2 hb}{2\sigma^2 cd}\right)^{2b/(d-2b)} \tag{48}$$

In the actual implementation of the method, whenever an unclassified set $\mathscr{S}$ is found such that $P\{\mathscr{S}\} < \alpha$, we immediately proceed to the Monte-Carlo phase rather than wait until the entire collection of unclassified subsets is obtained. Therefore, we need to know how many states γ should be selected from the unclassified subset $\mathscr{S}$. We let the number of states selected from $\mathscr{S}$ be proportional to the probability of the subset $\mathscr{S}$, i.e.

$$\gamma = \frac{N}{L} P\{\mathscr{S}\} \tag{49}$$

The optimal value of γ can be obtained by substituting equations (43), (47) and (48) into equation (49):

$$\gamma = P\{\mathscr{S}\}\frac{a}{4\sigma^2}\left(-\frac{a^2 hb}{2\sigma^2 cd}\right)^{b/(d-2b)} \tag{50}$$

VI. Implementation

The decomposition-Monte-Carlo approach has been incorporated into a production-grade program called Remain (Reliability Evaluation of Multiarea Interconnection). The software package includes an effective implementation of the Ford-Fulkerson maximal-flow algorithm[6]. Remain is capable of handling systems without any restrictions on the network topology. The number of areas and the number of states in each area generation and each tieline capacity distribution are only limited by the burdens they impose on the computational effort. The results for the seven-area system presented in the next section give an indication of typical computing times with this program.

Remain has the capability of studying multiarea reliability over extended periods. The study period is subdivided into a number of subperiods defined by the events determined by a discrete event simulator. The events simulated include new units coming online, old units retiring, the beginning and end of maintenance periods and changes in the generation and intertie capacity random variables' distributions caused by seasonal factors or by policy decisions. In the subperiods defined by two successive events, the probability

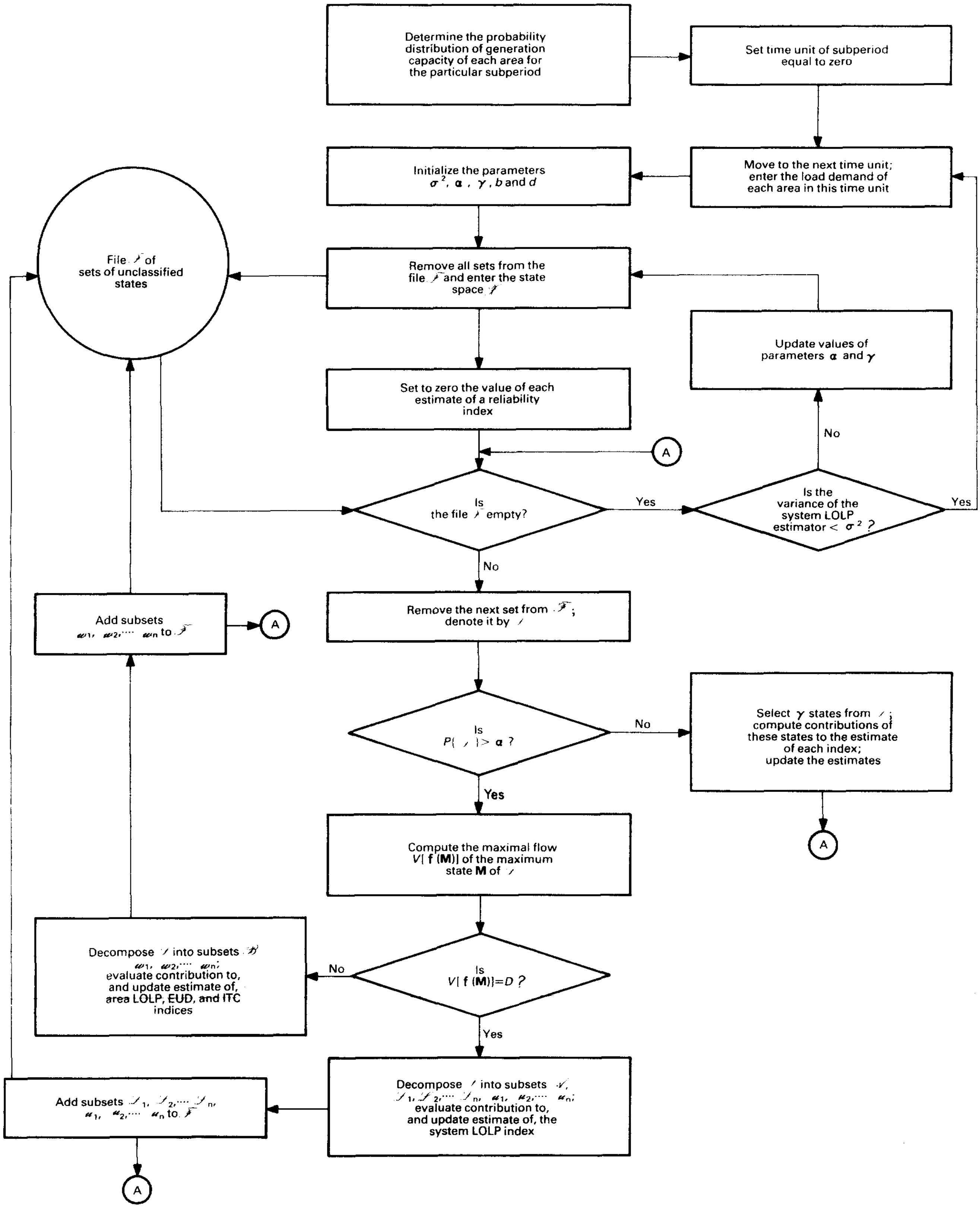

Figure 5. Flowchart of multiarea reliability evaluation program Remain

distributions of the generation capacity and tieline capacity random variables are fixed. These subperiods are further subdivided into shorter time units to account for changing load patterns in the areas and load diversity. For each of these time units, each area has a deterministic (fixed) load. The solution scheme evaluates the reliability indices for this set of fixed loads. The yearly values of the reliability indices are computed by weighted-averaging the values of these indices for each time unit with the weights being the ratio of the duration of the time unit to the total time duration.

A flowchart of Remain is presented in Figure 5. The following initial values are set: $\alpha = \gamma = 10^{-5}$, $b = 0.4$ and $d = -0.6$. The authors' experience shows that for only a small number of peak load demand time units, α and γ need to be recalculated. In actual implementation, a depth-first search scheme[9] is used so that the decomposition phase and the Monte-Carlo phase are completed along each branch of the search tree. For simplicity, it is assumed that, for each state, the minimal cut is unique so that $\mathcal{N}_s^*(\mathbf{x}) = \mathcal{N}_s\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$, $\mathcal{N}_t^*(\mathbf{x}) = \mathcal{N}_t\{\mathscr{C}[\mathbf{f}(\mathbf{x})]\}$, and the Ford-Fulkerson algorithm needs to be run only once.

VII. Application example

An important application of Remain is the study of existing power pools, in particular for the planning of tieline capacity. To illustrate the application of Remain to the study of interconnection enhancement and the performance of Remain, the reliability of a seven-area power system is evaluted as a function of intertie capacity.

The system configuration is shown in Figure 6. This system models an interconnection of four utilities which are represented by areas A, B, C and D. The remaining three areas E, F and G have no native load demand. Each of these areas represents generation resources that are jointly owned by two of the utilities. Table 1 gives the generation levels and associated probabilities for each of the seven areas. The interconnection data giving the tieline capacities and associated probabilities is presented in Table 2. In this

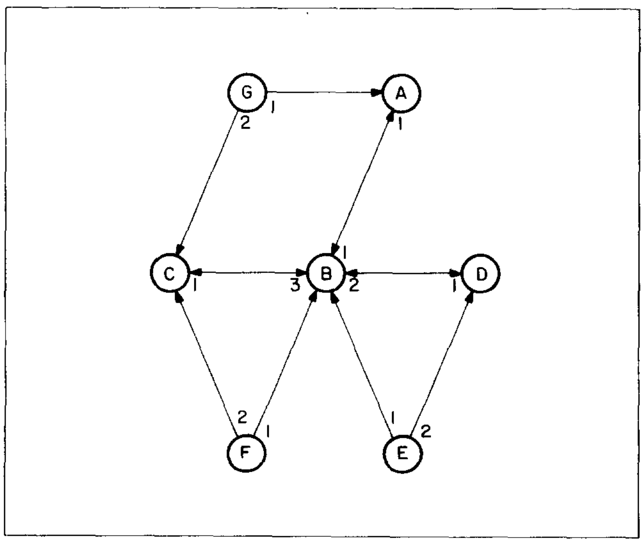

Figure 6. Seven-area power system configuration for the application example

example, for the sake of simplicity, all unidirectional ties are represented by deterministic variables and all bidirectional ties are represented by random variables. In Figure 6, the numbers on the directed branches leaving an area give the priority associated with assisting the neighbouring areas to which the area is directly connected. The ties leaving each area are numbered in decreasing priority order; when an area has excess capacity, the area connected by the tie labelled 1 receives assistance first. The assistance priority lists are needed for evaluating the area LOLP values under the NLLS policy.

Table 3 gives the area loads for the 11 hours corresponding to the 11 largest monthly peaks of the total pool-wide load for the planning year considered in this example. Consider the hour in which the annual peak system load occurs. The generation data in Table 1 is for the period to which this hour belongs. The reliability indices for this hour are evaluated and presented in Tables 4 and 5. The dramatic improvement in reliability brought about by interconnecting the four utilities is seen by comparing the reliability indices for the isolation policy with those of the interconnected systems under the LLS and NLLS policies.

It is clear from the data in Table 4 that most of the unreliability of the interconnected system is due to the high LOLP of area D (twice as severe as that of any other area). The values of the ITC index indicate the high probability that the two tielines into area D restrict power flow even though excess power is available from the other areas. It is concluded that any attempt to improve the reliability of the pool must concentrate on relief for area D. Either additional native generation capacity must be added, or the tieline capacity into area D must be expanded by increasing existing ties or establishing new ties. The possibility will now be considered of expanding the tie from area B to area D from 200 MW to 600 MW with the outage probability remaining unchanged. All other system parameters remain unchanged. This results in a considerable improvement in the reliability indices. For example, the first line in Table 6 gives the area LOLP values under the NLLS policy and the system LOLP and EUD values with the increased tie capacity. For comparison purposes, the last line in Table 6 gives these indices when the bidirectional ties between areas have unlimited and completely reliable capacity. All other system parameters remain unchanged. This case yields the maximum benefit that the system can derive from interconnection. Further improvements in system reliability can only come from increases in the generation capacity of the system.

The CPU time required on an IBM 3033 system for evaluating the reliability indices for the annual peak hour is given in Table 7. Less than 15% of this time was spent on determining maximum flows. For purposes of comparison, the times for exhaustive enumeration and pure decomposition are also presented. The computational times and accuracies of the ten other hours studied are given in Table 3. Remain has been tested with other multiarea power systems. The computation time is not just a function of the system dimension. As clearly indicated in Table 3, it depends very much on the systems under study and the load demands.

In extensive testing of Remain, it was observed that the contributions of relatively few unreliable hours overshadow

Table 1. Probability densities of area generation random variables

Generation capacity level, MW	Probability	Generation capacity level, MW	Probability
Area A			
21 000	0.100 000	4 700	0.012 500
20 500	0.020 000	4 400	0.002 200
20 000	0.300 000	4 100	0.000 250
19 500	0.200 000	3 900	0.000 045
19 000	0.140 000	3 500	0.000 005
18 500	0.040 000		
18 000	0.010 000	*Area D*	
17 500	0.006 000	3 300	0.280 000
17 000	0.003 500	3 100	0.430 000
16 500	0.000 450	2 900	0.250 000
16 000	0.000 045	2 700	0.025 000
13 500	0.000 005	2 500	0.012 500
		2 300	0.002 200
Area B		2 100	0.000 250
15 500	0.080 000	1 900	0.000 045
15 000	0.150 000	1 700	0.000 005
14 500	0.270 000		
14 000	0.250 000	*Area E*	
13 500	0.160 000	2 200	0.722 500
13 000	0.050 000	2 000	0.170 000
12 500	0.035 000	1 800	0.010 000
12 000	0.004 500	1 100	0.085 000
11 500	0.000 450	900	0.010 000
11 000	0.000 045	0	0.002 500
10 000	0.000 005		
		Area F	
Area C		2 500	0.600 000
6 500	0.080 000	2 000	0.200 000
6 200	0.200 000	1 500	0.150 000
5 900	0.300 000	1 000	0.049 500
5 600	0.230 000	0	0.000 500
5 300	0.150 000	*Area G*	
5 000	0.025 000	3 900	1.000 000

Table 2. Intertie capacities

Probability densities of bidirectional intertie capacity random variables

Between Areas A and B		Between Areas B and C		Between Areas B and D	
Intertie capacity, MW	Probability	Intertie capacity, MW	Probability	Intertie capacity, MW	Probability
2 000	0.980 1	1 000	0.999 9	200	0.999 9
1 000	0.019 8	0	0.000 1	0	0.001
0	0.000 1				

Unidirectional intertie capacities, MW

From Area E to B	From Area E to D	From Area F to B	From Area F to C	From Area G to A	From Area G to C
1 750	450	1 500	900	2 500	1 400

Table 3. Area loads, system LOLP and computation times for the planning period

Hour	Loads for Area, MW				System LOLP evaluation		CPU time for evaluation of all indices, s
	A	B	C	D	System LOLP (10^{-6})	Standard deviation (10^{-6})	
1	19 550	15 000	6 650	2 850	347	0.180	30.41
2	19 450	14 700	6 600	2 800	343	0.160	23.64
3	18 400	14 400	6 400	2 700	61	0.090	8.04
4	17 500	14 200	6 060	2 600	56	0.065	3.90
5	16 550	13 500	5 850	2 500	6	0.037	0.97
6	16 450	13 450	5 500	2 400	6	0.033	0.71
7	15 800	13 200	5 300	2 300	0.12	0.020	0.21
8	15 700	12 400	5 100	2 200	0.13	0.016	0.13
9	15 600	12 000	5 000	2 100	0.01	0.010	0.07
10	15 400	11 500	4 900	2 000	0.01	0.008	0.05
11	14 900	11 000	4 850	1 800	0.00	0.006	0.02

Table 4. Reliability indices for the annual peak system load hour

Interconnection policy	System		Area LOLP			
	LOLP	EUD, MW	Area A	Area B	Area C	Area D
Isolation	1.000 000	1 846.991	0.400 000	0.770 000	1.000 000	0.040 000
LLS	0.000 347	14.087	0.000 009	0.000 003	0.000 003	0.000 338
NLLS	0.000 347	14.087	0.000 008	0.000 001	0.000 001	0.000 337

Table 5. ITC index for the annual peak system load hour

From Area	To Area				
	A	B	C	D	System
A	0.000 000	0.000 000	0.000 001	0.000 337	0.000 337
B	0.000 006	0.000 000	0.000 000	0.000 337	0.000 343
C	0.000 006	0.000 000	0.000 000	0.000 337	0.000 343
D	0.000 008	0.000 003	0.000 002	0.000 000	0.000 009
E	0.000 007	0.000 001	0.000 001	0.000 299	0.000 307
F	0.000 006	0.000 000	0.000 000	0.000 337	0.000 343
G	0.000 006	0.000 001	0.000 001	0.000 337	0.000 344

Table 6. Impact of improved tie capacities on reliability

Case	System		Area LOLP under the NLLS policy			
	LOLP	EUD, MW	Area A	Area B	Area C	Area D
Increased tie capacity	0.000 015	0.406	0.000 007	0.000 001	0.000 001	0.000 006
Unlimited tie capacity	0.000 003	0.001	0.000 002	0.000 001	0.000 000	0.000 000

those of the remaining, generally reliable, hours. This is borne out by the data presented in Table 3. Note that the system LOLP of hour 10 is five times smaller than that in the annual peak hour. Data for hour 12 is not presented since the system LOLP is effectively 0.

VIII. Conclusion

A computationally efficient scheme has been presented for evaluating the reliability of multiarea power system interconnections. Also, a newly defined reliability index has been described that is used in planning enhancements of

Table 7. Computational effort required for the annual peak system load hour

Approach	Number of maximal flows	CPU time, s	Standard deviation
Enumeration	4 704 480	4 704*	(Exact)
Decomposition	22 700	82	0.000 000 2
Proposed method	17 480	30	0.000 000 2

the reliability of interconnections. Numerical results have been presented, illustrating the performance of this new analysis on a system of practical interest.

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