SYNCHRONOUS MACHINE AND TORSIONAL DYNAMICS SIMULATION IN THE COMPUTATION OF ELECTROMAGNETIC TRANSIENTS

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ABSTRACT

This paper presents the development of a new time domain simulation program for the computation of machine *and* network transients over a wide frequency spectrum. The program uses the extensive capability for the detailed representation of, and the computation of electromagnetic transients on, the power system network available in the BPA Electromagnetic Transients Program (EMTP). The model of the synchronous generator, in which both the electrical and the mechanical shaft torsional dynamics are represented, possesses a large degree of generality. The model is interfaced with the EMTP network and a newly developed and computationally efficient numerical integration scheme is used to compute the machine transients. The option for representing arbitrary excitation systems has been implemented. The program's capability to simulate a three-phase network with an arbitrary number of synchronous machines – generators or motors – and their auxiliary control equipment, allows the evaluation of network – machine – control systems interactions. The application of the program to investigate the subsynchronous resonance phenomenon in a realistic-sized system is given. Other typical applications include transmission line reclosure, independent pole switching, load rejection or unit tripping, loss of synchronism and multi-machine interaction problems. The program, in use for some time by a number of utilities, has been incorporated as a standard feature of the EMTP.

INTRODUCTION

For several years, western utilities have been confronted with a number of critical problems involving large synchronous machine – power system network interaction. Two examples of particular concern are the potentially hazardous torques on machines, caused by EHV transmission line reclosure, and the *subsynchronous resonance* (SSR) phenomenon that may occur in series compensated transmission systems due to the so-called *torsional interaction* between the mechanical turbine-generator shaft system and the electrical system. The two SSR incidents of 1970 and 1971 that resulted in shaft failures at the Mohave plant in Southern Nevada have been discussed at length in the literature [6]. In the wake of these incidents, utilities recognized the severe effects that SSR may cause and realized the need to develop analytical and computational tools to investigate such effects. Particularly pressing was the need to develop a practical and economical simulation tool to study in detail the transient behavior of the synchronous generators *and* the transmission network in the time range of interest [Fig. 1]. While good digital simulation packages were available for the study of the very fast electromagnetic transients on transmission networks (e.g. switching surge or transient recovery voltage studies) and of the slower transient behavior for transient stability [Fig. 1], no digital program in the public domain was available to utilities for the simulation of machine-network dynamics.

In 1974, a joint effort was initiated between PG&E and SCE to develop a program for simulating the transient interactions of the network, machines and their associated control equipment over a relatively wide frequency spectrum. To minimize development time and costs, it was decided to make use of the extensive capability for the detailed representation of the power system network available in the ElectroMagnetic Transients Program (EMTP). This is a widely used state-of-the-art digital transients program developed and maintained by BPA [1].

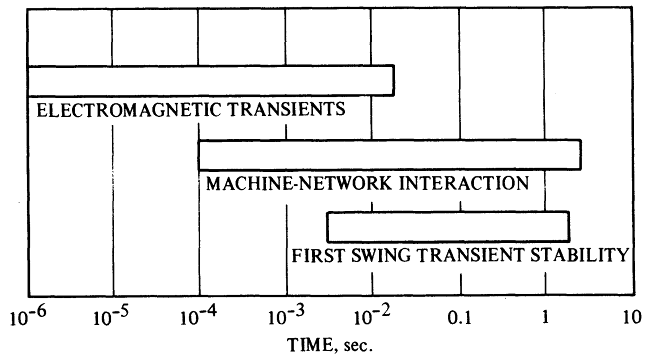


Fig. 1 Approximate time ranges of interest of certain transient phenomena

In order to reproduce the various phenomena in the frequency range of interest a sufficiently detailed model of a synchronous generator is required. To satisfy this and other requirements, the model adopted in this work:

(1) has sufficient generality to represent virtually any type of synchronous machinery
(2) utilizes the standard electrical machine data (ANSI section C42.10) supplied by manufacturers and the standard mechanical data used in SSR studies [6]
(3) incorporates as much detail as the available data permits, e.g., for the electrical system, two rotor circuits per axis
(4) allows the detailed evaluation of the *bilateral coupling* between the mechanical torsional system and the electrical system.*

The representation of machine saturation and governor-prime mover action is neglected, as these effects are of secondary importance for the program applications of interest. On the other hand, the option to represent user-defined excitation systems in as much detail as desired was implemented. The model in the program uses the rotor based *d,q,o* reference frame with all variables normalized. This is compatible with the form of the standard data and general industry practice. The per unit system adopted is described in Appendix B.

An important aspect of this work was the development of a new numerical integration method for the simulation of machine-control systems part of the power system. This scheme, based on the class of algorithm in [5], is computationally efficient and has good accuracy.

The model and program we developed are applicable to a broad range of machine-network interaction problems. Typical examples are the computation of the torque amplification caused by the torsional interaction in SSR studies, transmission line reclosure, independent pole switching, high-speed automatic line reclosure, load rejection or unit tripping, loss of synchronism, multi-machine interaction problems, and the study of various unbalanced system conditions.

In 1975, after thorough testing, the program was made available on an experimental basis to a number of utilities. After additional testing, the debugged version of the program has been incorporated as a standard feature of the EMTP in 1976. The results obtained give good agreement with field experience and other simulation programs. Ref. [7] compares the performance of several programs, including this one, on a benchmark test case. Since the inception of this project, other efforts to develop similar programs have been reported in the literature [3], [4].

*This is in contrast to some simpler models which consider only *unilateral coupling*: the feedback effect of the rotor oscillations on the electrical system is ignored with only the effect of the electromagnetic torque on the mechanical rotor motion considered. The simpler models are considerably less accurate especially for machine-network systems with relatively low overall damping.

F 77 529-1. A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE PES Summer Meeting, Mexico City, Mex., July 17-22, 1977. Manuscript submitted February 4, 1977; made available for printing March 30, 1977.

This paper reports on the results of our work and treats in detail the various aspects of the project – the formulation of the mathematical model, the interfacing of the model with the EMTP, and the development of the numerical solution scheme. A numerical example is included to illustrate a typical application.

INTERFACING THE SYNCHRONOUS MACHINE WITH THE EMTP

It is clear from the system equations (A2), (A6)–(A8) in Appendix A that the synchronous generator model is a nonlinear dynamical system. For purposes of EMTP interface, the synchronous generator represented by this model is treated as a three-phase network branch with nonlinear and time varying parameters that connects the three-phase generator terminal bus to ground. In the EMTP, branches with nonlinear and/or time-varying elements are handled by making use of the compensation theorem [2] to separate these elements from the linear part of the network [1]. For a synchronous machine we use a three-phase generalization of the single-phase compensation method outlined in [2].

At a synchronous machine three-phase bus of the network, the "injection currents" are the armature currents i_a, i_b, i_c and the bus voltages are the stator voltages v_a, v_b, v_c. At each instant t, the effects of the network on the machine may be replaced by the Thévenin equivalent "seen" by the machine at its terminals:

$$\underline{v}^{abc}(t) = \underline{\tilde{Z}}^{Th}(t)\underline{i}^{abc}(t) + \underline{\tilde{e}}^{Th}(t) \tag{1}$$

where, $\underline{v}^{abc} = [v_a, v_b, v_c]^T$ ($\underline{i}^{abc} = [\underline{i}_a, \underline{i}_b, \underline{i}_c]^T$)is the terminal voltage (current) vector. Without the generator branch the network is characterized [3] by a matrix $\underline{Y}(t)$ and an injection current $\underline{k}(t)$ which determine the nodal voltages $\underline{e}(t)$ by solving

$$\underline{Y}(t)\ \underline{e}(t) = \underline{k}(t) \tag{2}$$

The open circuit vector $\underline{\tilde{e}}^{Th} = [e_a, e_b, e_c]^T$ is obtained from $\underline{e}(t)$ where e_a, e_b, e_c are the components of $\underline{e}$ corresponding to the three phase voltages at the generator bus. The Thévenin equivalent impedance is

$$\underline{\tilde{Z}}^{Th} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \tag{3}$$

By solving repeatedly for $\underline{e}^{\nu}$ in

$$\underline{Y}(t)\,\underline{e}^{\nu} = \underline{k}^{\nu},\ \nu = a, b, c \tag{4}$$

where $\underline{k}^{\nu}$ is a vector with components

$$k_{\mu}^{\nu} = \begin{cases} 1 & \text{if } \mu \text{ corresponds to phase } \nu \text{ current injection into the bus} \\ & \text{at which the generator is connected} \\ 0 & \text{otherwise} \end{cases}$$

we obtain the componets of $\underline{\tilde{Z}}^{th}$ since

$Z_{\mu\nu} = e_{\mu}^{\nu}$ $\quad \nu, \mu = a, b, c$

where e_{μ}^{ν} is the component of $\underline{e}^{\nu}$ corresponding to the phase μ voltage of the generator bus. Note that $\underline{\tilde{Z}}^{Th}(t)$ remains constant as long as the simulation step and the network configuration remain unchanged.

Following the solution of the generator equations, $\underline{i}^{abc}$ is known. The compensation theorem [2] uses superposition to obtain the nodal voltage vector $\hat{\underline{e}}(t)$ of the network with the generator branch included

$$\hat{\underline{e}}(t) = \underline{e}(t) + \left[\underline{e}^{a}(t) \mid \underline{e}^{b}(t) \mid \underline{e}^{c}(t)\right] \underline{i}^{abc}(t) \tag{5}$$

The EMTP is written in such a manner that it can handle more than two branches with nonlinear/time-varying elements only if these branches are separated by distributed parameter lines. Consequently to simulate with the EMTP a multimachine network including possibly other branches having nonlinear/time-varying elements, all nonlinear branches, including the machines, must be separated from one another by distributed parameter transmission lines. The compensation theorem can be applied then to each nonlinear branch [1].

SOLUTION OF THE SYNCHRONOUS MACHINE EQUATIONS

The state equations of the synchronous machine model (s.m.m.) are given by eqs. (A2) and (A16) of Appendix A. The s.m.m. state vector is $[\underline{\theta}^m, \dot{\underline{\theta}}^m, \underline{i}]^T$ with components $\underline{\theta}^m$ ($\dot{\underline{\theta}}^m$), the M-vector of the *mechanical* angular displacements (velocities) of the M shaft-coupled masses, and $\underline{i}$, the vector of machine currents.

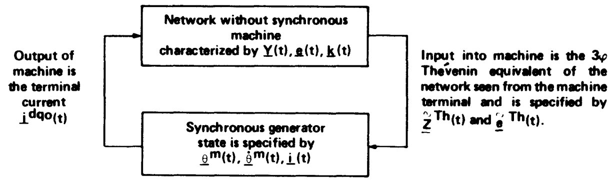


Fig. 2 The network – synchronous generator interface

The input and output of the s.m.m. can be viewed in terms of its interaction with the remainder of the network (see Fig. 2).

The solution of the machine equations consists of integrating the s.m.m. state equations followed by computing the modified network nodal voltages using eq. (5) to account for the effects of the machine on the remainder of the network. The EMTP integrates all the network d.e.s using the trapezoidal rule [1].This second order scheme is numerically stable for any stepsize Δt for a set of stable d.e.s and its accuracy is dependent on the stepsize [5]. Since many of the electromagnetic phenomena of interest are of rather short duration, their realistic simulation requires very small stepsizes, typically 0.1 ms. or shorter, so that the trapezoidal rule obtains good accuracy [1].

Several factors must be considered in the choice of a suitable numerical scheme for the machine d.e.s. Besides those factors discussed in [5], there is the additional consideration that the scheme chosen should solve the machine d.e.s in a manner consistent with the EMTP solution of the remainder of the network. Thus some "synchronization" of the EMTP and s.m.m. stepsizes is necessary. Also, there is little point in choosing a high-order integration scheme since this could lead to serious (numerical) interface errors between the network and the s.m.m. A natural choice is to also use the trapezoidal rule, with the same stepsize used in the EMTP, for the set of nonlinear machine d.e.s and solve the resultant nonlinear algebraic equation by some iterative method. Such an approach is used in [4]. We have developed an efficient numerical scheme which may be viewed as a *modified trapezoidal rule*. Our solution scheme, which is an extension of the class of algorithms proposed in [5], is derived by making detailed use of the structural properties of the machine equations [5].

We next discuss our solution scheme. For the sake of simplicity in this discussion $v_f(t)$ is assumed constant; the inclusion of exciter and power system stabilizer dynamics is considered in a later section. In addition, for a machine with a rotating exciter, T^e_{exc} is assumed to be held constant at its initial (steady-state) value.

Consider first the mechanical equation (A2). Let us define the state transition matrix

$$\underline{\Phi}(\Delta t) = \left[\begin{array}{c|c} \underline{\Phi}^{11}(\Delta t) & \underline{\Phi}^{12}(\Delta t) \\ \hline \underline{\Phi}^{21}(\Delta t) & \underline{\Phi}^{22}(\Delta t) \end{array}\right] \triangleq \exp\left\{\left[\begin{array}{c|c} \underline{0} & \underline{U}_M \\ \hline -\underline{J}^{-1}\underline{K} & -\underline{J}^{-1}\underline{D} \end{array}\right]\Delta t\right\} \tag{6}$$

with the partitioning being identical to that of the exponentiated matrix. If $\underline{\theta}^m(t)$, $\dot{\underline{\theta}}^m(t)$ are known and the torque vector $\underline{T}(t)$ were known for all values in the interval $[t, t+\Delta t]$ then a well known result in linear system theory states that we may compute $\underline{\theta}^m(t+\Delta t)$, $\dot{\underline{\theta}}^m(t+\Delta t)$ using

$$\left[\begin{array}{c} \underline{\theta}^m(t+\Delta t) \\ \hline \dot{\underline{\theta}}^m(t+\Delta t) \end{array}\right] = \underline{\Phi}(\Delta t) \left[\begin{array}{c} \underline{\theta}^m(t) \\ \hline \dot{\underline{\theta}}^m(t) \end{array}\right] + \int_t^{t+\Delta t} \underline{\Phi}(t+\Delta t-\xi) \left[\begin{array}{c} \underline{0} \\ \hline \underline{J}^{-1} \end{array}\right] \underline{T}[\underline{i}(\xi)]d\xi \tag{7}$$

By approximating the integrand in (7) by a first order polynominal in ξ and integrating, i.e., approximating the integral using the *trapezoidal rule* we obtain

$$\left[\begin{array}{c} \underline{\theta}^m(t+\Delta t) \\ \hline \dot{\underline{\theta}}^m(t+\Delta t) \end{array}\right] = \left[\begin{array}{c|c} \underline{\Phi}^{11}(\Delta t) & \underline{\Phi}^{12}(\Delta t) \\ \hline \underline{\Phi}^{21}(\Delta t) & \underline{\Phi}^{22}(\Delta t) \end{array}\right] \left\{ \left[\begin{array}{c} \underline{\theta}^m(t) \\ \hline \dot{\underline{\theta}}^m(t) \end{array}\right] + \frac{\Delta t}{2}\left[\begin{array}{c} \underline{0} \\ \hline \underline{J}^{-1} \end{array}\right] \underline{T}[\underline{i}(t)] \right\} + \frac{\Delta t}{2}\left[\begin{array}{c} \underline{0} \\ \hline \underline{J}^{-1} \end{array}\right] \underline{T}[\underline{i}(t+\Delta t)] \tag{8}$$

We next consider the electrical state equation (A16). Application of the trapezoidal rule to (A16) results in

$$\underline{i}(t+\Delta t) = q(t) - \frac{\Delta t}{2}\Big\{\Big[\underline{L}^{-1}\underline{R} + \dot{\theta}(t+\Delta t)\underline{L}^{-1}\underline{Q}\,\underline{L} - \underline{L}^{-1}\underline{Z}[\theta(t+\Delta t)]\Big]\cdot \underline{i}(t+\Delta t) - \underline{L}^{-1}\Big[v_f(t+\Delta t)\underline{u}_1 + \underline{e}[\theta(t+\Delta t)]\Big]\Big\} \tag{9}$$

where

$$\underline{q}(t) = \underline{i}(t) - \frac{\Delta t}{2}\Big\{\Big[\underline{L}^{-1}\underline{R} + \dot{\theta}(t)\underline{L}^{-1}\underline{Q}\,\underline{L} - \underline{L}^{-1}\underline{Z}[\theta(t)]\Big]\underline{i}(t) - \underline{L}^{-1}\Big[v_f(t)\underline{u}_1 + \underline{e}[\theta(t)]\Big]\Big\} \tag{10}$$

Eqs. (8) – (10) form the basis of our scheme. Suppose that we have solved the network *and* machine equations at time t. Then, all the network variables at time t, $\underline{\theta}^m(t)$, $\dot{\underline{\theta}}^m(t)$, $\underline{i}(t)$ and consequently $\underline{q}(t)$ are known. To compute the network and machine variables at time $t+\Delta t$ we first solve the network matrix eq. (2) at $t+\Delta t$ and then obtain the s.m.m. inputs $\underline{\tilde{Z}}^{Th}(t+\Delta t)$, $\underline{\tilde{e}}^{Th}(t+\Delta t)$. We use the following

Algorithm: To solve the machine equations at time $t+\Delta t$

Step 0. The quantities $\underline{\theta}^m(t)$, $\dot{\underline{\theta}}^m(t)$, $\underline{i}(t)$, $\underline{q}(t)$, $\underline{\tilde{Z}}^{Th}(t+\Delta t)$, $\underline{\tilde{e}}^{Th}(t+\Delta t)$ are known.

Step 1. Compute $\underline{\theta}^m(t+\Delta t)$ using (8)

$$\underline{\theta}^m(t+\Delta t) = \underline{\Phi}^{11}(\Delta t)\underline{\theta}^m(t) + \underline{\Phi}^{12}(\Delta t)\Big\{\dot{\underline{\theta}}^m(t) + \frac{\Delta t}{2}\underline{J}^{-1}\underline{T}[\underline{i}(t)]\Big\} \tag{11}$$

Step 2. Convert the generator mass mechanical angle into electrical angle using relation (A5)

$$\theta(t+\Delta t) = n_p\,\theta^m_{gen}(t+\Delta t) \tag{12}$$

Compute the Thévenin quantities in the *d,q,o* reference frame using the Park transformation $\underline{P}[\theta(t+\Delta t)]$ defined in (A4) and relations (A14), (A15):

$$\underline{Z}[\theta(t+\Delta t)] = \underline{S}\,\underline{P}[\theta(t+\Delta t)]\underline{\tilde{Z}}^{Th}(t+\Delta t)\underline{P}^T[\theta(t+\Delta t)]\underline{S}^T \tag{13}$$

$$\underline{e}[\theta(t+\Delta t)] = \underline{S}\,\underline{P}[\theta(t+\Delta t)]\underline{\tilde{e}}^{Th}(t+\Delta t) \tag{14}$$

Step 3. Use linear extrapolation on the values of $\dot{\theta}$ at t and $t-\Delta t$ to predict the value of $\dot{\theta}$ at $t+\Delta t$

$$\dot{\theta}^{(P)}(t+\Delta t) = 2\dot{\theta}(t) - \dot{\theta}(t-\Delta t) \tag{15}$$

Step 4. Compute $\underline{i}(t+\Delta t)$ by rearranging (9) and solving

$$\Big[\underline{U} + \frac{\Delta t}{2}\underline{L}^{-1}\Big\{\underline{R} + \dot{\theta}^{(P)}(t+\Delta t)\underline{Q}\,\underline{L} - \underline{Z}[\theta(t+\Delta t)]\Big\}\Big]\underline{i}(t+\Delta t) = \underline{q}(t) + \frac{\Delta t}{2}\underline{L}^{-1}\Big\{v_f(t+\Delta t)\underline{u}_1 + \underline{e}[\theta(t+\Delta t)]\Big\} \tag{16}$$

and calculate $T^e_{gen}(t+\Delta t)$ using (A7):

$$T^e_{gen}(t+\Delta t) = \frac{n_p}{3}\,\underline{i}^T(t+\Delta t)\underline{L}^T\underline{Q}^T\underline{i}(t+\Delta t) \tag{17}$$

Step 5. Compute $\dot{\theta}^m(t+\Delta t)$ using (8)

$$\dot{\underline{\theta}}^m(t+\Delta t) = \underline{\Phi}^{21}(\Delta t)\underline{\theta}^m(t) + \underline{\Phi}^{22}(\Delta t)\Big\{\dot{\underline{\theta}}^m(t) + \frac{\Delta t}{2}\underline{J}^{-1}\underline{T}[\underline{i}(t)]\Big\} + \frac{\Delta t}{2}\underline{J}^{-1}\underline{T}[\underline{i}(t+\Delta t)] \tag{18}$$

Step 6. If the electrical speed

$$\dot{\theta}(t+\Delta t) = n_p\cdot\dot{\theta}^m_{gen}(t+\Delta t) \tag{19}$$

satisfies

$$\left|\dot{\theta}(t+\Delta t) - \dot{\theta}^{(P)}(t+\Delta t)\right| < \varepsilon \tag{20}$$

where ε is some specified tolerance go to *Step 7*; otherwise replace $\dot{\theta}^{(P)}(t+\Delta t)$ with $\dot{\theta}(t+\Delta t)$ and return to *Step 4.*

Step 7. Adjust the nodal network voltages using eqs. (5), (A12) and the inverse Park transformation:

$$\hat{\underline{e}}(t+\Delta t) = \underline{e}(t+\Delta t) + \Big[\underline{e}^a(t+\Delta t) \,\vdots\, \underline{e}^b(t+\Delta t) \,\vdots\, \underline{e}^c(t+\Delta t)\Big]\underline{P}^T[\underline{\theta}(t+\Delta t)]\underline{S}^T\underline{i}(t+\Delta t) \tag{21}$$

COMMENTS ON THE NUMERICAL SCHEME

The algorithm presented has a number of features that make it computationally attractive and efficient. The split-form representation of the nonlinear s.m.m. state equations (A2), (A16) and the semilinearity property of this representation [5] are used advantageously in this scheme. For the mechanical part, an implicit numerical quadrature formula (the trapezoidal rule) is used; still, the system structure is such as to allow the computation of $\underline{\theta}^m(t+\Delta t)$ explicitly using (11). For the electrical part, an implicit backward differentiation formula (the trapezoidal rule) is used to compute $\underline{i}(t+\Delta t)$; however, the semilinearity and the use of the predicted value $\dot{\theta}^{(P)}(t+\Delta t)$, obtain $\underline{i}(t+\Delta t)$ as the solution of the linear system (16) in one iteration. The use of linear extrapolation for $\dot{\theta}^{(P)}(t+\Delta t)$ is reasonable since the mechanical system changes very little over the typical stepsize ($\Delta t \leq .1$ ms) used in the EMTP. The scheme computes $\dot{\underline{\theta}}^m(t+\Delta t)$ explicitly using $\underline{T}(t+\Delta t)$ computed from $\underline{i}(t+\Delta t)$ and compares the computed and predicted values of $\dot{\theta}(t+\Delta t)$ to see whether another $\underline{i}$, $\dot{\underline{\theta}}^m$ iteration is required. Results of our extensive numerical tests indicate that usually one, and at most two additional iterations are needed.

The trapezoidal rule is a second order scheme for both numerical quadrature and backward differentiation so that our algorithm, like the numerical scheme used for the network [1], is a second order implicit scheme; in addition, the same stepsize Δt is used for both the network and machine parts. This is in contrast to the scheme used in [3] where an explicit method of lower order and with smaller stepsize than that of the network is used to integrate the machine equations. A detailed discussion on the suitability of implicit and explicit integration schemes for a stiff set of d.e.s such as the electrical state equations (A16) is presented in [5].

We note that some computational simplification is possible. The Thévenin impedance matrix $\underline{\tilde{Z}}^{Th}$ is constant between two successive switchings in the network. Consequently, $\underline{\tilde{Z}}^{Th}$ need be computed only at each switching point before entering the time-step loop. The state transition matrix $\underline{\Phi}(\Delta t)$ is constant, if, as in this program, a fixed stepsize Δt is used and so $\underline{\Phi}(\Delta t)$ is computed only once outside the time-step loop.

DETERMINATION OF INITIAL CONDITIONS

For purposes of network initialization, each machine is represented as a balanced three-phase sinusoidal voltage source (a conventional – positive sequence – load flow may be used to determine voltage magnitude and phase angle at each generator bus) and in the network, all the nonlinearities are neglected but unbalance in the phases is considered. Once the voltage phasor at each three-phase machine bus is specified, the sinusoidal steady-state solution feature in the EMTP is used to determine the initial conditions for all energy-storage elements of the network including the initial, possibly unbalanced, three-phase network currents that correspond to the machine armature currents. The positive sequence component of the armature currents is then calculated and used as the *a*-phase component of a set of balanced three-phase armature currents. These balanced currents and the balanced armature voltages specified previously are used to determine in the conventional manner the initial values of all the machine electrical states in the *d,q,o* reference frame. The initial values of the electromagnetic torques T^e_{gen} and T^e_{exc} are established from (A7) and (A8).

For the mechanical system, all the steady-state angular velocities $(\dot{\theta}^m_i)^o$ are equal to $1/n_p$ in mechanical per unit, the mechanical synchronous speed. It is clear from Fig. A1 that the sum of the electromagnetic torques and the damping torques (all known) equals in the steady state the sum of the externally applied mechanical torques. For each mass, the *fraction* of the total mechanical torque on the mass is specified in the input data, and hence, the externally applied mechanical torque on each mass is determined. The steady state value $(\theta^{m}_{gen})^{o}$ of the mechanical rotor angle is obtained using the intial value of the electrical rotor angle and (A5). Finally, the remaining (M−1) components of the steady state value of $(\underline{\theta}^{m})^{o}$ are determined by solving any (M−1) equations in (A1).

EXCITATION SYSTEM DYNAMICS

Excitation systems such as the usual voltage regulators, exciters, power system stabilizers or some arbitrary control system providing field excitation to a machine are simulated by means of the newly added TACS (Transient Analysis of Control Systems) feature of the EMTP [8]. The TACS feature simulates the interaction between the control systems, network components and synchronous machine dynamics. The excitation systems are described in the usual block diagram form. The input can be specified to be either a network variable, e.g., bus voltage, current or frequency (available from a built-in frequency *transducer*), or a machine variable, e.g., rotor angle or speed. The field voltage is output and fed back to the machine model equations. TACS solves the control system equations using the trapezoidal rule. At each time step of the simulation, the evaluation of the control variables in TACS follows the computation of electrical network and synchronous machine variables, introducing a delay of one time step in the application of the control system's output signals to the machine or network. A one step delay is also introduced to handle a time-varying electromagnetic torque on a rotating exciter. Inaccuracies introduced by this delay, typically less than 100 μs, are negligible for most practical applications.

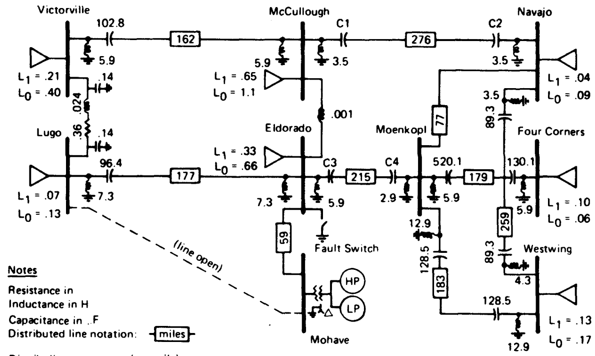
APPLICATION EXAMPLE

An important application of the model and program is to the study of subsynchronous oscillations that may occur in series compensated transmission systems – in particular, the simulation of the torque amplification caused by the so-called *torsional interaction* [6] between the rotating shaft system and the electrical system. To illustrate the performance of the new simulation tool, the behavior of the shaft torques as a function of the series compensation level in the network is investigated on a nine bus system.

A one-line diagram of the system configuration and data used in this example are found in Fig. 3. One cross-compound synchronous generator* is connected at Mohave. The effects of all other machines are modeled as sinusoidal voltage sources and equivalent impedances. The transient disturbance considered is a three-phase fault applied at Eldorado at t = 0 and removed at t=0.06 s. The Eldorado-Lugo and Eldorado-McCullough lines are opened with the first line current zero crossings after t=0.066 s. Two cases varying the series compensation levels in the Navajo-McCullough and Eldorado-Moenkopi lines are examined here for comparison: *Case A*: 35% compensation ($C_1 = C_2 = 92.3\mu F$ and $C_3 = C_4 = 118.2\mu F$) and *Case B*: 47% compensation ($C_1 = C_2 = 69.2\mu F$ and $C_3 = C_4 = 88.7\mu F$).

Several time plots selected to illustrate the results are shown in Fig. 4. The electromagnetic torque on the generator rotor of the HP unit is observed to decay more slowly as a result of the increase in series compensation. This reduction in machine-network damping is a manifestation of the torsional interaction phenomenon resulting from "tuning" the network so that the electrical network natural frequency more closely coincides with the complement of a mechanical shaft torsional frequency [6]. The increase in torsional interaction is further evidenced in the amplification of the shaft torques and the electrical oscillations as shown in the remaining six plots. The amplitude of the pulsating torque on the 3-4 shaft segment (generator-exciter) of the HP unit is amplified more than three times for a relatively small increase in series compensation. The other HP shaft segments 2-3 (turbine-generator) and 1-2 (turbine-turbine, not shown), experience similar but somewhat lesser amplification. The plots of voltage across capacitor C3 indicate the magnification of the subsynchronous oscillations which occur in the network. These results illustrate the sharp tuning characteristics of the bilateral coupling between the torsional modes of the shaft and the natural frequencies of the network, and demonstrate the capability of the simulation program to reproduce these phenomena.

*To increase the versatility of the simulation program, the capability of representing cross-compound machines has been implemented. Such units are modeled as two machines connected in parallel at the same node. The equations of the two paralleled machines are solved simultaneously using a modified form of the model and algorithm. Although the modifications required are extensive and detailed, and, hence, not presented here, the basic form of the model and algorithm remains unchanged.



Distrib. line parameters (per mile):

$R_1 = .0271$, $L_1 = 1.586$ mH, $C_1 = .0188$ μF, $R_0 = .169$, $L_0 = 5.63$ mH, $C_0 = .0118$.

Source equivalent ◁ pos. and zero sequence inductance specified

Mohave transformer: 525/22 kV, wye gnd./delta, magnetizing impedance neglected, leakage inductance and resistance on high side = .144 H and .22 ohm

Machine rating: HP Unit – 483 MVA, 22 kV, 2 pole
LP Unit – 426 MVA, 22 KV, 4 pole

Machine Electrical Data (per unit):

	X_d	X_q	X'_d	X'_q	X''_d	X''_q	R_a	X_l	T'_{do}	T'_{qo}	T''_{do}	T''_{qo}
HP Unit	1.8	1.72	.285	.49	.22	.22	.0027	.16	3.7s	.49s	.032s	.059s
LP Unit	1.76	1.65	.285	.49	.205	.205	.0036	.155	8.4s	.44s	.036s	.069s

Machine Mechanical Data (per unit):

	$J_{turb.1}$	$J_{turb.2}$	$J_{gen.}$	$J_{exc.}$	K12	K23	K34	
HP Unit	151.3	227.2	512.9	25.0	49.4	68.3	5.05	$D_{12} = D_{23} = D_{34} = .04$, $D_1 = D_2 = D_3 = D_4 = 0$
LP Unit	6700	6810	4417	12.1	123	188	1.0	$D_{12} = D_{23} = D_{34} = .024$, $D_1 = D_2 = D_3 = D_4 = 0$

Fig. 3 500 kv transmission network and data

CONCLUDING REMARKS

An efficient and flexible simulation tool, with which a large variety of network – machine – excitation system interaction problems can be studied, has been presented. The synchronous machine model developed, while possessing a large degree of generality, has been designed to be easy to use requiring only the standard data normally provided by manufacturers as input. Some topics related to the model which require further investigation include the representation of machine saturation, the introduction of optional lower and higher order rotor circuit models and the representation of nonlinearities in the torsional shaft model. The model and program have undergone extensive testing by several utilities. The results obtained compare favorably with field experience and results of digital and analog programs of similar capability. The program, incorporating the newly developed TACS feature, provides the utility engineer with a significant addition to his computational capability.

ACKNOWLEDGMENT

The authors wish to express their appreciation to Dr. W. S. Meyer of BPA, Mr. J. E. Alms of SCE and Mr. R. H. Webster of PG&E for their helpful advice and assistance in the various stages of this project.

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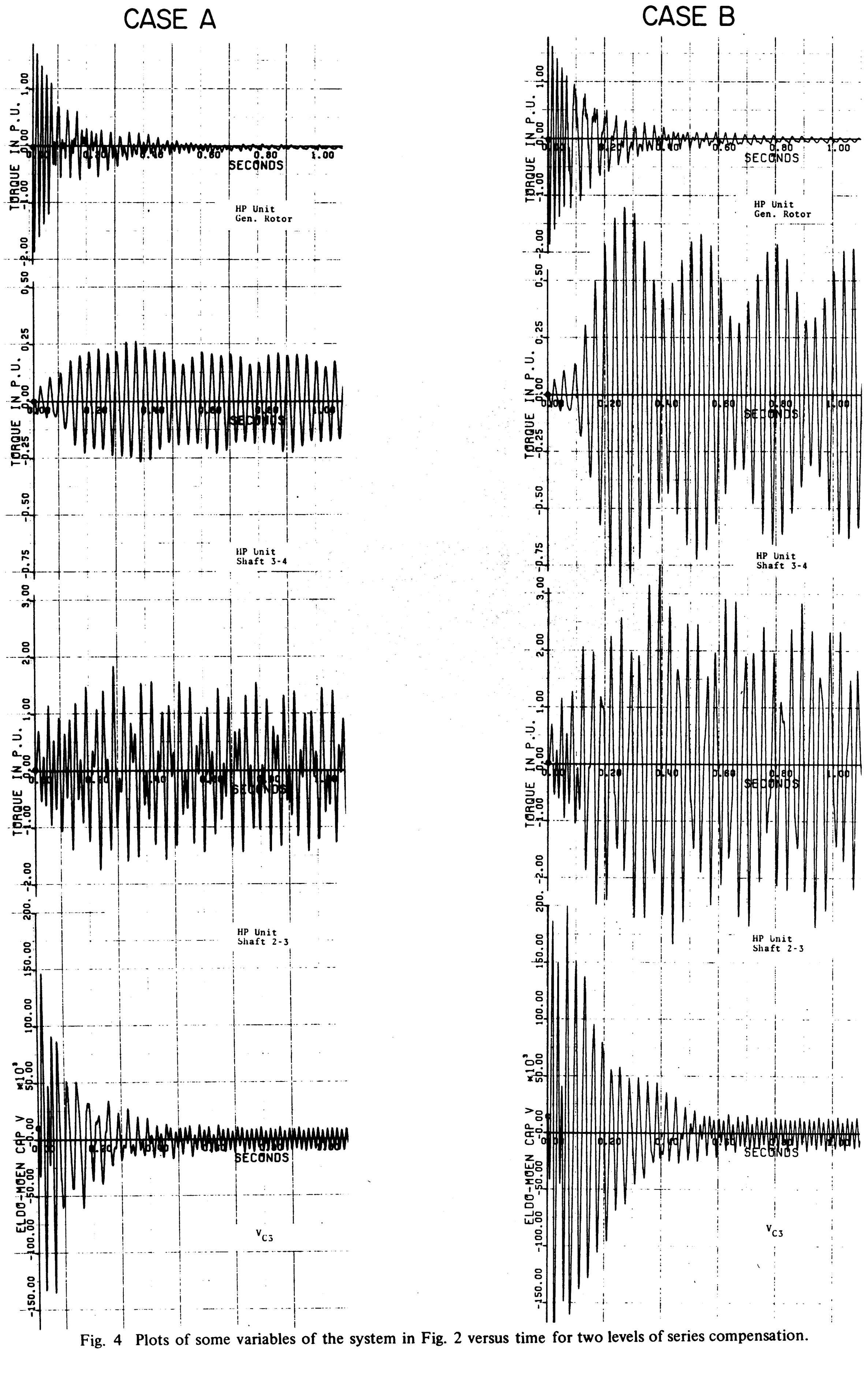

Fig. 4 Plots of some variables of the system in Fig. 2 versus time for two levels of series compensation.

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APPENDIX A: THE SYNCHRONOUS MACHINE MODEL

(i) Model of the Mechanical System

The synchronous machine mechanical system, consisting of various rotating elements coupled together by shafts in tandem, is modeled as a linear multimass-spring-dashpot system. The scope of application of this model is rather wide: it can be used to represent a turbine-generator, a system with a motor driving one or more mechanical loads (e.g. pumps), or any combination of a machine, turbines and mechanical loads on a shaft. In addition, a rotating shaft-driven exciter can be represented.

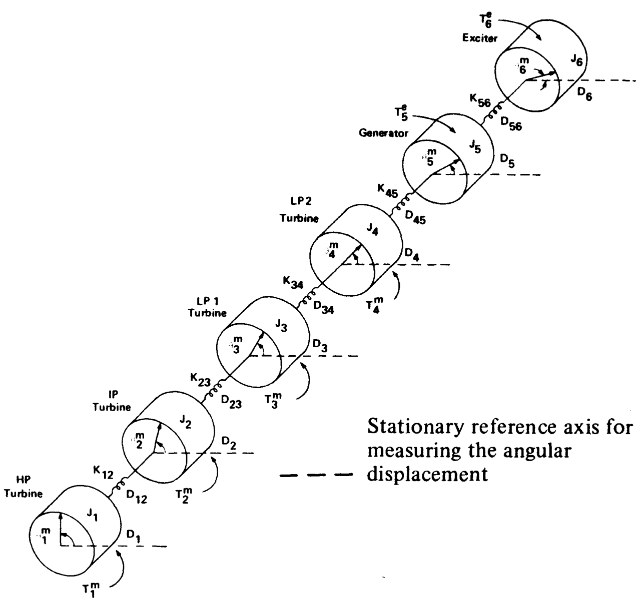

Fig. A1 Mass-spring-dashpot model of a typical turbine-generator unit

A typical turbine-generator is illustrated in Fig. A1. Each major rotating element is modeled as a lumped mass and each shaft segment is modeled as a massless rotational spring with its stiffness given by the spring constant. Viscous damping of each mass and shaft segment is represented.

The equation of motion of mass i of an M-mass rotating system is:

$$J_i\ddot{\theta}_i^m + D_i\dot{\theta}_i^m + D_{i,i-1}(\dot{\theta}_i^m - \dot{\theta}_{i-1}^m) + D_{i,i+1}(\dot{\theta}_i^m - \dot{\theta}_{i+1}^m) + K_{i,i-1}(\theta_i^m - \theta_{i-1}^m) + K_{i,i+1}(\theta_i^m - \theta_{i+1}^m) = T_i^m - T_i^e \quad \text{(A1)}$$

where, for mass i, J_i is the moment of inertia, D_i is the self-damping coefficient, θ_i^m is the *mechanical* angle of rotation measured with respect to a fixed reference, T_i^m is the applied mechanical torque and T_i^e is the electromagnetic torque; for masses i and j, $i \neq j$, $D_{i,j} = D_{j,i}$ and $K_{i,j} = K_{j,i}$ are the mutual damping coefficient and the spring constant of the shaft connecting the two masses, respectively. We assume that no governor action is represented so that the T_i^m are constant and equal to their initial (steady-state) values. The T_i^e are nonzero only for the generator and the exciter masses.

The equations of motion (A1) in matrix form become

$$\begin{bmatrix} \dot{\underline{\theta}}^m(t) \\ \ddot{\underline{\theta}}^m(t) \end{bmatrix} = \left[\begin{array}{c|c} \underline{0} & \underline{U}_M \\ \hline -\underline{J}^{-1}\underline{K} & -\underline{J}^{-1}\underline{D} \end{array}\right] \begin{bmatrix} \underline{\theta}^m(t) \\ \dot{\underline{\theta}}^m(t) \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \underline{J}^{-1} \end{bmatrix} \underline{T}(t) \quad \text{(A2)}$$

Here $\underline{J} = \text{diag}\{J_i: i = 1,2,\ldots,M\}$, $\underline{D}$ and $\underline{K}$ are the tridiagonal matrices defined by

$$\underline{K} = \begin{bmatrix} \hat{K}_1 & -K_{12} & & & \\ -K_{12} & \hat{K}_2 & -K_{23} & & \\ & -K_{23} & \ddots & & \\ & & & \hat{K}_{M-1} & -K_{M-1,M} \\ & & & -K_{M-1,M} & \hat{K}_M \end{bmatrix}$$

$$\hat{K}_1 = K_{12} \qquad \hat{K}_M = K_{M-1,M} \quad \text{(A3)}$$
$$\hat{K}_i = K_{i-1,i} + K_{i,i+1} \qquad i=2,\ldots,M-1$$

$$\underline{D} = \begin{bmatrix} \hat{D}_1 & -D_{12} & & & \\ -D_{12} & \hat{D}_2 & -D_{23} & & \\ & -D_{23} & \ddots & & \\ & & & \hat{D}_{M-1} & -D_{M-1,M} \\ & & & -D_{M-1,M} & \hat{D}_M \end{bmatrix}$$

$$\hat{D}_1 = D_{12} + D_1 \qquad \hat{D}_M = D_{M-1,M} + D_M$$
$$\hat{D}_i = D_{i-1,i} + D_i + D_{i,i+1} \qquad i=2,\ldots,M-1$$

$\underline{T}$ is a vector with components $T_i^m - T_i^e$ and $\underline{U}_M$ is the M x M unit matrix.

Note: In the model implemented we allow for the representation of the additional damping torque $D_i'[\dot{\theta}_i^m - (\dot{\theta}_i^m)°]$ on mass i due to the deviation from the synchronous mechanical speed $(\dot{\theta}_i^m)°$. In this case the diagonal element $\hat{D}_i$ of $\underline{D}$ is replaced by $\hat{D}_i' = \hat{D}_i + D_i'$ and the constant term $D_i'(\dot{\theta}_i^m)°$ is added to the i^{th} component of $\underline{T}$.

(ii) Model of the Electrical System

The model of the electrical system consists of three identical, symmetrically placed, lumped armature windings *a, b, c* and four unequal lumped rotor windings *f, g, kd, kq* (see Fig. A2). The *f* coil represents the rotor field winding. The fictitious *g* coil represents the effect of eddy currents which circulate in the solid steel of the rotor. The damper windings are represented by the *kd* and *kq* coils in the direct and quadrature axes, respectively. All windings except the field *f* are short circuited as they are not connected to voltage sources. The three stator windings are assumed to be Y-connected and either grounded through a neutral reactor L_n and resistance R_n or ungrounded.

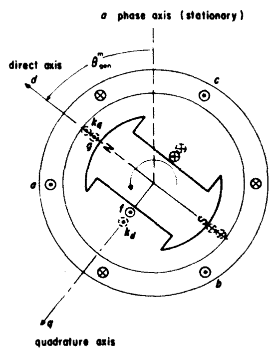

Fig. A2 The three-phase two-pole synchronous machine

Under the usual assumptions that (i) each self and mutual inductance may be expressed as a sum of a constant and a sinusoidal function of the rotor angle $\theta(t)$ and/or $2\theta(t)$, and (ii) saturation is neglected, application of Kirchhoff's voltage law results in a set of linear differential equations with time-varying coefficients. This time-varying system becomes considerably simplified upon applying the following modified form of Park's tranformation

$$\underline{P}(\theta) = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta-120°) & \cos(\theta+120°) \\ -\sin\theta & -\sin(\theta-120°) & -\sin(\theta+120°) \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \tag{A4}$$

$\underline{P}(\theta)$ transforms quantities $\underline{x}_{abc} = [x_a, x_b, x_c]^T$ (where "x" stands for current i, flux linkage λ or voltage v) in the *abc* reference frame to $\underline{x}_{dqo} = [x_d, x_q, x_o]^T$ in the *dqo* reference frame.

The *electrical rotor angle* $\theta(t)$ is related to mechanical rotor angle $\theta^m_{gen}(t)$ shown in Fig. A2 by

$$\theta(t) = n_p\, \theta^m_{gen}(t) \tag{A5}$$

where n_p is the number of pole pairs.

The voltage relations in the *d, q, o* reference frame are

$$v_f = \dot{\lambda}_f + R_f i_f$$
$$v_d = \dot{\lambda}_d - R_a i_d - \dot{\theta}\lambda_q$$
$$0 = \dot{\lambda}_{kd} + R_{kd} i_{kd}$$
$$0 = \dot{\lambda}_g + R_g i_g$$
$$v_q = \dot{\lambda}_q - R_a i_q + \dot{\theta}\lambda_d$$
$$0 = \dot{\lambda}_{kq} + R_{kq} i_{kq}$$
$$v_0 = \dot{\lambda}_0 - R_0 i_0$$

where $R_o = R_a + 3R_n$. The flux linkage equations are

$$\begin{bmatrix} \lambda_f \\ \lambda_d \\ \lambda_{kd} \end{bmatrix} = \begin{bmatrix} L_f & -L_{af} & L_{fkd} \\ L_{af} & -L_d & L_{akd} \\ L_{fkd} & -L_{akd} & L_{kd} \end{bmatrix} \begin{bmatrix} i_f \\ i_d \\ i_{kd} \end{bmatrix} \triangleq \underline{L}_d \begin{bmatrix} i_f \\ i_d \\ i_{kd} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_g \\ \lambda_q \\ \lambda_{kq} \end{bmatrix} = \begin{bmatrix} L_g & -L_{ag} & L_{gkq} \\ L_{ag} & -L_q & L_{akq} \\ L_{gkq} & -L_{akq} & L_{kq} \end{bmatrix} \begin{bmatrix} i_g \\ i_q \\ i_{kq} \end{bmatrix} \triangleq \underline{L}_q \begin{bmatrix} i_g \\ i_q \\ i_{kq} \end{bmatrix}$$

$$\lambda_o = -(L_o + 3L_n)\, i_o \triangleq \hat{L}_o\, i_o$$

In these equations we use the convention that positive voltage and positive current correspond to *generator action*. We derived these equations from the motor equations by reversing the armature currents reference directions to simplify the interface with the EMTP. This formulation consequently requires that all the inductances multiplying the armature currents be negative, as shown, to obtain the correct flux relations.

In matrix notation, with $\underline{\lambda} \triangleq [\lambda_f, \lambda_d, \lambda_{kd}, \lambda_g, \lambda_q, \lambda_{kq}, \lambda_o]^T$, $\underline{i} \triangleq [i_f, i_d, i_{kd}, i_g, i_q, i_{kq}, i_o]^T$, $\underline{v} \triangleq [v_f, v_d, 0, 0, v_q, 0, v_o]^T$, $\underline{R} \triangleq \mathrm{diag}\{R_f, -R_a, R_{kd}, R_g, -R_a, R_{kq}, -R_o\}$,

$\underline{L} \triangleq \mathrm{diag}\{\underline{L}_d, \underline{L}_q, \hat{L}_o\}$ and $\underline{Q} \triangleq \begin{bmatrix} \underline{0}_{3x3} & \underline{Q}_{dq} \\ \underline{Q}_{qd} & \underline{0}_{4x4} \end{bmatrix}$, where

$$\underline{Q}_{dq} = -\underline{Q}^T_{qd} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

the machine equations become

$$\underline{v} = \dot{\underline{\lambda}} + \underline{R}\,\underline{i} + \dot{\theta}\,\underline{Q}\,\underline{\lambda}$$
$$\underline{\lambda} = \underline{L}\,\underline{i}$$

In state-space form these equations become

$$\dot{\underline{i}}(t) = -[\underline{L}^{-1}\underline{R} + \dot{\theta}(t)\,\underline{L}^{-1}\underline{Q}\,\underline{L}]\,\underline{i}(t) + \underline{L}^{-1}\underline{v}(t). \tag{A6}$$

The electromagnetic torque on the generator rotor is given by

$$T^e_{gen} = \frac{n_p}{3}(\lambda_d i_q - \lambda_q i_d) = \frac{n_p}{3}\underline{i}^T\underline{L}^T\underline{Q}^T\underline{i} \tag{A7}$$

For machines with rotating exciters, the electromagnetic torque on the exciter rotor is given by

$$T^e_{exc} = \frac{1}{3}\frac{v_f i_f}{\dot{\theta}^m_{exc}} \tag{A8}$$

The model implemented allows the possibility of maintaining T^e_{exc} constant at its initial (steady-state) value. As the mechanical applied torques are constant it follows that

$$\underline{T}(t) = \underline{T}[\underline{i}(t)]. \tag{A9}$$

Note: The factor 1/3 in eq. (A7) and (A8) arises from the relationship between the electrical and mechanical base quantities. (See Appendix B)

(iii) Interface with the Network

The stator components of $\underline{v}$ in (A6), i.e., v_d, v_q, v_o, are obtained from the Thévenin equivalent of the network viewed from the machine terminals as discussed in Section 2. The synchronous machine-network interface, as stated in eq. (1), provides an expression of the stator voltages in terms of the stator currents. In (1) the Thévenin equivalent is given in terms of the network *a, b, c* variables. We restate (1) in terms of the machine rotor based *d, q, o* variables with $\underline{v}^{dqo} = [v_d, v_q, v_o]^T$ and $\underline{i}^{dqo} = [i_d, i_q, i_o]^T$:

$$\underline{v}^{dqo}(t) = \underline{P}[\theta(t)]\tilde{\underline{Z}}^{Th}(t)\underline{P}^T[\theta(t)]\underline{i}^{dqo}(t) + \underline{P}[\theta(t)]\tilde{\underline{e}}^{Th}(t) \tag{A10}$$

Let us define the 7 x 3 matrix

$$\underline{S} \triangleq [\underline{u}_2 \mid \underline{u}_5 \mid \underline{u}_7]$$

where $\underline{u}_j$ is the j^{th} column of the 7 x 7 unit matrix $\underline{U}_7$. Then

$$\underline{v} = v_f \underline{u}_1 + \underline{S}\ \underline{v}^{dqo} \tag{A11}$$

and

$$\underline{i}^{dqo} = \underline{S}^T \underline{i}. \tag{A12}$$

Upon substituting (A10) and (A12) into (A11) we have

$$\underline{v}(t) = v_f(t)\underline{u}_1 + \underline{Z}[\theta(t)]\underline{i}(t) + \underline{e}[\theta(t)] \tag{A13}$$

where,

$$\underline{Z}[\theta(t)] \triangleq \underline{S}\ \underline{P}[\theta(t)]\underline{\tilde{Z}}^{Th}(t)\underline{P}^T[\theta(t)]\underline{S}^T \tag{A14}$$

$$\underline{e}[\theta(t)] \triangleq \underline{S}\ \underline{P}[\theta(t)]\underline{\tilde{e}}^{Th}(t) \tag{A15}$$

Finally, we combine eqs. (A13) and (A6) to obtain

$$\begin{aligned}\dot{\underline{i}} = -\ (&\underline{L}^{-1}\underline{R} + \dot{\theta}(t)\underline{L}^{-1}\underline{Q}\ \underline{L} - \underline{L}^{-1}\underline{Z}[\theta(t)])\ \underline{i}(t) + \\ &\underline{L}^{-1}(v_f(t)\underline{u}_1 + \underline{e}[\theta(t)])\end{aligned} \tag{A16}$$

Then (A16) gives the equations of motion on the electrical side and (A2) on the mechanical side of a synchronous machine.

APPENDIX B: THE PER UNIT SYSTEM

The per unit system adopted for this work has the property that the form of the equations in dimensional and per unit form is the same. It is based largely upon the original work in [9] and the more recent contribution in [10]. A summary of the base quantities is given.

(i) Electrical System Base Quantities

(a) The stator base quantities, denoted by the subscript SB, are

$S_{SB} \triangleq$ rated r.m.s. volt-amps per phase, va

$V_{SB} \triangleq$ rated line-neutral r.m.s. voltage, v

$\omega_{SB} \triangleq$ rated synchronous speed of machine, rad./s.

$t_{SB} \triangleq 1/\omega_{SB}$, s.

The remaining stator base quantities I_{SB}, L_{SB}, λ_{SB} and R_{SB} are defined in terms of these four quantities using the well-known physical relationships.

(b) The rotor base quantities, denoted by the subscript RB, are chosen to be

$S_{RB} \triangleq S_{SB}$

$\omega_{RB} \triangleq \omega_{SB}$ or $t_{RB} \triangleq t_{SB}$

$I_{RB} \triangleq \frac{1}{\sqrt{3}} L_{af}$ (per unit) I_{FI}, a

For the electromagnetically coupled rotor and stator circuits, the first two relations ensure that the per unit mutual inductances are reciprocal. I_{FI} is that field current that produces rated voltage on the air gap line. I_{RB} is defined so that the mutual flux in the d-axis it produces is equal to that produced by I_{SB} flowing in the d-axis stator winding. The remaining base quantities are again obtained from their defining physical relations.

(ii) Mechanical System Base Quantities

The mechanical base quantities, denoted by subscript MB, are

$t_{MB} \triangleq t_{SB}$ or $\omega_{MB} \triangleq \omega_{SB}$

$S_{MB} \triangleq S_{SB}$

$T_{MB} \triangleq 3\, S_{MB}/\omega_{MB} \triangleq$ base torque, Newton -m.

$\theta_{MB} \triangleq \omega_{MB}\ t_{MB} = 1$ rad., base *mechanical* angle

$J_{MB} \triangleq T_{MB}/\omega_{MB}^2$ = base moment of inertia, kg. m^2

$K_{MB} \triangleq T_{MB}/\theta_{MB}$ = base spring constant, Newton -m

$D_{MB} \triangleq T_{MB}/\omega_{MB}$ = base damping coefficient, Newton -m-s.

Discussion

B. L. Agrawal and **R. G. Farmer** (Arizona Public Service Co., Phoenix, Ariz.): The authors are to be congratulated for a concise and compact presentation of the EMPT program. As mentioned by the authors, the program was made available to several utilities on an experimental basis. Arizona Public Service was one of these utilities. The program has been used extensively for transient torque studies related to SSR, including many correlation studies with other commercially available programs. We have concluded that the program is efficient and can be applied with relative ease considering the program complexity and data requirements. The program has considerable flexibility for data input and we have been able to accurately represent a 4-stage mutually coupled series filter connected to the neutral end of the high voltage winding of a generator stepup transformer. The availability of this program to the utilities is very welcome. We would offer a word of caution and state that 60 to 90 man-days may be required to become proficient in the data preparation and application techniques of this program.

This program is particularly valuable when used in conjunction with a scanning type program which can identify, at a small cost, system conditions which have the potential for high transient torques. The EMTP program, of course, was designed for transient analysis but since the bilateral coupling between mechanical and electrical systems is included, "steadystate" stability can be investigated. We have experienced limited success in this application for two major reasons. First, the mechanical model is a spring-mass model which requires mechanical damping input as dashpot damping for each mass and material damping for each shaft. At the present, mechanical damping is measured and estimated best as modal damping. It is difficult, at best, to convert modal damping to the damping modeled in the program. Secondly, a long time simulation is required to determine the decay or growth rates of the torque or rotor motion such that the degree of stability or instability can be determined.

In the footnote on page 1 the authors imply that the "unilateral coupling" models are much less accurate especially for machine network systems with relatively low overall damping. Would the authors expand on this and give an example. We have performed numerous correlation studies for EMTP and unilateral program on existing and planned power systems and have found reasonable correlation.

The program provides for two forms of input for the synchronous machine electrical data. The first is the synchronous, transient, and subtransient data normally supplied by manufacturers for stability and short circuit studies. The second is in the form of direct and quadrature rotor circuits. To accurately transform the first (stability data) to rotor circuit data requires the solution of 28 simultaneous equations [1]. The technique used by the program is a shortcut method which can lead to inaccuracies. Therefore, for the most accurate representation, we would recommend that the rotor circuit models be obtained from the machine manufacturer, if possible. Have the authors considered a more exacting method or data conversion?

There seems to be a typographical error in the second paragraph under Application Example. Should the timing for fault removal and line clearing be .060 and .066 seconds respectively.

We want to thank the authors for their efforts in developing this program and preparing this paper which helps in program application.

REFERENCE

[1] IEEE Subsynchronous Resonance Task Force Report, "First benchmark model for computer simulation of subsynchronous resonance." IEEE paper F 77 102-7, presented at the Power Engineering Society Winter Meeting, January 1977, New York.

Manuscript received July 11, 1977.

Hiroshi Suzuki and **Katsuhiko Uemura** (Mitsubishi Electric Corp., Amagasaki, Hyogo, Japan): The authors proposed contributable tool to analyze the electromagnetic transients of machine and network of power systems. The program EMTP is excellent in the point of its general purpose. We have just a few comments on certain particulars. To analyze the electromagnetic transient problem, for example subsynchronous resonance problem, torsional damping values of turbine generator shafts are important parameters. Generally modal damping data are measured from oscillographs of torsional angles at shop tests. In this program, however, the authors chose self and mutual damping instead of modal damping for input data of damping coefficients. And the authors assumed very small values (i.e. 0.04 and 0.024 p.u.) for them. Do the authors have a method to estimate the self and mutual damping data?

Manuscript received July 19, 1977.

C. E. J. Bowler, D. N. Ewart (General Electric Company, Schenectady, NY) and **K. Carlsen** (ERDA, Washington, DC): This paper is important to the industry in view of its potential widespread usage, by utilities, for the analysis of phenomena where both network and turbine generator dynamics are involved. From the standpoint of the turbine generator representation, this program is likely to be used in the computation of duty cycles imposed by a variety of system operations and disturbances involving both compensated and uncompensated ac and dc transmission systems. Prompted by such usage, we feel the following comments are appropriate.

First we would like to clarify comments made about the MANTRAP (Machine and Network Transients Program) program developed by General Electric using the Bonneville Power Administration EMTP and referred to by the authors in their reference [3].

The MANTRAP program developed in 1972 has been in commercial use since 1973. The solution and interface scheme utilized in MANTRAP is quite different from the authors'. It does use a 1st order explicit integration scheme for the turbine generator electromechanical solution, but due to the dynamics typically encountered in turbine generators and associated equipment, the time step may be significantly larger than that dictated by the network dynamics. This relationship is made possible in MANTRAP by the method of providing the machine network interface. The interface between phase and rotor reference frames is performed at the voltage behind the apparent infinite frequency reactance of the generator, avoiding an artificially short stator circuit time constant and therefore eliminating the need for separation of the machine from the network with a distributed parameter line.

The need in the authors' program to use a short distributed parameter line for the multimachine plant and on some occasions for the single unit plant, such as in transformer saturation, switched resistors and reactors and breaker operations, and the need to recalculate a new Thevenin equivalent at each switching, seems counter to the requirements of efficiency and accuracy. First it fixes the integration step size and second, may preclude the required coupled coil transformer representation with saturation.

The quotation by the authors of a computationally efficiency interface algorithm appears inappropriate considering the need to iterate the generator velocity between time steps. The question naturally arises as to the element that controls the number of such iterations. If it is the magnitude of generator velocity excursion, the resulting computer time may be expensive in the analysis of weak transmission systems.

Concerning the turbine generator representation, we have the following comments and cautions. The synchronous machine model as presented does not represent a large degree of generality as it does not allow for saturation either in the mutual coupling or the leakage paths. The present restrictions to two rotor coils per axis should be removed for greater flexibility. While we agree that better data is not generally available to the industry through a lack of experiment, such data we believe is important in the analysis, at least of subsynchronous resonance corrective devices.

The assumption of constant mechanical torque may pose limitations, such representation requires the modeling of separate dashpots to ground at each turbine mass as the authors have indicated. The analysis of load rejection and also large power swings would require change in the damping coefficients with time. The assumption of constant power would remove this necessity, and at such time that governor models be impleted would require only minor program changes.

We have the following questions: What computer resources (memory requirements and time) expense is associated with synchronous machine representation compared with the network? What field experience correlation do the authors quote, and would they make the comparison in their closure? How short is the distributed parameter line typically linking the machine to the network in the case where transformers modeling saturation is required?

What is the availability of the program and what support might the typical user expect in his use of the program?

Manuscript received August 1, 1977.

V. Brandwajn and **H. W. Dommel** (University of British Columbia, Vancouver, B.C., Canada): The authors are to be congratulated for presenting an excellent paper on the simulation of synchronous machine dynamics with an electromagnetic transients program. The algorithm developed by the authors has been successfully applied to many practical problems, and has already proved its usefulness in a number of electric utility companies. The first-named discusser has had the opportunity to become very familiar with the details of the program in connection with contractual work done for the Bonneville Power Administration, and therefore appreciates the excellent work done by the authors. The following remarks are partly based on this detailed knowledge, and are intended to compliment the authors' ideas.

Manuscript received August 8, 1977.

The determination of initial conditions from positive sequence values only, as suggested by the authors, is reasonable for problems with small unbalances in terminal voltages and currents. This approach has been used by the discussers as well [4]. Sometimes, cases do start from unbalanced ac steady-state conditions, e.g., if the system contains untransposed lines. It is very easy to modify the initialization to come closer to the initial unbalanced ac steady state in the following way:

1. Find the initial conditions, including the rotor position, from the positive sequence values as suggested by the authors (the discussers believe that the rotor position is not influenced by negative and zero sequence values).
2. The primary effect of the negative sequence currents is a magnetic field which rotates at synchronous speed in opposite direction to the rotation of the rotor. This field induces currents of double power frequency (120 Hz) in the rotor circuits. Rewrite the rotor circuit equations as steady-state phasor equations with ω = twice power frequency, and solve them for the rotor current phasors for given negative sequence currents (which are also phasors at twice power frequency in d,q,0-coordinates). Superimpose the instantaneous values of all currents on the values found in step 1.
3. Calculate the electromagnetic torque created by positive and negative sequence currents.
4. Superimpose the instantaneous values of zero sequence currents and voltages.

This initialization is still not completely accurate because it ignores the fact that the double frequency currents in the rotor circuits induce 3rd, 5th, ... harmonics in the stator, which in turn induce 4th, 6th, ... harmonics in the rotor. It would be extremely difficult, however, to modify the ac steady-state network calculation to include the harmonics. It is much easier to let the transients solution build up these harmonics effects, and to hope that these secondary effects are built up to steady-state conditions within a few cycles. Fig. 1 shows the stator current of phase C and the electromagnetic torque of a generator, which is connected through a step-up transformer and a transmission line to an infinite bus, when a permanent single-line-to-ground fault is applied to the high side of the delta/wye-connected step-up transformer. The data for this case is taken from [A]. This is a severe test for unbalanced conditions. The curves on the left were obtained with initialization from positive sequence values only, and the curves on the right with the modified initialization described above. The incorrect fast ripples with positive sequence initialization clearly disappear with the modified initialization, but now a dc-offset appears in the current curve which the discussers did not expect and which they cannot fully explain yet (influence of higher harmonics or programming bug?). Steady-state conditions are obtained in approximately 3 to 4 cycles in the curves on the right, however. The initial values of the electromagnetic torque differ by about 60% ($1.82 \cdot 10^5$ N·m for curve at left, $3.11 \cdot 10^5$ N·m for curve at right). It is also worth mentioning that the physically unacceptable jump in the current of phase C disappears for the modified initialization.

The authors calculate the state transition matrix for the mechanical system through a forward summation of the Taylor series expansion. If the parameters of this subsystem are such that it becomes more stiff, then this Taylor series expansion may cause problems of numerical stability, since the 4-th order series expansion of the state transition matrix is identical to a fourth order explicit Runge-Kutta method [B]. It may then be safer to use a Crank-Nicholson rational approximation to the state transition matrix [C]. (The expansion to 3-rd order terms in [C] can be extended to any order.) Such a rational approximation is not only more stable, but approximately twice as fast. To check this problem of numerical stability, relatively large damping constants were added to the benchmark model [7]. Then the Taylor series expansion did not converge, but began to diverge after 11 terms, whereas the rational approximation gave sufficiently accurate results with 5 terms.

The authors modified their program to handle cross-compound machines as well. It is easy to modify the authors' algorithm in such a way that any number of generators can be connected in parallel to the same network bus by simply representing the generators as current sources instead of voltage sources. The network is represented by its three-phase Thevenin equivalent circuit:

$$\underline{v}^{abc}(t) = \underline{\tilde{z}}^{Th}(t)\ \underline{i}^{abc}(t) + \underline{\tilde{e}}^{Th}(t) \tag{1}$$

and each of the n generators as a current source after reduction to stator quantities:

$$\underline{i}_i^{abc}(t) = \underline{Y}_i\ \underline{v}^{abc}(t) + \underline{I}(t) \tag{2}$$

where i = 1, ..., n

and

$$\underline{i}^{abc}(t) = \sum_{i=1}^{n} \underline{i}_i^{abc}(t) \tag{3}$$

Substitution of (2) and (3) into (1) yields the following results:

$$[\underline{U}-\underline{\tilde{z}}^{Th}(t)\cdot(\sum_{i=1}^{n}\underline{Y}_i)]\underline{v}^{abc}(t)=\underline{\tilde{e}}^{Th}(t)+\underline{\tilde{z}}^{Th}(t)\cdot\sum_{i=1}^{n}\underline{I}(t) \quad (4)$$

In this way, it should be possible to handle any number of generators connected to the same bus efficiently.

The above remarks do not, in any way, reduce the importance and value of the algorithm developed by the authors, but simply suggest possible improvements.

Note: By the time of proofreading, the cause of the dc offset in the current and of the incorrect first peak in the torque has been traced to incorrect voltage initialization (symmetrical voltages were used at the terminal rather than behind subtransient reactances).

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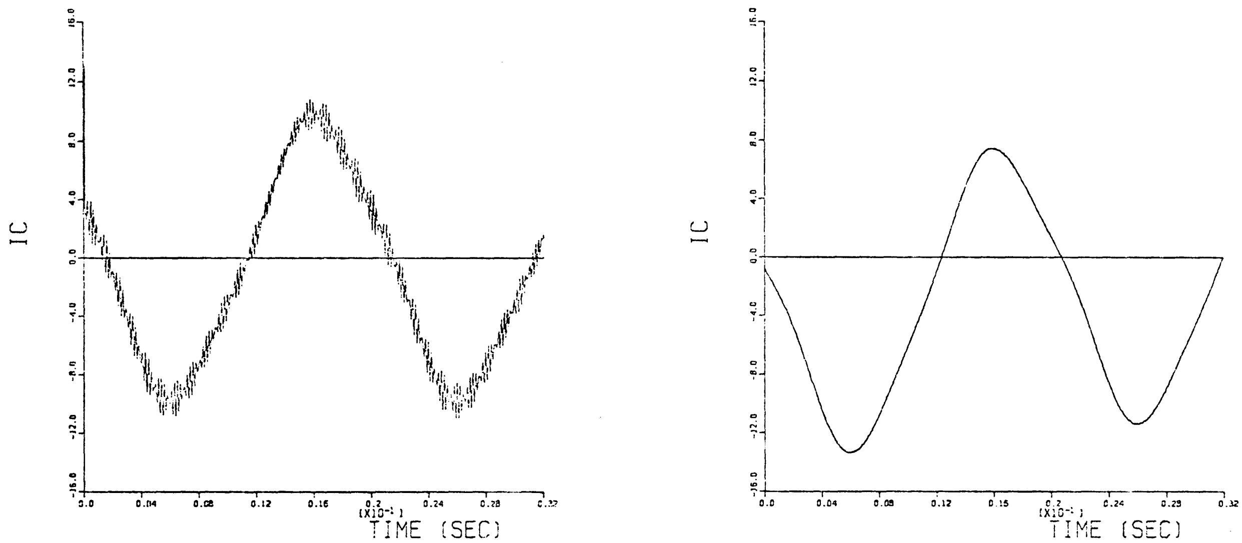


Fig. 1.a. Comparison of the simulated generator current in phase C.

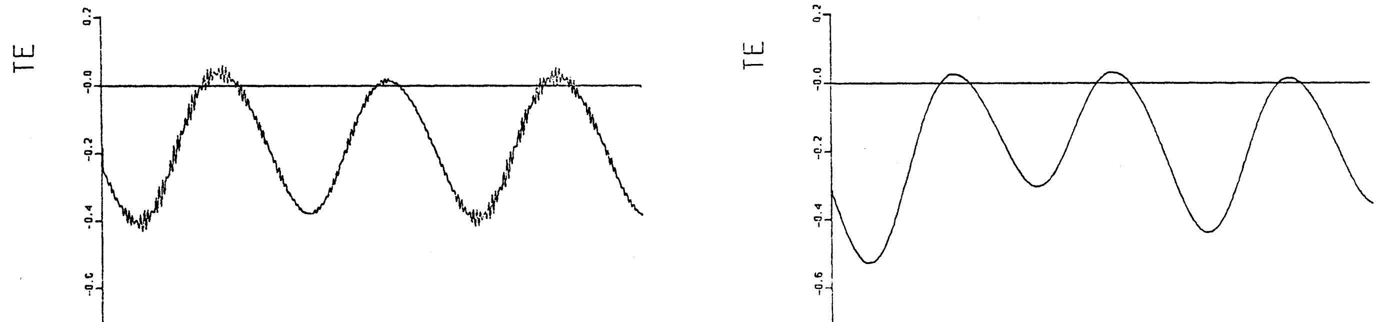


Fig. 1.b. Comparison of the simulated electromagnetic torque of the generator.

P. C. Krause (School of Electrical Engineering, Purdue Unversity, West Lafayette, IN): The discussor is presently studying shaft torsionals in the system shown in Fig. A on the hybrid computer at Purdue. This system consists of three machines, an infinite bus and six transmission lines. The prime mover of each machine has a high pressure, intermediate pressure, and two low pressure turbines. All three machines are represented in detail using Park's equations with two damper windings in the quadrature axis. Each of the three prime movers is represented as a five mass system. The excitation of each machine is held constant during each study. Each machine is equipped with a step up transformer. The network is represented as equivalent three phase series R-L circuits with provisions to fault and to switch the faulted line (Fig. A). In a typical fast closing study, for example, a three phase fault is applied at the point indicated in Fig. A and the faulted line is switched out after three cycles. After 20 to 35 cycles, the line is reclosed into the same fault, whereupon, the line is switched out permanently after three cycles.

The simulation of this system requires three EAI analog computers (3 consoles) and it took two mandays and about four console hours to patch and to check out. Computation is done at 20 times slower than real time for convenience purposes even though computation could be done as fast as real time or faster. Therefore, to make a fast reclosing study spanning 60 cycles or 1 second of system time requires 20 seconds of computer time. It then takes approximately one or two minutes to change the time of reclosing or make minor parameter changes and reestablish initial conditions, ready for another study. Hence, one person could easily make 20 to 25 such studies in an hour. Each console hour costs $15; therefore, these 20 to 25 studies would cost a total of about $50. Would the authors please give a similar manhour and computer time requirement to perform this study using their digital program?

Manuscript received August 12, 1977.

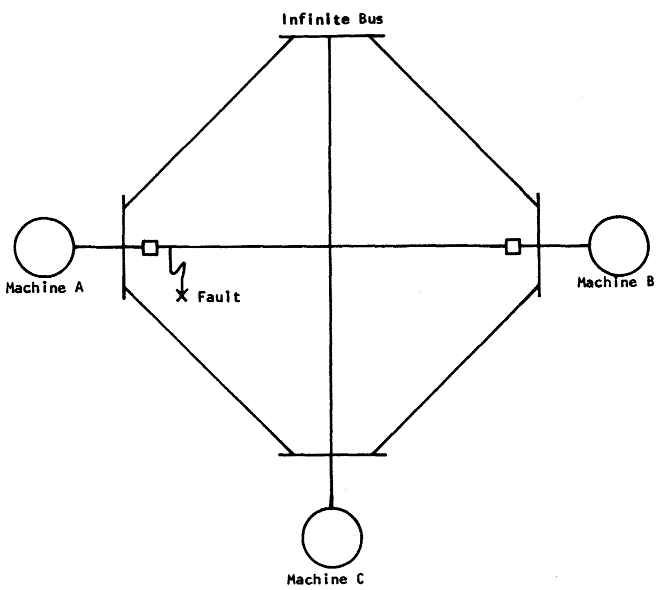


Fig. A. System Studied.

Adam Semlyen (University of Toronto, Toronto, Ontario, Canada): The authors are to be commended for their pioneering work in synchronous machine modelling for long range electromagnetic transients calculation, using the general purpose Electromagnetic Transients Program (EMTP). Clearly, rotor and shaft dynamics are important in reproducing field results such as sub-synchronous resonance and dynamic overvoltages. In particular, selection of proper numerical values for the elements of the matrix $\underline{D}$, representing damping, is not only important but apparently difficult.

Off-diagonal damping terms seem to be insufficiently defined physically, and the authors' comments on this matter would be welcome. Is it justified to equate these coefficients to zero? The diagonal terms include, in the formulation of the paper, a term representing the additional torque due to deviation from the synchronous mechanical speed. This seems to be well justified, considering that the mechanical torque results due to the existence of a relative speed of the steam with respect to the turbine blades. Including friction, the torque in p.u. is $T_i^m \cong 2 - \dot{\theta}_i^m$, where the angular velocity $\dot{\theta}_i^m$, is also in p.u. This equation yields for the p.u. power $P_i^m \cong \dot{\theta}_i^m (2 - \dot{\theta}_i^m)$ i.e. a maximum (constant) value for $\dot{\theta}_i^m \cong 1$ p.u., as expected. The slope of the torque characteristic is clearly -1 p.u. which produces well defined and rather large damping coefficients. It would be appreciated if the authors would indicate their experience regarding the above approach to taking into account the damping in the individual turbine units.

Manuscript received June 6, 1977.

G. Gross and **M. C. Hall**: We appreciate very much the interest of the discussers in this work. We thank them for their valuable comments and welcome the opportunity provided to clarify certain points in our paper.

Several discussions have dealt with questions concerned with various aspects of damping in the mechanical system. The *linear multimass-spring-dashpot system* model used in this work for the representation of the mechanical system has had wide use in the analysis of shaft torsional dynamics. In our formulation, the *self-damping* torque on each lumped mass consists of a term that is proportional to the angular velocity of the mass and another term that is proportional to the deviation of this velocity from the *mechanical* synchronous speed. The first term is used for modeling friction and windage effects, and the second term is for representing the damping internal to the steam turbine. This second term expresses the torque on the turbine blades that is proportional to the difference between the steam and blade velocities. Analysis of the limited test data available indicates that this internal turbine damping torque varies with the turbine load, with very little internal damping present under no-load conditions.

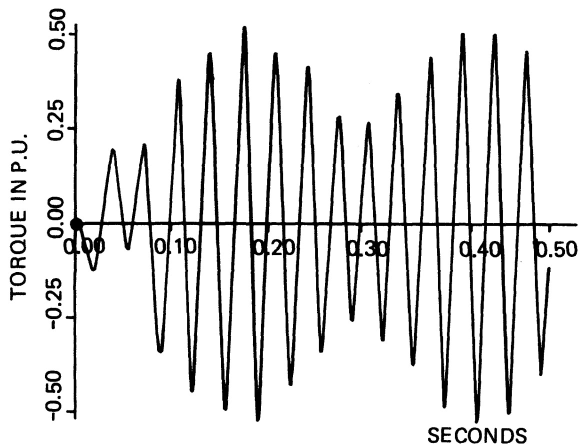
The off-diagonal elements in the $\underline{D}$ matrix in the equations of motion describing the torsional dynamics referred to in Prof. Semlyen's discussion are the *mutual* damping terms. These represent the damping effect associated with the energy dissipated from the *cyclic twisting* of the shaft material. Such damping is present when two adjacent masses on a shaft have unequal angular velocities and, in effect, is a *viscous* damping torque which is proportional to the difference in the velocities of the two masses.

The elements of the $\underline{D}$ matrix cannot, in general, be obtained from direct measurement; rather, damping of torsional oscillations is measured by observing the modal behavior of the torsional system. In this way, the rate of decay of the shaft oscillations when the shaft is oscillating at one of its natural frequencies or modes, i.e., the modal damping, can be measured for each mode, as was pointed out by Messrs. Suzuki and Uemura. We agree with Messrs. Agrawal and Farmer that it is difficult, in general, to convert modal damping quantities into the nonzero self- and mutual-damping terms in $\underline{D}$. We have had reasonable success using the following approach. We first obtain the *no-load* modal damping from test data. Under the assumption of zero self damping for each mass ($D_{ii} = 0$), we iteratively compute a set of mutual-damping coefficients. We start with an initial guess for the values of the D_{ij} and compute the eigenvalues of the mechanical system with this $\underline{D}$. If the real parts of the eigenvalues do not coincide (sufficiently closely) with the modal-damping quantities, we repeat this procedure with a new set of D_{ij} until a good match is obtained. The self-damping coefficients can then be added to represent the damping of the masses under load conditions. Again, eigenvalue analysis can be used to compare with the test results under load conditions to determine the proper magnitude of the self-damping coefficients. While we have found this procedure to be adequate for our purposes, much work concerning the development of better data for the elements of $\underline{D}$ remains to be done.

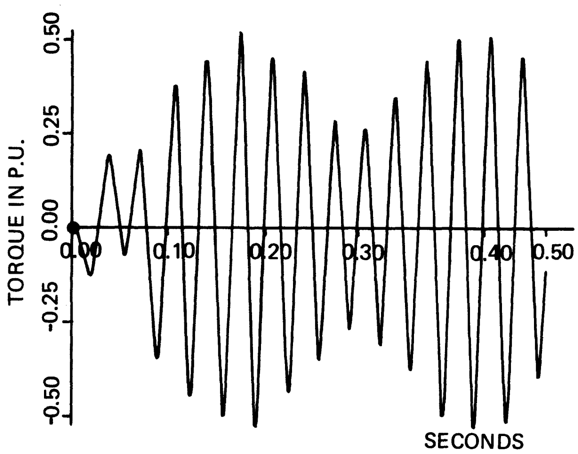
It is interesting to note that simulation results show that the peak shaft torques in the first few cycles following a disturbance are relatively insensitive to the values of the damping coefficients. For example, for the test system of the application example, a typical shaft torque response to three significantly different values of mutual damping coefficients D_{ij} is illustrated in Fig. A. Note that there is virtually no difference between the zero damping case (i) and no-load damping case (ii). A fifty-fold increase in the values of the D_{ij} in case (iii) results in, approximately, only a 15% decrease in the peak torque of the first few cycles. Thus, if the *primary objective* of an analysis is to study the transient peak torques that are developed in the first few cycles following the onset of a disturbance, the lack of accuracy in the damping coefficients will have relatively minor effects.

Messrs. Agrawal and Farmer indicated that they have found reasonable correlation in simulation results using the *bilaterally-coupled model* in this work and those using a simpler *unilaterally-coupled model.* Basically, by a unilateral coupling model, we mean one in which the feedback of the mechanical part of the synchronous machine into the electrical system is not considered (see, for example, [A]). While our experience in studying comparisons of simulation using the two model types is rather limited, we are of the opinion that the *unilateral coup-*

Manuscript received December 19, 1977.



(i) ZERO DAMPING CASE
ALL $D_{ii} = 0$; ALL $D_{ij} = 0$



(ii) NO-LOAD DAMPING CASE
ALL $D_{ii} = 0$; ALL D_{ij} = VALUES CORRESPONDING TO NO-LOAD TESTS AND GIVEN IN FIG. 3

-0.50 (iii) HEAVY DAMPING CASE
ALL $D_{ii} = 0$; ALL D_{ij} = 50 TIMES THE NO-LOAD VALUES [CASE (ii)]

FIG. A THE TORQUE ON THE GENERATOR-EXCITER SHAFT SEGMENT OF THE HP UNIT FOR THREE VALUES OF MECHANICAL SHAFT DAMPING. THE SYSTEM CONFIGURATION IS THAT GIVEN IN FIG. 3. THE FAULT IS ON THE ELDORADO-MOENKOPI LINE AND THE CLEARING TIME IS .092 SEC.

ling model is a useful tool and provides reasonably accurate results in certain cases. As is the case with models in general, however, its limitations must be recognized and understood to enable one to properly apply it. The model with unilateral coupling cannot reproduce the unstable condition characterized by growing oscillations due to the *torsional interaction effect,* nor can it reproduce multimachine interactions. If the time constants of the electromagnetic torques are small compared with the time constants associated with torsional oscillations, then the accuracy of the results obtained using a model with unilateral coupling should be acceptable. As the time constants associated with the electromagnetic torques increase to become of the same order of magnitude as those of the mechanical system, the accuracy of the simulation decreases. The accuracy of simulation results obtained using the unilaterally-coupled model become very poor as the overall machine-network damping approaches zero (corresponding to very long time constants). Consequently, some care is required in the use of the model with unilateral coupling; it should be used with caution for systems with low overall damping, or in cases where the system damping is not known a priori.

Another point raised by Messrs. Agrawal and Farmer concerns the machine model data conversion. We chose the standard approximation technique used in transient stability work for calculating the machine model parameters from the machine constants provided by equipment manufacturers (see [B], Chapter 6). The modest inaccuracies introduced by this simple approach were considered acceptable in the initial stages of program development with a more accurate determination of the parameters remaining a future objective. In fact, the implementation of a technique for fitting machine parameters of the model from available data using nonlinear optimization is currently underway at BPA as an extension of the work reported here.

Regarding the comment by Messrs. Agrawal and Farmer concerning the application of the EMTP described in this paper for investigating *steady-state stability*, we believe that a more suitable tool would be an eigenvalue analysis of the linearized equations of motion of a detailed system model including torsional and network dynamics.

Messrs. Bowler, Ewart, and Carlsen raised a number of interesting points. Some of the comments dealing with the need for a distributed parameter line to connect the machine with various network elements, require clarification. The only present restriction is in linking a machine to another "true" nonlinear and/or time-varying element. This restriction does not apply to circuit breakers which are modeled as switches, and to saturated transformers which are treated as pseudo-nonlinear elements. The latter are represented as piecewise linear elements using a standard EMTP feature. The EMTP requires separation of machines connected at *different* buses by distributed parameter lines. The program, however, provides the capability of representing a cross-compound unit or any two machines connected at the same bus without this restriction.

Messrs. Bowler, Ewart, and Carlsen also raised a number of questions concerning the solution algorithm. An important aspect of the computational technique is that it uses a scheme with the same order and the same step size for solving both the machine d.e.s. and the network part. This ensures that there cannot arise any *numerical interface* problems between the two parts as could in the case in which the machine and network parts are solved by methods of different order and step size. The discussers expressed concern about the effect that the $\underline{i}$-θ^m iterative loop may have on the computational efficiency of the solution scheme. Extensive tests of the program on both weak and stiff transmission systems indicate that no more than two additional iterations were required. A factor that has an effect in determining the number of such iterations required is the relative size of the step size Δt to that of the time constant associated with the mechanical motion of the generator mass.

These discussers expressed concern on the computational burden associated with calculating a Thévenin equivalent. As long as the network structure remains fixed and the stepsize Δt is unchanged, $\tilde{\underline{Z}}^{Th}$ is constant. At each switching point $\tilde{\underline{Z}}^{Th}$ must be recalculated. The only work associated with the computation of a new $\tilde{\underline{Z}}^{Th}$ is *two* additional back substitutions. As a typical study usually involves less than ten switching points, the additional computational expense is not significant.

Another topic discussed by Bowler, Ewart, and Carlsen concerned the synchronous machine model. We can introduce the magnetic saturation effects into this model when the latter are represented as functions of the rotor flux linkages or the rotor currents. In the presence of such effects, the value of the $\underline{L}$ matrix may change in each iteration; however, no basic change in the computational structure is introduced. Also, it is possible to include in the formulation presented as many rotor circuits as the available data permits again without affecting the structure of the solution algorithm. If data for more than two rotor coils per axis were available, the effects of "dynamic saliency" [C] could be represented. Furthermore, the restriction of constant mechanical torque used in the paper has been removed in the latest version of the EMTP produced by BPA. The program now allows the specification of arbitrary governor systems by means of the TACS feature.

In response to the questions concerning the computer resource requirements, we have the following comments. Simulation out to 2 sec. of the sample system in Fig. 3 using a stepsize of .0002 sec. requires approximately 600 CPU sec. when the cross-compound unit is represented in detail and about 400 CPU sec. when it is modeled as a constant sinusoidal voltage source. We have not actually run the simple test system in Prof. Krause's discussion; however, we hope that the timings given above can provide a basis for carrying out comparisons with an analog computer. The memory requirements vary widely depending on the size at which the program arrays are dimensioned and the overlay structure used, if any. For example, a version of the EMTP with ability to simulate up to three cross-compound units is installed on the PGandE TSO system and needs a memory of 380 K (bytes). A version with up to ten cross-compound units requires about 600 K (bytes) using an overlay structure. At SCE a virtual storage machine is available and, consequently, no overlay in the program is used.

The discussion by Bowler, Ewart, and Carlsen also touches on the question of correlation of simulation results with field experience. Our major difficulty in making comparisons of simulation runs with historical events is in defining the actual sequence of disturbances and initial conditions. We have, however, obtained good qualitative comparison between measurements obtained from oscillographs following system disturbances and simulation. For example, the simulation results of the Mohave SSR incidents closely match the frequencies observed on the oscillographs from one of the incidents. In addition as mentioned in the paper and in some of the discussions, the model and program have been tested with success in a wide range of applications by a large number of utilities including most recently BPA. This testing has included studies comparing the performance of the EMTP with both hybrid and digital programs of similar capability.

We concur with Messrs. Brandwajn and Dommel that there are better means available to compute the state transition matrix than by truncated series expansion. We deemed this scheme to be acceptable in the initial stage of program development with an improved evaluation scheme remaining a future objective. Whether to accomplish this using the Crank-Nicholson-Davison technique referenced by the discussers, or some other Padé approximant technique, we cannot state at this time. In fact, even a technique based on the determination of the eigensystem may turn out to be suitable for the relatively low-dimensional mechanical system. Our experience with the truncated series expansion has been very good so far with usually no more than six or seven terms used in the expansion.

The question of program support has been raised. The EMTP continues to be well-supported and documented by BPA. The latter organization is supporting a wide range of activities towards the improvement and expansion of the program.

We wish to bring to the attention of the readers a number of extensions to the work described here that have been implemented by BPA since the writing of this paper. These include the modifications mentioned above and the initialization routine changes described in the Brandwajn-Dommel discussion. Additional modifications include features that permit the use of delta-connected armature windings and the omission of either the q-axis damper winding, or the d-axis damper winding, or both.

The typographical errors pointed out by the discussers have been corrected in the final version of the paper.

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